Is the "curse of natural resources" really a curse?

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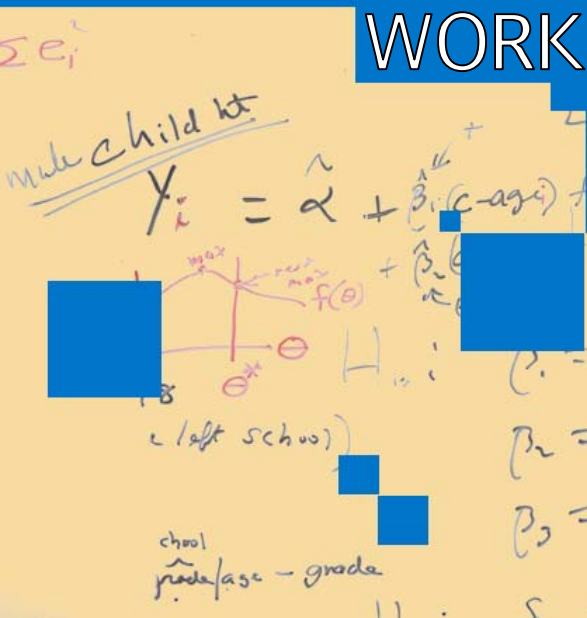
Duke University

March 20, 2008

Economic Research Initiatives at Duke
WORKING PAPERS SERIES

Is the "curse of natural resources" really a curse?

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Abstract

This paper takes a new look at the long-run implications of resource abundance. Using a Schumpeterian growth model that yields an analyitical solution for the transition path, it derives conditions under which the "curse of natural resources" occurs and is in fact a curse, meaning that welfare falls, conditions under which it occurs but it is not a curse, meaning that growth slows down but welfare rises nevertheless, and conditions under which it does not occur at all. An effective way to summarize the results is to picture growth and welfare as hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance, and is an important reminder that the welfare effect of resource abundance depends on the whole path of consumption, not on a summary statistic of its slope. Growth regressions that ignore the endogeneity of initial income do not provide sufficient information to assess whether resource abundance is bad even if one could prove beyond reasonable doubt that the relation is indeed negative and causal. Recent evidence that the correlation is actually positive should make us even more skeptical of policy advice based on the "curse" logic.

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1 Introduction

The debate on the role that natural resources play in the long-run fortunes of the economy is as lively as ever. Currently it focuses on the influential work of Sachs and Warner (1995, 2001), who find a negative cross-country correlation between the share of natural resources exports in GDP and the growth rate of GDP per capita, and interpret the export share as a measure of natural resource abundance. This finding poses tough questions for economic theory and policy — the idea that abundant resources are bad for the economy strikes many as a paradox. Not surprisingly, then, it has spurred a large amount of work that has added to an already large literature.[1]

Roughly speaking, the current debate pitches explanations of the negative correlation based on institutional deterioration, whereby natural resources create the opportunity for rent-seeking, corruption and conflict, against explanations based on sectoral reallocation, whereby activities intensive in natural resources crowd out other, more technologically dynamic activities (e.g., manufacturing, knowledge-based services).[2] Recently, something akin to a third side has emerged as some researchers find a positive correlation between growth and resource abundance and conclude that there is no paradox to resolve.[3]

The motivation for this paper is what all sides of the debate have in common. Empirical contributions take as given that the slower growth associated to resource abundance is bad — hence referring to it as the "curse of natural resources" is accepted practice — and that dispelling the "curse" requires proving that the correlation is, in fact, positive; theoretical contributions discuss specific mechanisms producing slower growth, not whether slower growth yields lower welfare. Underlying this literature, thus, seems to be the tacit assumption $growth = welfare$.

This is puzzling since the notion that growth and welfare are not the same is the cornerstone of the intertemporal trade-off at the heart of modern

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[1] A comprehensive review is beyond the scope of the paper. The interested reader can consult Gylfason (2001a), Stevens (2003) or Lederman and Maloney (2007).

[2] From this viewpoint, of particular interest is the evidence in Papyrakis and Gerlagh (2007) and Boyce and Emery (2007), who document a negative correlation across U.S. states between the growth rate of the Gross State Product (GSP) and, respectively, the share of the primary sector in GSP and the share of employment in mining. Because they focus on U.S. states, these studies raise important issues for institution-based stories.

[3] The positive correlation is actually not new; see Sala-i-Martin (1997), Sala-i-Martin, Doppelhofer and Miller (2004). What is new is the explicit assertion that the "curse" does not exist; see Alexeev and Conrad (2007), Brunneschweiler and Bulte (2007), Lederman and Maloney (2007b).

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growth theory. Moreover, since the correlation between resource abundance and the initial level of GDP per capita is positive it is not at all obvious that resource abundant countries are worse off.[4] Similarly puzzling is that empirical papers only rarely state explicitly whether "curse" denotes a permanent or a temporary slowdown. Yet, such a property is surely of consequence for the welfare effects that we can expect. If the slowdown is permanent, it can easily dominate the level effect of resources abundance; if it is temporary, it can be dominated by the level effect. It follows that the specifics of the growth model that one uses to interpret the regressions are of paramount importance and should be discussed explicitly.[5]

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[4]See Rodriguez and Sachs (1999), Bravo-Ortega and De Gregorio (2007) and, especially, Alexeev and Conrad (2007) for cross-country evidence; see also Boyce and Emery (2007) for U.S. states

[5]It is worth discussing examples of papers that tackle these issues to some extent to illustrate what I have in mind. Bravo-Ortega and de Gregorio (2007) develop a theoretical model whose predictions they take to the data. However, they don't use the model to draw welfare implications — surprisingly so, since they emphasize the positive relation between resource abundance and the whole path of log-income (see Figure 4.1, p. 80) and thus forewarn the reader that not only the intertemporal trade-off plays a crucial role in interpreting the regressions, but that the predicted welfare effect is *positive*. The most striking example of this practice is Rodriguez and Sachs (1999), who develop a fully specified simulation model of the Venezuelan economy and document that that country's growth experience is due to the fact that the discovery of oil allows it to live beyond its means — meaning that it converges to the steady state *from above*. Despite having all the ingredients, they do not calculate the discounted integral of utility to show that such a path is in fact associated to lower welfare; they only remark in the conclusions that experiencing falling income and consumption is bad. It surely is, but it is not clear that looking at *flow utility* answers the key question: Would Venezuela be better off if it hadn't found oil? After all, it converges from above because it overshoots the steady state in the first place. To argue that the slower growth subsequent to the inital overshooting period produces lower welfare, at a minimum they need to argue that the economy converges to a *lower* steady state (convergence from above to the *same* steady state, as it seems to be the case in the data reported in Figure 3, p. 289, yields *higher* welfare). Among purely empirical papers, Gylfason (2001b) documents the negative relation between growth and the share of natural wealth in national wealth and then writes: "Shaving one percentage point of any country's annual growth rate is a serious matter because the (weighted) average rate of per capita growth in the world economy since 1965 has been about 1.5 percent per year" (p. 849). Thus, the benchmark for infering that a high share of natural wealth is a bad thing is the world's average growth rate, not the economy's own welfare. Papyrakis and Gerlagh (2003, 2007) derive the long-run income effect of resource abundance from the transition dynamics of the standard reduced-form, log-linear representation of growth models. However, they are never explicit about (i) how resource abundance affects steady-state growth and initial income and (ii) how the long-run income effect that they compute relates to welfare. Welfare usually depends on the whole path of consumption (as in the papers just discussed) and the long-run income effect alone is not sufficient to assess whether it rises or falls with resource abundance. The paper that is most explicit about

In this paper I take to heart these observations and take a new look at the long-run implications of resource abundance through the lens of modern Schumpeterian growth theory. In particular, I use a model of the latest vintage that allows me to study welfare analytically. I derive conditions under which the "curse" occurs and is in fact a curse, meaning that welfare falls, conditions under which it occurs but it is not a curse, meaning that growth slows down but welfare rises nevertheless, and conditions under which it does not occur at all.

