INNOVATION AND EQUILIBRIUM?

Martin Shubik

September 2008

INNOVATION AND EQUILIBRIUM?

Martin Shubik

September 2008

Abstract

A discussion is given of the problems involved in the formal modeling of the innovation process. The link between innovation and finance is stressed. The nature of how the circular flow of funds is broken and the role of finance in evaluation and control is discussed.

Keywords: Innovation, Invention, Circular flow, Finance

JEL Classifications: D5, G2, O3

1 Introduction

This essay is written in fond memory of Hyman Minsky whose writings on the possibilities for instability in a modern economy with many financial institutions have had considerable influence on the way economists view the strengths and weaknesses of our increasingly finance oriented economies (see Minsky 1986).

Hy had a considerable skepticism about the uses and abuses of mathematical economic theory, especially general equilibrium theory's emphasis on equilibrium. His skepticism has been shared by many macroeconomists of both left and right persuasion.

Starting as far back as Schumpeter in the early part of the twentieth century a provocative picture of the capitalist innovating economy was painted. Although the proponents of Schumpeter's approach have grown considerably over the last twenty to thirty years, primarily in Europe (see, for example, Nelson and Winter 1982 and Dosi et al. 1988), so, since the late 1950s have the general equilibrium theorists. The thesis presented here is that the reconciliation of equilibrium microeconomics and disequilibrium financial economics and macroeconomics is both feasible and depends far less on correcting errors or misperceptions in either than on understanding the highly different questions being answered by each; and identifying the gaps in the models utilized by both that need to be filled if a reconciliation is to take place.

Keynes described and dealt with disequilibrium. Jim Tobin regarded the overall macroeconomic system as having a general equilibrium structure but with many changing parameters that needed to be re-estimated frequently. Lejonhufvud, Davidson and many others have stressed money, financial control and disequilibrium as the characteristics of the macroeconomic system. In all instances the stress is on process, in contrast with the general equilibrium stress on efficient static equilibrium.

2 Innovation, Money, Equilibrium or Disequilibrium?

Much of economic activity involves the use of money and credit. The populist essays from the right may sing the virtues of the capitalist structure in promoting economic activity in general and innovation in particular. The essays on the left suggest the evils of the system and the accrual of unwarranted power by the financiers.

The analysis of general equilibrium is carried out far more precisely and mathematically than the literature on innovation. There are some simulations, but the predominant approach to innovation is via the essay. In spite of its elegance and abstraction, as was noted by Koopmans (1977), general equilibrium theory is pre-institutional. Because the economic world is highly complex and multivariate, radical simplification is called for in the mathematization of the models studied. Thus, considerable emphasis has been placed on equilibrium conditions as the mathematics of processes in disequilibrium is more or less intractable except in special cases. Even in the low dimensional models of Lucas (1980), Shubik (1972) and Karatzas, Shubik and Sudderth (1992) convergence to equilibrium from positions out of equilibrium has not been proved.

Although originally written nearly a hundred years ago, Schumpeter's work on *The Theory of Economic Development* (1934, 1911) provided an insightful description (in essay form) of a plausible dynamic process involving the interaction of the financial and physical processes of the economy intermixed with the socio-psychological factors of optimism and pessimism. No formal mathematical model was developed.

There is a growing literature involving the simulation of the dynamics of competition as characterized by the early work of the Carnegie Institute and the work of Nelson and Winter (1982) and others subsequently. This essay is directed to considering the analysis among the mechanisms for the creation of money and credit, the selection of innovation and the formation of price in an extremely simplified economy. In doing so a link is formed between equilibrium analysis and the writings on the essentially disequilibrium dynamics of innovation. It also becomes possible with this approach to begin to examine precisely what is meant by the power lack of power to control innovation, of those who control the money and credit supply in the financial system.

2.1 On simple well-defined models

The approach adopted here is to construct simple but detailed models specifying every feasible move and all information conditions. Even with the microeconomic detail of economic reality they represent a gross oversimplification. Our approach is to try to construct the simplest mechanism for which the phenomena of relevance appear. As they are well defined models they should either manifest the properties ascribed to "more realistic models" or otherwise they should serve to indicate why some phenomena do not appear until a higher level of complexity is attained.