The model has two factors of production in exogenous supply, labor and a natural resource, and two sectors, primary production (or resource processing) and manufacturing. I define resource abundance as the endowment of the natural resource relative to labor. The primary sector uses labor to process the raw natural resource; the manufacturing sector uses labor and the processed natural resource to produce differentiated consumption goods. Because both sectors use labor, its reallocation from manufacturing to primary production drives the economy's adjustment to an increase in resource abundance. The manufacturing sector is technologically dynamic: firms and entrepreneurs undertake R&D to learn how to use factors of production more efficiently and to design new products. Importantly, the process of product proliferation fragments the aggregate market into submarkets whose size does not increase with the size of the endowments and thereby sterilizes the scale effect. This means that the effect of resource abundance on growth is only temporary. The resulting structure is extremely tractable and yields a closed-form solution for the transition path.

The core mechanism is how the pattern of factor substitution in the two sectors determines the response of the natural resource price to an increase in the endowment ratio. The price response determines the income earned by the owners of the resource (the households) and thereby their expenditure on manufacturing goods. This mechanism links resource abundance to the size of the market for manufacturing goods. Since manufacturing is the economy's innovative sector, it drives how resource abundance affects incentives to undertake R&D.

Substitution matters because it determines the price elasticity of demand for the natural resource. Inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity; elastic

the importance of accounting for the effect of resource abundance on initial income is Alexeev and Conrad (2007) who show, convincingly in my view, that doing so changes dramatically most of the results reported in the literature. However, they don't discuss growth explicitly, they only argue that we can infer the growth effect from the long-run level effect, and never mention welfare.

demand means that the adjustment requires a mild drop in the price. Importantly, the two cases are part of a continuum because I use technologies with factor substitution that changes with quantities. Now, if the economy exhibits substitution, demand is elastic, the price effect is mild, the quantity effect dominates, and resource income rises, spurring more spending on manufacturing and a temporary growth acceleration. If, instead, the economy exhibits complementarity, demand is inelastic, the price effect is strong, resource income falls, and we have a temporary growth deceleration — a "curse". Whether the economy experiences a growth acceleration or deceleration, however, is not sufficient to determine what happens to welfare, since, given technology, the lower resource price makes consumption goods cheaper. This means that to assess the welfare effect of the change in the endowment ratio I need to resolve the trade-off between short- and long-run effects if they differ in sign. As said, I can do this analytically since I have a closed-form solution for the transition path.

My main result is that I identify threshold values of the equilibrium price of the natural resource — and therefore threshold values of the endowment ratio — that yield the following sequence of scenarios as we gradually raise the endowment ratio from tiny to very large.

1. There is no curse – with or without quotation marks. This happens when the resource price is high, because the endowment ratio is low, demand is elastic, the quantity effect dominates over the price effect, and, consequently, resource income rises. In other words, starting from a situation of scarcity, the increase in resource abundance generates a growth acceleration associated to an initial jump up in consumption that yields higher welfare.

2. The "curse" is not a curse. That is, there is a "curse", in that the rise in the endowment ratio causes a growth deceleration, but the deceleration is offset by the fact that resource abundance raises the initial level of consumption. This happens when the endowment ratio is in the intermediate range in between the two thresholds and, correspondingly, the resource price is in its intermediate range. Relative to the previous case, as we enter this range demand becomes inelastic and the price effect dominates over the quantity effect, with the result that resource income falls as the endowment ratio rises.

3. The "curse" is a curse. This happens when the resource price is low, because the endowment ratio is high, demand is inelastic, the price effect dominates over the quantity effect, and resource income falls.

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Differently from the previous case, the fall in resource income is now sufficiently large to cause the growth deceleration to dominate over the initial jump up in consumption.

One way to summarize this pattern is to picture growth and welfare as hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance.

Before getting into the details of the paper, it is worth discussing some features that help evaluate its strenghts and weaknesses in relation to the literature. First, I work with a closed economy and thus rule out trade-induced specialization (i.e., there is no Dutch disease) and dependence, in the traditional sense of primary exports being a crucial component of the national economy.[6] The main advantage of this approach is that there is no confusion about the definition of resource abundance. Moreover, it assigns a central role to endogenous price adjustments. This is important, in my judgment, because the assumption that prices are fixed (common in work that focuses on small open economies) removes drivers of income effects that should be part of the analysis. The main disadvantage is obvious: the paper's link to the empirical literature is not as direct. However, the recent emphasis of empirical researchers on relative endowments suggests that the problem might be more the literature's early focus on the primary exports share rather than this paper's focus on a closed economy.

The second feature is that I work with two sectors to capture the crucial aspect of specialization induced by resource abundance: the sectoral shares of activity and employment change and drive the economy's adjustment path and the associated welfare level. Importantly, I assume, in line with the rest of the literature, that one sector is technologically progressive and the other is not (or less so in a more general version of the model). This, of course, is necessary to obtain the reallocation and associated crowding-out mechanism that drives the "curse". Overall, then, the model emphasizes substitution and technological change (also diminishing returns to scale, but this is not essential) as the driving forces of the economy's adjustment to changes in the relative endowment.[7] I thus appear to side with reallocation-based explanations in the debate characterized earlier. In a sense I do,

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[6] I am working on an extension of the analysis of this paper to the open economy case. The preliminary results are in line with what I present here.

[7] See Smulders (2005) for an insightful discussion of the role ot the "neoclassical trinity" — substitution, technological change, diminishing returns — in the study of the interaction of economic growth and natural resources scarcity

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mainly because I believe that technological change is a prime driver of growth and reallocation-based models assign to it the starring role. However, the main message of the paper is not that reallocation-based stories are superior; rather, it is that closer attention should be paid to welfare and that the $growth = welfare$ implicit assumption is grossly misleading. This insight applies to all explanations of the "curse" — institution-based, reallocation-based or any other type that future researchers will develop.[8]

The paper's organization is as follows. Section 2 sets up the model. Section 3 constructs the general equilibrium of the market economy. Section 4 discusses the key properties of the equilibrium that drive the paper's main results. Section 5 derives the main results. It first studies the conditions under which the "curse" occurs and then studies the conditions under which it is, in fact, a curse. It also discusses interesting implications for our reading of the empirical literature. Section 6 concludes.