The stress is on process analysis. The economy is viewed as a game of strategy and an attempt is made to fully define a game in extensive form. This reductionist approach has

many benefits, but as is noted below in attempting to describe innovation, at some point it may have its limitations.

2.2 The Playable Game Test

> If after ten minutes at the Poker table you do not know who is the patsy, you are the patsy.

Although in the past twenty-five years there has been an explosion of the uses of game theory in economics, even at the middlebrow level, nevertheless much of game theory writing is at a high level of abstraction with little attention paid to context. An important part of the approach here is based on the belief that many economic institutions and instruments are designed to be operated more or less efficiently by average, non-specialist individuals. There are some institutions and instruments such as the markets for sophisticated derivatives that require professionals, where PhDs in mathematics, physics or mathematical economics or finance may help. In these markets, the amateurs may be slaughtered. But, for many institutions if one wishes to gain insight into rheir operation it should be possible to build a playable experimental game whose rules can be explained with little difficulty to a student or a non-sophisticated small businessman.

3 Economic Behavior and Behavioral Types

Much of work in mathematical economics and in game theory has been based explicitly or implicitly on an abstract *homo economicns* or von Neumann man. This individual has perfect recall and an ability to compute everything. I suggest that for the development of economic dynamics it is probably worth while to recognize around eight different behavioral types. They are the:

- random player with the state space unknown;
- random player with the state space known;
- optimal response player with global scope;
- optimal response player with local scope;
- non-specialist human;
- specialist human;
- expert specialist human;
- von Neumann player

The lower and upper bounds on intelligence are usually the easiest to study and serve as useful benchmarks when studying behavior in experimental games. Market such as a one commodity double auction market provides an example of institutional design where the random player with state space known does approximately as well as the von Neumann player (see Huber, Shubik and Sunder, 2007).

The first player noted is the random player with the state space unknown. This is noted to point out that statement that a player chooses randomly, to be made precise requires the

assumption that the player knows the domain over which she is randomizing. If the domain is not specified an individual must provide some subjective closure to take care of the uncertainty.

In experimental gaming considerable use has been made of the random player. This is seen in the work of Gode and Sunder (1993), and others.

Even the concept of a know-nothing cannot be modeled easily without taking some context into consideration. The know-nothing can be so ignorant that he/she does not even know the bounds on the choice to be made. Fortunately in many economic situations the bounds are given by context. Thus the economically naïve knows that she cannot bid a negative amount and cannot bid more than she has plus the amount she can borrow; and in spite of some mathematical economics theory, the amount that can be borrowed tends to be finite. Given these bounds the naïve may act randomly within these bounds.

Much of the study of repeated games has utilized an optimal response player with global response. The player bases her response on maximization given the belief that others will continue to do what they have recently done. There is a big picture and a small picture version of this behavioral assumption. Assuming that she has "the big picture" implies that she is capable of searching over the whole domain of outcomes to select an optimal response.

In contrast with the global player the optimal response player with local scope only has the ability to search in a limited domain. This presupposes that there is some sort of metric, such as space, on the set of choices. A simple example is provided by a 5 x 5 matrix game where each individual can only see and move to adjacent squares. If the agents were at (10, 10) this is a local optimum and the local optimal response player would not be motivated to move, but the global optimal response player would move to the unique pure strategy non-cooperative equilibrium point (at (20, 30). There are many several local noncooperative equilibria in this game.

	1	2	3	4	5
1	0,0	0,0	9,1	0,0	0,0
2	0,0	0,0	0,0	0,0	10,1
3	0,70	5,4	10,10	5,5	20,30
4	0,0	0,0	0,0	0,0	0,0
5	0,0	0,0	2,2	0,0	0,0

A 5 x 5 Matrix Game

The non-specialist human is what Keynes and many others in macro-economic modeling consider the "average" consumer to be. This individual is assumed to be more or less an economic agent, but is constrained by habit and limited by lack of expertise and detailed knowledge in many areas. Furthermore the same individual who as a consumer

may be a creature of habit may also be a specialist in some body of knowledge such as production, engineering or medicine.

The specialist has been trained in some body of knowledge. He explicitly knows the rules of the game and if he is not a high expert will tend to operate with a high degree of conscious thought and calculation.

The high expert specialist, as chess studies have shown, appears to have routinized much of his knowledge, thus many of his moves do not require explicit calculation. Furthermore where the beginner or the ordinary player sees few alternatives, the expert sees many.

In specialized markets such as commodity hedging markets or various derivative markets the experts live with risk, but understand clearly how to perform and profit from the economically useful function of matching individuals with different risk profiles, thereby laying off much of the risk. Amateurs who wish to venture into this type of game are merely the welcome contributors to professional profits.

4 On Institutions, Innovation and Analysis

> Like all men in Babylon, I have been proconsul; like all I have been a slave. I have known omnipotence, ignominy, imprisonment. I owe that almost monstrous variety to an institution — the Lottery — which is unknown in other nations, or at work in them imperfectly or secretly
>
> — Jorge, L. Borges

The Lottery in Babylon

> His success may be great, but be it ever so great the Wheel of Fortune may turn again and bring him down into the dust
>
> — Gautama Siddhartha

The economic analysis of economies without exogenous uncertainty is far easier both conceptually and mathematically than the analysis of an economy with uncertainties. The presence of uncertainty can be made to disappear by waving the wand of complete markets; but this assumption useful for establishing an extreme bound on the class of economic market models, is far from economic reality. An economy with uncertainty is in constant flux.

In spite of the observations of Keynes (1921), Knight (1921), De Finetti (1970), Savage (1954), Ellsberg, (1961) and others a completely satisfactory understanding of the distinction between risk and uncertainty still does not exist. The postulating of a priori numerical values to a subjective probability for any event provides a tidy way to set up a Bayesian updating analysis that has tucked away the fundamental difficulties in the model in the formulation of the initial conditions. The setting up of the betting odds market seems to be about as good a mechanism as we can propose at this time for eliciting subjective quantitative estimation of the unknown. The problem seems to lie

more within the realms of the psychology of perception and in the social psychology of crowd behavior than in conventional economic theory.

The processes of innovation do not fit in a comfortable manner into either formal mathematical micro-economic theory, or into game theory. It fits more closely to the biological processes of mutation than into the economics of equilibrium.

4.1 Bankruptcy and Dynamics

Highly associated with dynamics of the firm and individual are the bankruptcy and reorganization laws of a society. Money, debt and the bankruptcy laws are the three critical items required to break out of the static equilibrium framework of general equilibrium and to provide the flexibility to accommodate innovation and evolutionary processes.

The general equilibrium system calls for the mathematics of convex programming subject to linear constraints, where in equilibrium the constraints generically take the form of equality of the overall budget constraints. As soon as one attempts to model the system as a fully described strategic game it becomes necessary to specify both the cash flow conditions and what happens if, through error or intent, the conditions of equality are violated.

A supply equals demand equation can fail in two ways. There may be a gap between the two, or an overlap. If someone has sold more than she has bought, the balance can be settled in money. If some one has bought more than she has sold and is unable to settle the difference in money or credit, default and bankruptcy rules are required.

If the dynamic process is viewed as part of a game of strategy, bankruptcy laws are required to prevent strategic default. They are not just a happenstance institutional fact; they are a logical necessity. Bankruptcy laws are often accompanied with reorganization conditions, thus the death process of a firm is not like that of a human, as under reorganization it may have a rebirth like that of the Phoenix.

4.2 Innovation and Mutation

It is one matter to understand the genesis of originality and the motivation for invention and innovation, and another matter to characterize the economic aspects of innovation. It is an error to attribute a sole economic goal to an inventor. Often it appears that the desire by the inventor to see “the baby born” dominates the economic motive.

Probably through some biological process a given small percentage of the population may be potential innovators. Given the pool of talent, how are the innovators financed? The main aspects of the financing are considered below. Here only the role of default and bankruptcy is considered. Sweeping across the range from no penalty to a Draconian penalty such as death we observe that if there were no penalty for default there would be no lending as the lenders would have no protection against a non-paying borrower. If the

penalty were enormous the borrowers would be afraid to borrow. If we assume continuity there should be a zone or a point with an optimal penalty. The interpretation of society's selection of the level of penalty is that it reflects the level of risk due to firm failure that society as a whole is willing to absorb. When a firm or venture fails, money and other assets are not destroyed they are reassigned. Some credit may be annihilated.

As the bankruptcy penalty is relaxed more individuals are willing to risk financing new ventures and along with possibly more successful innovations, there are more failures. The analogy with mutation is that the bankruptcy penalty plays the role of the controlling device over the economic mutation rate, trading off the cost of failure against the value of success.