2 The model

2.1 Overview

The basic model that I build on is developed in Peretto and Connolly (2007), who build on Peretto (1998). A representative household supplies labor services in a competitive market. It also borrows and lends in a competitive market for financial assets. The household values variety and buys as many differentiated consumption goods as possible. Manufacturing firms hire labor to produce differentiated consumption goods, undertake R&D, or, in the case of entrants, set up operations. Production of consumption goods also requires a processed resource, which is produced by competitive suppliers using labor and a raw natural resource. The introduction of this up-stream primary sector is the main innovation of this paper. The economy starts out with a given range of goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to

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[8]Arguably, institution-based stories have an easier time explaining why an economy might experience a collapse of *both* the level and growth of income than reallocation stories. This implies that initial income is a critical ingredient not only in assessing welfare but also in discriminating between the two approaches. Alexeev and Conrad (2007), for example, use their evidence on the positive effect of resource abundance on initial income to assess the causal link between economic performance and institutional quality. Interestingly, they conclude that there is none.

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losses. Once in the market, firms establish in-house R&D facilities to produce cost-reducing innovations. As each firm invests in R&D, it contributes to the pool of public knowledge and reduces the cost of future R&D. This allows the economy to grow at a constant rate in steady state.

2.2   Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-(\rho-\lambda)(s-t)} \log u(s)ds, \quad \rho > 0 \tag{1}$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + p\Omega + \Pi_M - Y, \tag{2}$$

where ρ is the discount rate, A is assets holding, r is the rate of return on financial assets, W is the wage rate, $L = L_0 e^{\lambda t}$, $L_0 \equiv 1$, is population size, which equals labor supply since there is no preference for leisure, and Y is consumption expenditure. In addition to asset and labor income, the household receives rents from ownership of the endowment, Ω, of a natural resource whose market price is p and dividend income from resource-processing firms, Π_M. The household takes these terms as given.

The household has instantaneous preferences over a continuum of differentiated goods,

$$\log u = \log \left[\int_0^N \left(\frac{X_i}{L} \right)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3}$$

where ϵ is the elasticity of product substitution, X_i is the household's purchase of each differentiated good, and N is the mass of goods (the mass of firms) existing at time t.

The solution for the optimal expenditure plan is well known. The household saves according to

$$r = r_A \equiv \rho + \frac{\dot{Y}}{Y} - \lambda \tag{4}$$

and taking as given this time-path of expenditure maximizes (3) subject to $Y = \int_0^N P_i C_i di$. This yields the demand schedule for product i,

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}. \tag{5}$$

With a continuum of goods, firms are atomistic and take the denominator of (5) as given; therefore, monopolistic competition prevails and firms face isoelastic demand curves.

2.3 Manufacturing: Production and Innovation

The typical firm produces one differentiated consumption good with the technology

$$X_i = Z_i^\theta \cdot F_X \left(L_{X_i} - \phi, M_i \right), \quad 0 < \theta < 1, \ \phi > 0 \tag{6}$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, M_i is processed resource use (henceforth "materials" for short), and Z_i^θ is the firm's TFP, a function of the stock of firm-specific knowledge Z_i. The function $F_X (\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. Hence, the production technology exhibits constant returns to rival inputs, labor and materials, and overall increasing returns. (6) gives rise to total cost

$$W\phi + C_X(W, P_M)Z_i^{-\theta}X_i, \tag{7}$$

where the function $C_X (\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments.

The elasticity of unit cost reduction with respect to knowledge in (7) is the constant θ. The firm accumulates knowledge according to the R&D technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \tag{8}$$

where $\dot{Z}_i$ measures the flow of firm-specific knowledge generated by an R&D project employing L_{Z_i} units of labor for an interval of time dt and αK is the productivity of labor in R&D as determined by the exogenous parameter α and by the stock of public knowledge, K.

Public knowledge accumulates as a result of spillovers. When one firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, an R&D project that produces $\dot{Z}_i$ units of proprietary knowledge also generates $\dot{Z}_i$ units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A

simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the technological frontier is determined by the average knowledge of all firms.[9]

The R&D technology (8), combined with public knowledge K, exhibits increasing returns to scale to knowledge and labor, and constant returns to scale to knowledge. This property makes constant, endogenous steady-state growth feasible.

2.4 The Primary or Resources Sector

In the primary sector competitive firms hire labor, L_R, to extract and process natural resources, R, into materials, M, according to the technology

$$M = F_M (L_M, R), \tag{9}$$

where the function $F_M (\cdot)$ is a standard neoclassical production function homogeneous of degree one in its arguments. The associated total cost is

$$C_M (W, p) M, \tag{10}$$

where C_M is a standard unit-cost function homogeneous of degree one in the wage W and the price of resources p.

This is the simplest way to model the primary sector for the purposes of this paper. Materials are produced with labor and a natural resource. The natural resource is in fixed endowment and earns rents. The primary sector competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy.

3 Equilibrium of the Market Economy

This section constructs the symmetric equilibrium of the manufacturing sector. It then characterizes the equilibrium of the primary sector. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the economy. The wage rate is the numeraire, i.e., $W \equiv 1$.

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[9]For a detailed discussion of the microfoundations of a spillovers function of this class, see Peretto and Smulders (2002).

3.1 Partial Equilibrium of the Manufacturing Sector

The typical manufacturing firm is subject to a death shock. Accordingly, it maximizes the present discounted value of net cash flow,

$$V_i(t) = \int_t^\infty e^{-\int_t^s [r(v)+\delta]dv} \Pi_i(s)ds, \quad \delta > 0$$

where $e^{-\delta t}$ is the instantaneous probability of death. Using the cost function (7), instantaneous profits are

$$\Pi_{Xi} = [P_i - C_X(1, P_M)Z_i^{-\theta}]X_i - \phi - L_{Z_i},$$

where L_{Z_i} is R&D expenditure. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the R&D technology (8), the demand schedule (5), $Z_i(t) > 0$ (the initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' innovation paths), and $Z_j(t') \geq 0$ for $t' \geq t$ (innovation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, I assume that upon payment of a sunk cost $\beta P_i X_i$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.[10] Once in the market, the new firm implements price and R&D strategies that solve a problem identical to the one outlined above. Hence, entry yields value V_i and a free entry equilibrium requires $V_i = \beta P_i X_i$.

The appendix shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$L_X = Y\frac{\epsilon - 1}{\epsilon}\left(1 - S_X^M\right) + \phi N; \tag{11}$$

$$M = Y\frac{\epsilon - 1}{\epsilon}\frac{S_X^M}{P_M}, \tag{12}$$

where

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M)Z_i^{-\theta}X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Associated to these factor demands are the return to cost reduction and entry, respectively:

$$r = r_Z \equiv \alpha\left[\frac{Y\theta(\epsilon - 1)}{\epsilon N} - \frac{L_Z}{N}\right] - \delta; \tag{13}$$

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[10] See Etro (2004) and, in particular, Peretto and Connolly (2007) for a more detailed discussion of the microfoundations of this assumption.

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$$r = r_N \equiv \frac{1}{\beta} \left[\frac{1}{\epsilon} - \frac{N}{Y} \left(\phi + \frac{L_Z}{N} \right) \right] + \hat{Y} - \hat{N} - \delta. \qquad (14)$$

The dividend price ratio in (14) depends on the gross profit margin $\frac{1}{\epsilon}$. Anticipating one of the properties of the equilibria that I study below, note that in steady state the capital gain component of this rate of return, $\hat{Y} - \hat{N}$, is zero. Hence, the feasibility condition $\frac{1}{\epsilon} > (r + \delta)\beta$ must hold. This simply says that the firm expects to be able to repay the entry cost because it more than covers fixed operating and R&D costs.

3.2 General equilibrium

As mentioned, the resources sector is competitive. Hence, firms produce up to the point where $P_M = C_M (1, p)$ and demand factors according to:

$$R = S_M^R \frac{MP_M}{p} = Y \frac{\epsilon - 1}{\epsilon} \frac{S_X^M S_M^R}{p}, \qquad (15)$$

$$L_M = \left(1 - S_M^R\right) MP_M = Y \frac{\epsilon - 1}{\epsilon} S_X^M \left(1 - S_M^R\right), \qquad (16)$$

where

$$S_M^R \equiv \frac{\partial \log C_M(W, p)}{\partial \log p}$$

and I have used (11) and (12) to obtain the expressions after the second equality sign. These factor demands yield that the competitive resource firms make zero profits.

Equilibrium of the primary sector requires $R = \Omega$. One can thus think of (15) as the equation that determines the price of the natural resource, and therefore resource income for the household, given the level of economic activity measured by expenditure on consumption goods Y.

The remainder of the model consists of the household's budget constraint (2), the labor demands (11) and (16), the returns to saving, cost reduction and entry in (4), (13) and (14). The household's budget constraint becomes the labor market clearing condition (see the appendix for the derivation):

$$L = L_N + L_X + L_Z + L_M,$$

where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and R&D operations of existing firms and L_M is aggregate employment of resources-processing firms. Assets market equilibrium requires equalization of all rates of return (no-arbitrage),

$r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta Y$.

A useful feature of the model is that I can combine the equilibrium conditions for the labor and assets markets to obtain an equation that describes how the resource price affects household income and thus expenditure on consumption goods. Specifically, substituting $A = \beta Y$ into (2) and using the rate of return to saving (4), I obtain after rearranging terms

$$\frac{Y - L - p\Omega}{Y} = \beta(\rho - \lambda).$$

Notice how all dynamic terms dropped out. Recall that I am focussing on a closed economy where the fixed domestic resource supply Ω implies that the price of the resource reflects scarcity. Then, letting $y \equiv \frac{Y}{L}$ denote expenditure per capita and $\omega \equiv \frac{\Omega}{L}$ denote the endowment ratio, I can use (15) and the condition $R = \Omega$ to study the instantaneous equilibrium (p^*, y^*) as the intersection in (p, y) space of the following two curves:

$$y = \frac{1 + p\omega}{1 - \beta(\rho - \lambda)}; \tag{17}$$

$$y = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon} S_M^R(p) S_X^M(p)}. \tag{18}$$

The first describes how resource income determines expenditure on consumption goods; the second how expenditure drives demand for the factors of production and thereby determines resource income.

Before proceeding, notice how population growth implies that a solution with constant p^* and y^* fails to exist if the endowment Ω is constant and the ratio ω shrinks over time. There are four ways of handling this problem. The first is to assume zero population growth. The second is to allow for population growth and assume that Ω grows at the same rate so that ω stays constant. The third is to posit Cobb-Douglas technologies so that the shares S_X^M and S_M^R are exogenous constants. This approach allows for explosive growth of p^* due to scarcity but has the drawback that while y^* remains constant it is independent of the endowment ratio (see below). The fourth way is to allow for population growth and deal with the fact that S_X^M and S_M^R are time-varying. The first three approaches retain the desirable feature that y^* is constant and consequently that the interest rate is $r = \rho$ at all times; see the Euler equation (4). The fourth requires to handle a time-varying interest rate. To keep things as simple as possible, and retain the feature that the endowment ratio affects y^* through prices, I follow the

second approach and assume that ω is constant because Ω and L grow at the same rate,[11] *possibly zero*. I discuss in detail existence and the comparative statics properties of the equilibrium (p^*, y^*) with respect to ω in Section 5. For the reminder of this section and the next, I posit that it exists and study its implications for the economy's dynamics.

3.3 Dynamics[12]

It is useful to work with the variable $n \equiv \frac{N}{L}$. Taking into account the non-negativity constraint on R&D, the fact that y^* is constant and implies $r^* = \rho$ allows me to solve (8) and (13) for

$$\hat{Z} = \alpha \frac{L_Z}{N} = \begin{cases} \frac{y^*}{n} \frac{\alpha\theta(\epsilon-1)}{\epsilon} - \rho - \delta & n < \bar{n} \\ 0 & n \geq \bar{n} \end{cases}, \tag{19}$$

where

$$\bar{n} \equiv y^* \frac{\alpha\theta\,(\epsilon - 1)}{(\rho + \delta)\,\epsilon}.$$

Substituting into (14) yields

$$\hat{n} = \begin{cases} \frac{1}{\beta}\left[\frac{1-\theta(\epsilon-1)}{\epsilon} - \left(\phi - \frac{\rho+\delta}{\alpha}\right)\frac{n}{y^*}\right] - (\rho+\delta) & n < \bar{n} \\ \frac{1}{\beta}\left[\frac{1}{\epsilon} - \phi\frac{n}{y^*}\right] - (\rho+\delta) & n \geq \bar{n} \end{cases}.$$

The general equilibrium of the model thus reduces to a single differential equation in the mass of firms per capita.[13] The economy converges to

$$n^* = \begin{cases} \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi - \frac{\rho+\delta}{\alpha}}y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi\alpha - (\rho+\delta)} < \frac{\theta(\epsilon-1)}{(\rho+\delta)\epsilon} \\ \frac{\frac{1}{\epsilon} - (\rho+\delta)\beta}{\phi}y^* & \frac{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}{\phi\alpha - (\rho+\delta)} \geq \frac{\theta(\epsilon-1)}{(\rho+\delta)\epsilon} \end{cases}. \tag{20}$$

These solutions exist only if the feasibility condition $\frac{1}{\epsilon} > (\rho + \delta)\beta$ holds. The interior steady state with both vertical and horizontal R&D requires

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[11] This assumption is actually quite common in empirical work. See Brunneschweiler and Bulte (2007) for a summary of the arguments that justify the practice.

[12] This section draws on Peretto and Connolly (2007) where the model that I build on in this paper is developed in full.