4.3 An Aside on Premarket and Nonmarket Innovation

> Necessity, who is the mother of invention.
> — Plato, *The Republic*

Invention and discovery have existed long before markets and the elaborate mechanisms for financing invention in the modern economy. The first uses of fire are still not known with great accuracy. The wheel was possibly invented around 8,000 years ago but no hard evidence appears to go back beyond Sumeria. The early inventions were probably by individuals and spread through families and tribes. Once social organization appeared it is a likely conjecture that government played a role.

It is probable that the talent for originality and invention is essentially exogenously given in any society. The difference that an organized society and economy can make is in the utilization of the idea generators, not the intrinsic generation of ideas. Searching for talent among the masses can increase the crop of innovators, but in a certain sense good ideas are cheap. It is the implementation of the ideas that count.

With the market structure of medieval times it is likely that the prince or powerful families served as patrons. To this day the family and the private rich still play a role in the sponsorship of innovation.

4.4 Institutions Form and Function

Part of the theme here is that even an elementary minimal model of innovation requires a complexity of organization far beyond that of general equilibrium (GE) theory. A listing of the relevant institutions and their functions is noted. The primary functions are stated first. Then in parentheses some subfunctions are noted.

4.4.1 Form and Function

- Markets: exchange (information aggregation and disaggregation)
- Firms: production (marketing, accounting, inventorying)

- Government: Legislation and enforcement of the laws (standards, commercial code)
- Entrepreneurs: Locating and promoting inventors, raising funds (evaluating markets and products)
- Inventors: Invention
- Venture capitalists and Investment banks Feasibility and profitability evaluation, assessment of management, financing
- Private banks Lending and accepting deposits (credit assessment)
- Central bank: Management of money supply and interest rates
- Government other financial institutions: taxation subsidies and national debt
- Stock market: liquidity for extant stock and accommodation for new issues
- Bond market: A market for long term debt
- Money market: A market for short term debt
- Clearing house: Clearing facility to minimize speed in exchange and the need for monetary settlement.

The ultimate agents are real persons who are manifested in multiple roles as employees, consumers, owners and a variety of specialists generally attached to one of the organizations. The organizations are all legal persons owned in one form or another by the real persons. A loose biological analogy is that the real persons are the cells (who come in many manifestations) and the corporate persons are the organs of the body economic. For simplicity we refer only to corporations, rather than indulge in the other distinctions such as the various forms of partnership.

Among the more relevant rules are: Corporate law, contract law, the commercial code, patent law, tax law, and accounting conventions.

An important evolutionary feature of financial and other economic institutions is that they are often created for a specific function and later may serve many different purposes that were not foreseen at the time of their creation.

4.4.2 Economic Structures

Two types of agent have been noted, real persons and corporate persons and around ten types of institutions have been listed. In this section three simple diagrams are given to suggest that there is a considerable difference in basic complexity between the pre-institutional static general equilibrium models and a structure complex enough to encompass a process model of the financing of innovation.

The first is the old textbook warhorse usually illustrated by the Edgeworth–Bowley diagram of barter exchange between two traders exchanging two commodities

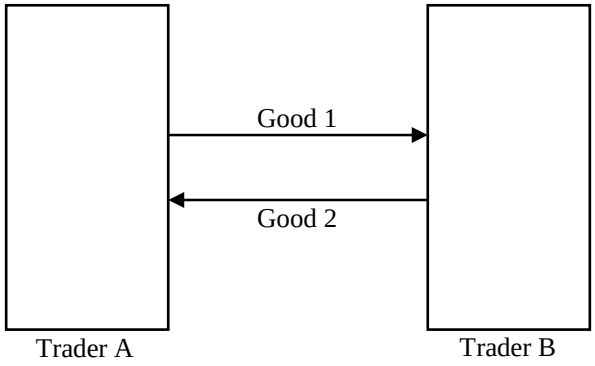

Figure 1
Bilateral Exchange

In Figure 1, Individual A exchanges Good 1 directly for Good 2 with Individual B.

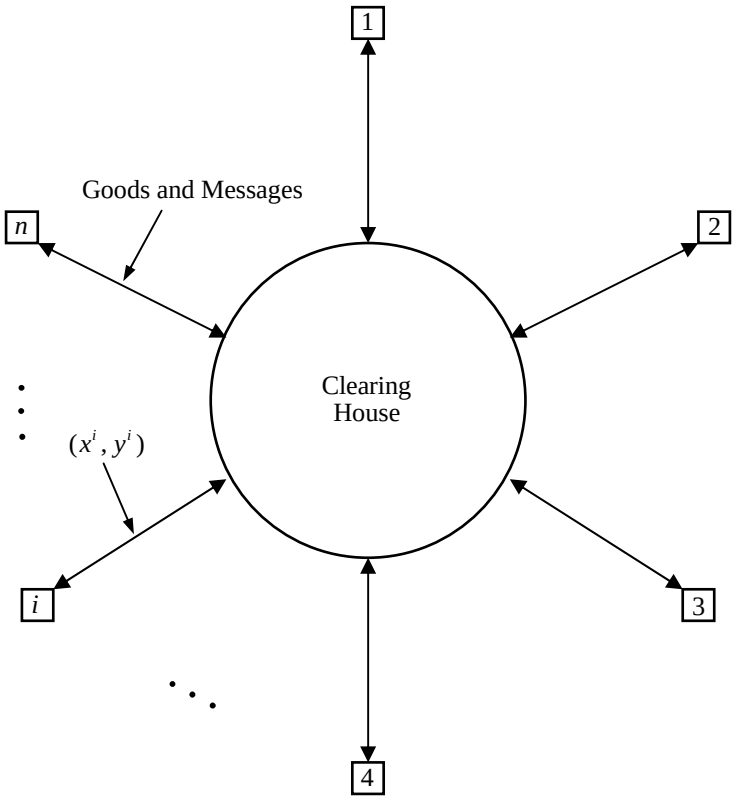

Figure 2
General Equilibrium Exchange with a Clearinghouse

Figure 2 represents the simplest structure that can achieve a general equilibrium outcome for multilateral exchange with n individuals trading in m commodities. The x^i and y^i are

to be interpreted as arrays of m goods offered and received in exchange. All individuals send their goods and bids denominated in their own credit money to a central clearing house that clears both goods and balances budgets by adjusting the relative prices of individual credit. No government money is called for. The work of Sahi and Yao (1989) and Sorin (1996) show that this is mathematically tractable, even though this perfect clearing house does not exist in practice. In Figure 2 the clearinghouse receives goods and messages from all individuals in the economy and ships the appropriate bundles of goods back.

Although mathematically elegant, the perfect clearinghouse and fiat moneyless central price calculating system exists primarily as a Soviet central planner's dream. Somewhat closer to reality is the institutional structure shown in Figure 3. Although even here production and innovation institutions have not been made explicit.