[13] For simplicity I ignore the non-negativity constraint on $\dot{N}$. I can do so without loss of generality because population growth implies that the mass of firms grows all the time. See Peretto (1998) for a discussion of this property. Also, I have posited a death shock so that negative net entry is feasible. This allows one to check that the model's qualitative implications for the case of zero population growth remain the same.

the more stringent conditions $\alpha\phi > \rho + \delta$ and

$$(\rho + \delta)\beta + \frac{\theta(\epsilon - 1)}{\epsilon} < \frac{1}{\epsilon} < (\rho + \delta)\beta + \frac{\alpha\phi}{\rho + \delta}\frac{\theta(\epsilon - 1)}{\epsilon}.$$

It then yields

$$\frac{y^*}{n^*} = \frac{\phi - \frac{\rho + \delta}{\alpha}}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - (\rho + \delta)\beta} \tag{21}$$

so that

$$\hat{Z}^* = \frac{\phi\alpha - (\rho + \delta)}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - (\rho + \delta)\beta}\frac{\theta(\epsilon - 1)}{\epsilon} - (\rho + \delta). \tag{22}$$

Notice how this steady-state growth rate is independent of the endowments L and Ω because there is no scale effect.

To perform experiments, I shall focus on this region of parameter space and work with the equation

$$\hat{n} = \nu - \left(\phi - \frac{\rho + \delta}{\alpha}\right)\frac{n}{\beta y^*}, \quad \nu \equiv \frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - (\rho + \delta).$$

This is a logistic equation (see, e.g., Banks 1994) with growth coefficient ν and crowding coefficient $\left(\phi - \frac{\rho + \delta}{\alpha}\right)\frac{1}{\beta y^*}$. Using the value n^* in (20), also called carrying capacity, I can rewrite it as

$$\hat{n} = \nu\left(1 - \frac{n}{n^*}\right), \tag{23}$$

which has solution

$$n(t) = \frac{n^*}{1 + e^{-\nu t}\left(\frac{n^*}{n_0} - 1\right)}, \tag{24}$$

where n_0 is the initial condition.

4 Properties of the equilibrium: technology, the path of consumption and welfare

The main advantage of this model is that one can solve explicitly for the level of welfare associated to the economy's transition to the steady state starting from any initial condition. In this section I bring this feature to the forefront as it provides the building blocs that I use in the derivation of the paper's main results.

In symmetric equilibrium (3), (5) and the fact that manufacturing firms set prices at a markup $\frac{\epsilon}{\epsilon-1}$ over marginal cost (see the Appendix) yield

$$\log u^* = \log\left(\frac{\epsilon-1}{\epsilon}\frac{y^*}{c^*}Z^\theta N^{\frac{1}{\epsilon-1}}\right),$$

where

$$c^* \equiv C_X\left(1, C_M\left(1, p^*\right)\right)$$

One can reinterpret the utility function (3) as a production function for a final homogenous good assembled from intermediate goods, so that u is indeed a measure of output, and define aggregate TFP for this economy as

$$T \equiv Z^\theta N^{\frac{1}{\epsilon-1}}. \tag{25}$$

Taking logs and time derivatives this yields

$$\hat{T}(t) = \theta\hat{Z}(t) + \frac{1}{\epsilon-1}\hat{N}(t) = \theta\hat{Z}(t) + \frac{1}{\epsilon-1}\lambda + \frac{1}{\epsilon-1}\hat{n}(t),$$

where $\hat{Z}(t)$ is given by (19), $\hat{n}(t)$ by (23) and $n(t)$ by (24). In steady state this gives

$$\hat{T}^* = \theta\hat{Z}^* + \frac{1}{\epsilon-1}\lambda \equiv g^*,$$

which is independent of the endowments L and Ω, and thus of ω.

Observe now that according to (21) in steady state we have

$$\frac{y^*}{n^*} = \frac{y_0}{n_0} \Rightarrow \frac{y^*}{y_0} = \frac{n^*}{n_0},$$

and define

$$\Delta^* \equiv \frac{n^*}{n_0} - 1 = \frac{y^*}{y_0} - 1.$$

This is the percentage change in expenditure that the economy experiences in response to changes in fundamentals and/or policy parameters. It fully summarizes the effects of such changes on the scale of economic activity. Using this information, I obtain the following result.

Proposition 1 *Let $\log u^*(t)$ and U^* be, respectively, the instantaneous con-sumption index (3) and the welfare function (1) evaluated at y^*. Then, a path starting at time $t = 0$ with initial condition n_0 and converging to the steady state n^* is characterized by:*

$$\log T(t) = \log T_0 + g^*t + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon-1}\right)\Delta^*\left(1 - e^{-\nu t}\right), \tag{26}$$

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where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon-1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\alpha\theta(\epsilon-1)}{\epsilon} \frac{\phi - \frac{\rho+\delta}{\alpha}}{\frac{1-\theta(\epsilon-1)}{\epsilon} - (\rho+\delta)\beta}.$$

Therefore,

$$\log u^*(t) = \log \frac{y^*}{c^*} + g^* t + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon-1}\right)\Delta^*\left(1 - e^{-\nu t}\right), \qquad (27)$$

where without loss of generality $\frac{\epsilon-1}{\epsilon}T_0 = 1$. This yields

$$U^* = \frac{1}{\rho-\lambda}\left[\log\left(\frac{y^*}{c^*}\right) + \frac{g^*}{\rho-\lambda} + \mu\Delta^*\right], \qquad (28)$$

where

$$\mu \equiv \frac{\gamma + \frac{\nu}{\epsilon-1}}{\rho-\lambda+\nu}.$$

Proof. See the Appendix. ∎

The transitional component of the TFP operator in (26) summarizes the cumulated gain/loss due to above/below steady-state cost reduction and product variety expansion. The expression for flow utility (27) allows one to see separately the initial jump due to y^*/c^* and the gradual evolution due to T. Accordingly, the first term in (28) captures the role of steady-state real expenditure calculated holding technology constant; the second captures the role of steady-state growth; the third is the contribution from the gain/loss due to the transitional acceleration/deceleration of TFP relative to the steady state path. Following Peretto and Connolly (2007) we define the term μ as a *welfare multiplier* summarizing the transitional effects of the change in market size Δ^*.

5 The "curse": does it occur, is it a curse?

This section analyzes the effects of a change in the endowment ratio. It begins with a discussion of the conditions under which it raises or lowers consumption expenditure so that the market for manufacturing goods expands or contracts. It then shows how the interaction of the initial change in consumption and the transition dynamics after the shock produce a change in welfare whose sign can be assessed analytically.

5.1 Expenditure and prices

The first step in the assessment of the effects of resource abundance is to use (17) and (18) to characterize expenditure and prices. The following property of the demand functions (12) and (15) is useful.

Lemma 2 *Let:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M};$$

$$\epsilon_M^R \equiv -\frac{\partial \log R}{\partial \log p} = 1 - \frac{\partial \log S_M^R}{\partial \log p} = 1 - \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R}.$$

Then,

$$\frac{\partial \left(S_M^R(p) \, S_X^M(p) \right)}{\partial p} = \Gamma(p) \frac{S_M^R(p) \, S_X^M(p)}{p}, \tag{29}$$

where

$$\Gamma(p) \equiv \left(1 - \epsilon_X^M(p) \right) S_M^R(p) + 1 - \epsilon_M^R(p).$$

Proof. See the Appendix. ∎

$\Gamma(p)$ is the elasticity of $S_M^R(p) \, S_X^M(p)$ with respect to p. According to (12), therefore, it is the elasticity of the demand for the resource R with respect to its price p, holding constant expenditure per capita y. It thus captures the partial equilibrium effects of price changes in the resource and materials markets for given market size and regulates the shape of the income relation (18). To see how, differentiate (18), rearrange terms and use (15) to obtain:

$$\frac{d \log y(p)}{dp} = \frac{\frac{\epsilon - 1}{\epsilon} \frac{d \left(S_M^R(p) S_X^M(p) \right)}{dp}}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon} S_M^R(p) \, S_X^M(p)} = \omega \Gamma(p).$$

This says that the effect of changes in the resource price on expenditure on manufacturing goods depends on the overall pattern of substitution that is reflected in the price elasticities of materials and resource demand and in the resource share of materials production costs. To see the pattern most clearly, I pretend for the time being that $\Gamma(p)$ does not change sign with p. I comment later on how allowing $\Gamma(p)$ to change sign for some p makes the model even more interesting. The following proposition states the results formally, Figure 1 illustrates the mechanism.

Proposition 3 *Suppose that* $\Gamma(p)$ *is positive, zero or negative for all* p. *Then, there are three cases.*

1. **Substitution.** *This occurs when* $\Gamma(p) > 0$ *and the income relation (18) is a monotonically increasing function of* p *with domain* $p \in [0, \infty)$ *and codomain* $y \in [y^*(0), y^*(\infty))$, *where*

$$y^*(0) = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{\epsilon}S_M^R(0)S_X^M(0)},$$

$$y^*(\infty) = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{\epsilon}S_M^R(\infty)S_X^M(\infty)}.$$

Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ *with the property:*

$$p^*(\omega) \quad : \quad (0, \infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega;$$

$$y^*(\omega) \quad : \quad (0, \infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} < 0 \quad \forall \omega.$$

2. **Cobb-Douglas-like economy.** *This occurs when* S_M^R *and* S_X^M *are exogenous constants,* $\Gamma(p) = 0$ *and the income relation (18) is the flat line*

$$y = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{\epsilon}S_M^R S_X^M} \equiv y_{CD}^*.$$

Then there exists a unique equilibrium $(p_{CD}^*(\omega), y_{CD}^*)$ *with the property:*

$$p_{CD}^*(\omega) : (0, \infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega.$$

3. **Complementarity.** *This occurs when* $\Gamma(p) < 0$ *and the income relation (18) is a monotonically decreasing function of* p *with domain* $p \in [0, \infty)$ *and codomain* $y \in (y^*(\infty), y^*(0)]$, *where*

$$y^*(\infty) = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{\epsilon}S_M^R(\infty)S_X^M(\infty)},$$

$$y^*(0) = \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon-1}{\epsilon}S_M^R(0)S_X^M(0)}.$$

Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ *with the property:*

$$p^*(\omega) \quad : \quad (0, \infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega;$$

$$y^*(\omega) \quad : \quad (0, \infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} > 0 \quad \forall \omega.$$

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Proof. See the Appendix. ∎

I refer to the second case as the Cobb-Douglas-like economy because this seemingly special configuration, requiring $\epsilon_M^R = 1 + \left(1 - \epsilon_X^M\right) S_M^R$, is in fact quite common in the literature as it occurs when both technologies are Cobb-Douglas and $\epsilon_X^M = \epsilon_M^R = 1$.

Notice how the effect of resource abundance on the resource price is negative in all cases while the effect on expenditure changes sign according to the substitution possibilities between labor and materials in manufacturing and between labor and the natural resource in materials production. This brings us to the observation that if $\Gamma(p)$ changes sign for some p, the model generates endogenously a switch from, say, complementarity to substitution. I illustrate this case in Figure 2 where the income relation (18) is a hump-shaped function of p. The pattern is best captured by looking at the properties of the function $\Gamma(p)$. Using the definitions in Lemma 2, we have

$$
\begin{aligned}
\frac{d\Gamma(p)}{dp} &= -\frac{d\epsilon_X^M(P_M)}{dP_M}\frac{dP_M}{dp}S_M^R(p) \\
&\quad + \left(1 - \epsilon_X^M(P_M)\right)\left(1 - \epsilon_M^R(p)\right) - \frac{d\epsilon_M^R(p)}{dp}.
\end{aligned}
$$

This derivative is negative if the following two conditions hold:

- the elasticities ϵ_X^M and ϵ_M^R are increasing in P_M and p, respectively;

- the terms $1 - \epsilon_X^M$ and $1 - \epsilon_M^R$ have opposite sign.

The hump-shaped income relation in Figure 2 then obtains if $\Gamma(0) > 0$ and $\Gamma(\infty) < 0$. Notice that the second condition says that if demand in one sector is elastic, say $1 < \epsilon_X^M$, then demand in the other sector is inelastic, $1 > \epsilon_M^R$. Below I show that these conditions hold quite naturally in an economy with CES technologies. Here I discuss the general property.

Proposition 4 *Suppose that there exists a price $\bar{p}$ where $\Gamma(p)$ changes sign, from positive to negative, so that the income relation (18) is a hump-shaped function of p with domain $p \in [0, \infty)$ and codomain $y \in [y^*(0), y^*(\infty))$ or $y \in [y^*(\infty), y^*(0))$, where*

$$
\begin{aligned}
y^*(0) &= \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon}S_M^R(0)S_X^M(0)}, \\
y^*(\infty) &= \frac{1}{1 - \beta(\rho - \lambda) - \frac{\epsilon - 1}{\epsilon}S_M^R(\infty)S_X^M(\infty)}.
\end{aligned}
$$

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Then there exists a unique equilibrium $(p^*(\omega), y^*(\omega))$ with the property:

$$p^*(\omega) \quad : \quad (0,\infty) \to (\infty, 0), \quad \frac{dp^*(\omega)}{d\omega} < 0 \quad \forall \omega;$$

$$y^*(\omega) \quad : \quad (0,\infty) \to [y^*(\infty), y^*(0)), \quad \frac{dy^*(\omega)}{d\omega} \gtreqless 0 \quad \omega \lesseqgtr \bar{\omega},$$

where $\bar{\omega}$ is the value of ω such that $p^*(\bar{\omega}) = \bar{p}$.

Proof. See the Appendix. ∎

The main message of this analysis is that resource abundance raises expenditure, and thereby results into a larger market for manufacturing goods, when the economy exhibits overall substitution between labor and resources (processed and raw) in the manufacturing of consumption goods and the processing of the natural resource into materials. Conversely, when the economy exhibits overall complementarity resource abundance results into a smaller market for manufacturing goods. More importantly, whether the economy exhibits substitution or complementarity depends on equilibrium prices and thus on the endowment ratio itself. In other words, there are solid reasons to expect that the effect of the endowment ratio on the path of consumption is non-monotonic.

5.2 The path of consumption and welfare

For concreteness, consider an economy in steady state (p^*, y^*) and imagine an increment $d\omega$ in its endowment ratio. Then, by construction we have

$$\Delta^* = \frac{y^*(\omega) + \frac{dy^*(\omega)}{d\omega}}{y^*(\omega)} - 1 = \frac{1}{y^*(\omega)} \frac{dy^*(\omega)}{d\omega}$$

and we can use the results of the previous section in a straightforward manner. Let us look first at the initial effect on consumption.