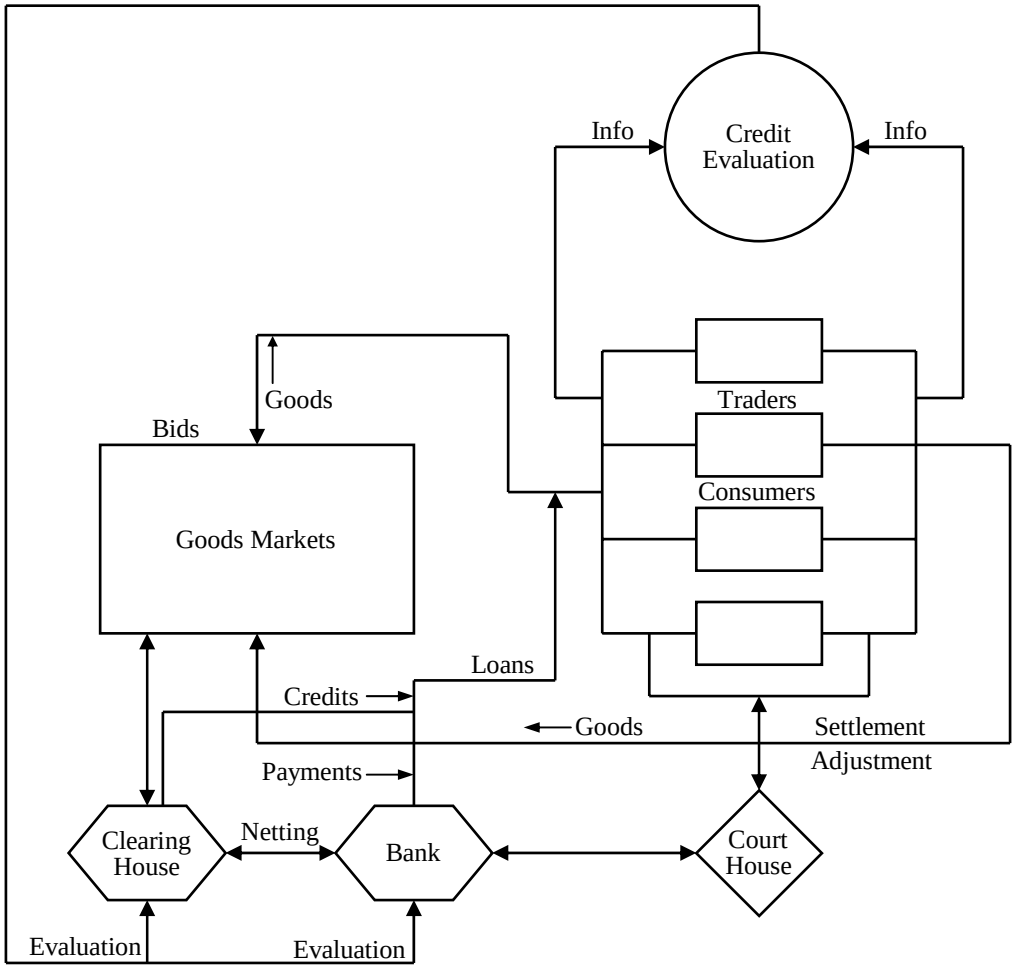

Figure 3
An Economy with Loans and Credit Evaluation

In Figure 3 emphasis is on evaluation, enforcement and information aggregation and disaggregation is presented by the explicit introduction of a credit evaluation agency, a clearing house and the courts. The consumers utilize both the clearing house and markets and, when needed, the courts. The consumers both supply goods and services to the markets and obtain goods and services from them.

5 Types of Innovation

There are at least three types of innovation that are worth considering independently. They are:

- Radically new product innovation
- Engineering variation of current product, and
- Cost, primarily organization or other process innovation.

In terms of uncertainty there are highly different. The most difficult to handle by conventional economic analysis is radical product innovation. Both the production procedures and the demand acceptance are unknown. There is little, if any precedence. The subjective probabilities, if any, may be cooked up by stretched analogy with other products that succeeded; and only quantified for the purpose for the construction of imaginary or *pro forma* financial statements used to persuade potential investors. An interesting discussion of the problems and approaches to managing innovation is given in Bechtel. et al. (1996).

If a new product succeeds and there is a competing similar product then it is by no means clear who will capture the market. The insightful work of Brian Arthur (1989) develops a probabilistic increasing returns to scale model where chance determines who inherits the market and the best technology does not necessarily emerge.

More or less standard product variation fits reasonably well into the current theory of oligopolistic competition. The large firms selling, say, refrigerators have products that are close to being identical. It is the job of marketing and the production engineers to have a spice shelf full of technically known modifications or additions that can help to differentiate the product of firm A from that of firm B. Both costs and demand can be reasonably estimated for such innovations.

By far the most prevalent form of innovation in most modern economies is process innovation involving organization and most frequently costs. New inventions call for expensive prototypes. Even if the market for the new product is clearly there, over the first few years, especially with mass market possibilities there is a considerable focus on unit cost reduction The prototype is highly expensive and the first batch for sale though cheaper than the prototype, is usually nowhere near the intended cost. The possibility to quantify a reasonable gaming experiment with cost innovation and to provide a reasonable scenario appears to be far easier than trying to construct an experimental game to illustrate basic product innovation.

6 Varieties in Financing Innovation

As is often the case with economic activities to be successful they require that specific functions be performed; but these functions may be manifested in many different institutions. Thus there may be many configurations of institutions that are sufficient for the activity. So it is with the financing of innovation. Among the possibilities are:

- The folk lore of family financing of the individual inventor in the garage
- The self-financed corporation
- Commercial bank financing
- Public or central bank/government sponsorship
- Financing via investment bankers, a private patron.

Any combination of these sources is also possibly depending on the social and economic details of the society.

In spite of the brilliance of Schumpeter's general insights on innovation, the process he sketched may have been more relevant to Austria of 1900 than to the United States or China of 2000. The same functional requirements are more or less still in place but the institutions supplying them have changed.

6.1 A Comment on Circular Flow and Innovation

Key to all of the various methods is the need to break the circular flow established at a general economic equilibrium. Mathematically the circular flow is required by the equilibrium budget balancing conditions and robs the overall system of n degrees of freedom, one for each budget constraint.

The introduction of money, be it gold or fiat, provides a system that in equilibrium preserves the circular flow and gives the illusion that money is irrelevant; but this is an artifact of the equilibrium conditions. In disequilibrium money and bankruptcy conditions permit the violation of circular flow and the institutions created to control the availability of money and to enforce the laws of default.

Money and financial institutions open the general equilibrium structure to dynamics. The natural way to describe a dynamic exchange and production economy is via a strategic market game with money, and private and government financial institutions. In equilibrium the money and financial institutions disappear but in disequilibrium they are available to play their role in control.

The financial institutions provide a control mechanism to guide the evolution of the economy. In particular they help to guide innovation. The basic assumption is that an attempt to innovate requires that resources are diverted from the current economy. Although we believe that the economy is always in a disequilibrium state, as a first crude approximation we assume that it is in equilibrium, it is disturbed from equilibrium by innovation and then resumes equilibrium after the innovation succeeds or fails. This enables us to concentrate on the violation of circular flow and on the redistribution of

resources during and after the attempt at innovation. In a separate paper Shubik and Sudderth (2008) provide and study a simple mathematical model of innovation. For simplicity we limit the investigation to cost innovation and note that even with this simple form of innovation the form of financing may control the welfare consequences and the nature of the disturbance to the circular flow.

7. Innovation, Evolution and Complexity

The argument above suggests that to a certain extent the static and dynamic equilibrium models of general equilibrium are consistent with evolutionary models as implied in the works of Schumpeter (1911, 1954), Keynes (1936, 1957), Boulding (1950), and many others, but basically at a different level of complexity. Implicitly all of these, essentially verbal treatments discuss disequilibrium states. Although the observations on disequilibrium are attractive, they are not well defined. Currently they appear to defy adequate mathematization. This is so because there is no sufficiently formal agreed on representation of the new product processes from invention to execution. A small step towards this appropriate mathematization can be achieved by first studying the far easier condition of cost innovation which still permits a comparative statics study as way out of studying the full dynamics.[1]

7.1 The Value and Limits of Reductionism

> Innovation, mutation and life itself are off-balance sheet assets.

The approach considered here is heavily reductionist. As soon as the masterful simplifyications of general equilibrium theory are given up one can be lost in a welter of special cases and institutional forms. Thousands, if not millions of detailed institutional structures appear calling for micro-micro economic or operations research detail. This appears to go with the economic facts of life of a modern economy. The study of myriads of special cases is the price to be paid to be able to say much about the dynamic behavior of an economic institution.

The hortatory analogies between economics and biology are attractive and undoubtedly contain elements of truth. But to operationalize them is another matter. In the approach followed here the stress has been on reductionism. Finer and finer details over and above the general equilibrium structure are added to obtain minimal structure to carry economic processes. As we add the requirements of complex structures capable of the aggregation of information and the evaluation of risk, minimal versions of the basic economic and financial institutions appear. Yet, when we start to address subfunctions of functions it appears that reductionism has no bounds. Each level of detail may be perfectly adequate to answer some operational questions, yet at each level more questions appear that are not

[1] Technically, it is modeled as a non stochastic parallel dynamic program with one controlled stochastic move at the start. In reality there are many random variables with unknown distributions constantly bombarding the system.

adequately answered. In particular, in spite of the spectacular growth of network theory we do not have a satisfactory representation of the organizational, morale and management aspects of the firm. This is why the accountant must deal with off-balance sheet assets and why the distinction between the firm as an ongoing entity and its liquidation value must be made. These are practical operational decisions but do not answer the basic distinction between organized life and a pile of assets.