Proposition 5 *The impact effect of the change in the endowment ratio is*

$$\frac{d \log \left(\frac{y^*(\omega)}{c^*(\omega)} \right)}{d\omega} = [\Gamma(p^*(\omega)) - \Psi(p^*(\omega))] \omega \frac{dp^*(\omega)}{d\omega}, \qquad (30)$$

where

$$\Psi(p^*(\omega)) \equiv \frac{\epsilon}{(\epsilon - 1) y^*(\omega)} = \kappa - S_M^R(p^*(\omega)) S_X^M(p^*(\omega)),$$

$$\kappa \equiv \frac{\epsilon}{\epsilon - 1} [1 - \beta(\rho - \lambda)].$$

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Proof. See the Appendix. ∎

Equation (27), Proposition 5 and the fact that $\frac{dp^*(\omega)}{d\omega} < 0 \; \forall \omega$ then yield three possible configurations for the path of consumption:

1. $\Gamma(p^*(\omega)) < 0 < \Psi(p^*(\omega))$. The initial jump is up and then we have a growth acceleration. There is no "curse" and welfare rises.

2. $0 < \Gamma(p^*(\omega)) < \Psi(p^*(\omega))$. The initial jump is up and then we have a growth deceleration. The welfare change is ambiguous since we have an intertemporal trade-off. Is the "curse" a curse?

3. $0 < \Psi(p^*(\omega)) < \Gamma(p^*(\omega))$. The initial jump is down and then we have a growth deceleration. Welfare falls. The "curse" is a curse.

Figure 3 illustrates these possibilities. Case 2 highlights the need to look at welfare directly to resolve the ambiguity.

Proposition 6 *The welfare effect of a change in the endowment ratio is*

$$\frac{dU^*(\omega)}{d\omega} = \frac{1}{\rho - \lambda} \left[(1 + \mu) \Gamma(p^*(\omega)) - \Psi(p^*(\omega)) \right] \omega \frac{dp^*(\omega)}{d\omega}. \qquad (31)$$

Proof. See the Appendix. ∎

We can then characterize four scenarios for welfare.

1. There is no curse – with or without quotation marks. This happens when
$$(1 + \mu) \Gamma(p^*(\omega)) < \Gamma(p^*(\omega)) \leq 0 < \Psi(p^*(\omega))$$
and the rise of the endowment ratio generates a growth acceleration associated to an initial jump up in consumption.[14]

2. There is a "curse", in that the rise of the endowment ratio causes a growth deceleration, but the deceleration is offset by the fact that resource abundance reduces prices and raises the level of consumption. This happens when

$$0 < \Gamma(p^*(\omega)) < (1 + \mu) \Gamma(p^*(\omega)) < \Psi(p^*(\omega)).$$

In this case the "curse" is not a curse.

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[14] Notice that this includes the Cobb-Douglas economy since in that case $1 = \epsilon_X^M = \epsilon_M^R$ and growth does not respond to ω at all while lower prices yield higher utility.

3. The "curse" is a curse. This happens when

$$0 < \Gamma\left(p^*\left(\omega\right)\right) < \Psi\left(p^*\left(\omega\right)\right) < \left(1+\mu\right)\Gamma\left(p^*\left(\omega\right)\right)$$

and the rise of the endowment ratio generates a growth deceleration that dominates over the initial jump up in consumption.

4. The "curse" truly is a curse. This worst-case scenario happens when

$$0 < \Psi\left(p^*\left(\omega\right)\right) < \Gamma\left(p^*\left(\omega\right)\right) < \left(1+\mu\right)\Gamma\left(p^*\left(\omega\right)\right)$$

and the rise of the endowment ratio generates a growth deceleration associated to an initial fall of consumption.

One of course is interested in mapping these scenarios into values of the endowment ratio itself. To do this, it is useful to impose some more structure on the model.

5.3 A CES economy

Consider the following CES economy:

$$X_i = Z_i^{\theta}\left[\psi_X\left(L_{X_i} - \phi\right)^{\sigma_X} + \left(1 - \psi_X\right)M_i^{\sigma_X}\right]^{\frac{1}{\sigma_X}}, \quad \sigma_X \leq 1;$$

$$M = \left[\psi_M L_M^{\sigma_M} + \left(1 - \psi_M\right)R^{\sigma_M}\right]^{\frac{1}{\sigma_M}}, \quad \sigma_M \leq 1.$$

The associated unit-cost functions are:

$$C_{X_i} = Z_i^{-\theta}\left[\psi_X^{-\frac{1}{\sigma_X-1}}W^{\frac{\sigma_X}{\sigma_X-1}} + \left(1 - \psi_X\right)^{-\frac{1}{\sigma_X-1}}P_M^{\frac{\sigma_X}{\sigma_X-1}}\right]^{\frac{\sigma_X-1}{\sigma_X}};$$

$$C_M = \left[\psi_M^{-\frac{1}{\sigma_M-1}}W^{\frac{\sigma_M}{\sigma_M-1}} + \left(1 - \psi_M\right)^{-\frac{1}{\sigma_M-1}}p^{\frac{\sigma_M}{\sigma_M-1}}\right]^{\frac{\sigma_M-1}{\sigma_M}};$$

From these one derives (recall that $W \equiv 1$):

$$S_X^M = \frac{1}{1 + \left(\frac{\psi_X}{1-\psi_X}\right)^{\frac{1}{1-\sigma_X}}P_M^{\frac{\sigma_X}{1-\sigma_X}}}; \quad \epsilon_X^M = 1 + \frac{\sigma_X}{1-\sigma_X}\left(1 - S_X^M\right);$$

$$S_M^R = \frac{1}{1 + \left(\frac{\psi_M}{1-\psi_M}\right)^{\frac{1}{1-\sigma_M}}p^{\frac{\sigma_M}{1-\sigma_M}}}; \quad \epsilon_M^R = 1 + \frac{\sigma_M}{1-\sigma_M}\left(1 - S_M^R\right).$$

As is well known, the CES contains as special cases the linear production function ($\sigma = 1$) wherein inputs are perfect substitutes, the Cobb-Douglas ($\sigma = 0$) wherein the elasticity of substitution between inputs is equal to 1, and the Leontief ($\sigma = -\infty$) wherein inputs are perfect complements. The advantage of the CES specification, then, is that we can make things quite explicit and at the same time general.