8 Institutions, Mathematics and Hy Minsky

In the essay above a sketch has been presented of an approach aimed at producing mathematical models capable of connecting equilibrium and disequilibrium models of the economy. This required models capable of violating the circular flow; but to be able to demonstrate how to construct formal mechanisms to do so is not the same as being able to construct models immediately pertinent to the current economy. Nor do these models enable us to estimate the size of the potential instabilities the financial institutions may introduce, especially when they may be utilized by individuals who do not fit into the narrow mould of the rational economic agent.

The power of Hy Minsky was in his insightful analysis in being able to offer an overview utilizing essay combined with selected statistics and little formal mathematics to outline clearly the nature of the instabilities inherent in the modern financially guided economies. He understood that the financial institutions are a creation of human society and hence can be managed and redesigned to provide the economic stability needed.

The economic functions required by a healthy economy are more or less invariant, but the institutions that provide them are in constant flux as the society evolves. At some basic level possibly human behavior can be regarded as a given (although social structure, wealth and education impose different behavioral profiles). The operational challenge is to produce further insights to influence the evolution of stable and efficient institutions. Combined with this the role of the formal micro-economic theorist is to be able to extend our understanding of the formal structure of the financial control system and to try to characterize what functions and behavior may be regarded as invariant and how they can be reconciled with the disequilibrium of the evolving economies and societies we live in.

References

Arthur, W. B. 1989. "Competing Technologies, Increasing Returns and Lock-in by Historical Events," *Economic Journal* 99, 106–131.

Boulding. K. E. 1950. *A Reconstruction of Economics*. New York: Wiley.

Bechtel, S.D. et al. 1996. "Managing Innovation," *Daedelus* 125:2.

Ellsberg, D. 1961, "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics* 75(4): 643–669.

DeFinetti, B. 1970. Teoria delle probabilita. Torino: Guilio Einaudi.

Dosi, G., C. Freeman, R. Nelson, G. Silverberg and L. Soete. 1988. *Technical Change and Economic Theory*. London and New York: Pinter.

Gode, D. K. and S Sunder, 1993, "Allocative Efficiency of Markets with Zero Intelligence Traders: Market as a Partial Substitute for Individual Rationality," *Journal of Political Economy* 101: 119–137.

Huber, J, M. Shubik, and S. Sunder. 2007 "Three Minimal Market Games: Theory and Experimental Evidence," CFDP 1623. Yale University, New Haven.

Karatzas, I., M. Shubik, and W. Sudderth, 1994. "Construction of Stationary Markov Equilibria in a Strategic Market Game," *Journal of Mathematical Operations Research* 19(4): 975–1006.

Keynes, J. M. 1921. *Treatise on Probability*. London: MacMillan.

Keynes, J. M. [1936] 1957. *The General Theory of Employment, Interest and Money*. Reprint London: MacMillan.

Knight, F. H. 1921. *Risk, Uncertainty and Profit*. Boston and New York: Houghton Mifflin.

Koopmans, T. C. 1977a. "Concepts of Optimality and Their Uses," *American Economic Review* 67(3): 261–274.

Lucas, R. E. 1980. "Equilibrium in a Pure Currency Economy," *Economic Enquiry* 18: 203–220.

Minsky, H. 1986. *Stabilizing an Unstable Economy*. New Haven, Yale University.

Nelson, R. 1996. *The Sources of Economic Growth*. Cambridge: Harvard University Press.

Nelson, R. R. and S. G. Winter. 1982. *An Evolutionary Theory of Economic Change*. Cambridge: Harvard, Belknap.

Sahi, S. and Yao, S. 1989. "The Noncooperative Equilibria of a Trading Economy with Complete Markets and Consistent Prices," *Journal of Mathematical Economics* 18: 325–346.

Savage, L. J. 1954. The Foundations of Statistics. New York: Wiley.

Schumpeter, J. A. 1934. *The Theory of Economic Development*. Cambridge: Harvard University Press, original in German 1911.

Shubik, M. 1972. "A Theory of Money and Financial Institutions: Fiat Money and Noncooperative Equilibrium in a Closed Economy," *International Journal of Game Theory* 1(4): 243–268.

Shubik, M. and W. Sudderth. 2008. "Product Innovation: Schumpeter and Equilibrium," in preparation.

Sorin, S. 1996. "Strategic Market Games with Exchange Rates," *Journal of Economic Theory* 68: 431–446

Von Hippel, Eric. 1988. *The Sources of Innovation*. New York amd Oxford: Oxford University Press.