Specifically, define:

$$\Gamma\left(\omega\right) \equiv \Gamma\left(p^*\left(\omega\right)\right) = \left(1 - \epsilon_X^M\left(p^*\left(\omega\right)\right)\right) S_M^R\left(p^*\left(\omega\right)\right) + 1 - \epsilon_M^R\left(p^*\left(\omega\right)\right);$$

$$\Psi\left(\omega\right) \equiv \Psi\left(p^*\left(\omega\right)\right) = \kappa - S_M^R\left(p^*\left(\omega\right)\right) S_X^M\left(p^*\left(\omega\right)\right).$$

Observe that if $\sigma_X > 0$ and $\sigma_M > 0$, then $\Gamma\left(p\right) < 0$ for all p and the "curse" never occurs. If $\sigma_X < 0$ and $\sigma_M < 0$, instead, $\Gamma\left(p\right) > 0$ for all p and the "curse" always occurs. Interestingly, if σ_X and σ_M have opposite signs we can capture how the economy moves from one case to the other as the endowment ratio changes. Specifically, let $\sigma_X > 0$ and $\sigma_M < 0$ so that the manufacturing sector exhibits gross substitution between labor and materials while the resource sector exhibits gross complementarity between labor and raw resources. Recall that this means that demand for processed resources, i.e., materials, in the manufacturing sector is elastic, while demand for raw resources in the primary sector is inelastic. We then have:

1. There is no "curse" and hence no curse. This happens when

$$\Gamma\left(\omega\right) \leq 0 \Leftrightarrow 0 < \omega < \bar{\omega}.$$

2. The "curse" is not a curse. This happens when

$$0 < \left(1 + \mu\right)\Gamma\left(\omega\right) < \Psi\left(\omega\right) \Leftrightarrow \bar{\omega} < \omega < \tilde{\omega}.$$

3. The "curse" is a curse. This happens when

$$0 < \Psi\left(\omega\right) < \left(1 + \mu\right)\Gamma\left(\omega\right) \Leftrightarrow \tilde{\omega} < \omega < \infty.$$

Figure 4 illustrates this analysis. The appendix establishes formally the properties of the curves $\Gamma\left(\omega\right)$ and $\Psi\left(\omega\right)$ used in the figure to find the threshold values of the endowment ratio. Here I focus on the economics.

First, and most important: Why does the overall pattern of substitution/complementarity matter so much for growth and welfare? Because it regulates the reduction in the price p required for the economy to absorb

the extra endowment of R relative to L. Notice, moreover, that the primary and manufacturing sectors are vertically related so the adjustment involves the interdependent responses of P_M to the increase in supply of processed resources and of p to the increase in supply of raw resources. Now, inelastic demand means that the price has to fall drastically to induce the market to absorb the additional quantity. Hence, if demand is inelastic in both sectors the overall adjustment requires drastic drops in both P_M and p. What matters is that the drastic fall of p results in a fall of resources income $p\omega$ which depresses expenditure y. This is the case $\Gamma > 0$. In contrast, if demand is elastic in both sectors the overall adjustment requires mild drops in prices. The crucial difference is that in this case $\Gamma < 0$ so that resources income $p\omega$ rises because the quantity effect dominates the price effect.

With this intuition in hand, we can now interpret the analytical results of the model. The function $\Gamma(\omega)$ starts out negative and increases monotonically, changing sign at $\bar{\omega}$ and converging to its positive upper bound as $\omega \to \infty$. To see this, write

$$\Gamma'(\omega) = \frac{d\Gamma(p^*)}{dp^*}\frac{dp^*}{d\omega}$$

and observe that the CES economy with $\sigma_X > 0$ and $\sigma_M < 0$ satisfies the conditions for $d\Gamma(p)/dp < 0$ $\forall p$ discussed above, namely: $\epsilon_X^M > 1$ and increasing in P_M; $\epsilon_M^R < 1$ and increasing in p.

The function $\Psi(\omega)$ is hump-shaped with a peak exactly at $\bar{\omega}$, where $\Gamma(\omega)$ changes sign. The reason is that this is the value where the derivative of $S_M^R(p)S_X^M(p)$ with respect to p equals zero. Notice also that $\Gamma(\omega) < \Psi(\omega)$ for all ω (see the appendix for the proof) so that Case 3 from figure 3 and Scenario 4 from the analysis of welfare no longer apply because initial consumption cannot fall.

The resulting pattern is the following: The endowment ratio is initially very low and prices are very high, that is, $\omega \to 0 \Rightarrow p \to \infty \Rightarrow P_M = C_M(1,p) \to \infty$. Under these conditions, $\Gamma(0) < 0 < \Psi(0)$ so that an increase in ω produces a growth acceleration associated to a jump up in initial consumption. Intuitively, this says that in a situation of extreme scarcity and extremely high price a helicopter drop of natural resource is good. As the relative endowment grows, the overall pattern of substitution changes. In particular, $\Gamma(\omega)$ changes sign at $\omega = \bar{\omega}$ and we enter the region where we get a "curse of natural resources" because an increase in resource abundance yields a decrease in expenditure on manufacturing goods that triggers a slowdown of TFP growth. This, "curse", however, is not really a curse since we are to the left of $\tilde{\omega}$ and the slowdown is associated to an

initial jump up in consumption that dominates in the intertemporal trade-off. As we keep moving to the right and enter the next region, $\tilde{\omega} < \omega < \infty$, the growth deceleration is again associated to an initial jump up in consumption but now the deceleration dominates in the intertemporal trade-off and welfare falls. It is here that we have a curse.

An effective summary of this pattern is to picture growth and welfare as hump-shaped functions of resource abundance. To do this, we first must translate the path of the log of TFP into a measure of average growth in line with what is considered in the empirical literature. Specifically, let

$$g(0,t) \equiv \frac{1}{t} \left[\log T(t) - \log T_0 \right]$$

be the average growth rate of TFP between time 0 and time t. Then, using (26) we have:

$$g(0,t) = g^* + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1} \right) \Delta^* \frac{1 - e^{-\nu t}}{t}. \tag{32}$$

This says that the an increase in the endowment ratio is associated to a transitory period of above (below) trend average growth if it generates an increase (decrease) in per capita expenditure on consumption goods. Notice that as $t \to \infty$ this measure converges to g^*, which does *not* depend on ω because there is no scale effect. Notice also, that since it depends on ω only through Δ^* it is hump-shaped in ω with the maximum at $\omega = \bar{\omega}$.

For welfare we have two options. We can use the measure U^* calculated in (28), which is clearly hump-shaped in ω with the maximum at $\omega = \tilde{\omega}$. We then conclude directly that the peak of growth occurs earlier than the peak of welfare. This property captures the crucial role of initial consumption, which rises with resource abundance, in disposing of the $growth = welfare$ tacit assumption. As equation (28) makes clear, welfare depends on the whole path of consumption, not on a summary statistic of its slope.

One could argue that using (32) and (28) is inappropriate as they are summary statistics computed over different time intervals. The alternative is to compute welfare as the discounted integral of the same part of the consumption path that we use to compute (32), that is,

$$U^*(0,t) = \int_0^t e^{-(\rho - \lambda)s} \log u^*(s) \, ds.$$

Using (26), (27) and (32), we can write

$$U^*(0,t) = (a_1 - a_3) g^* + a_2 \cdot \log \left(\frac{y^*}{c^*} \right) + a_3 \cdot g(0,t),$$

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where

$$a_1 \equiv \int_0^t e^{-(\rho - \lambda)s} s \, ds; \ \ a_2 \equiv \int_0^t e^{-(\rho - \lambda)s} ds; \ \ a_3 \equiv \frac{t \int_0^t e^{-(\rho - \lambda)s} \left(1 - e^{-\nu s}\right) ds}{1 - e^{-\nu t}}.$$

This more algebra-intensive exercise does not change the conclusion: the two summary statistics differ because the former excludes the initial jump in consumption. It is then obvious that at $\omega = \bar{\omega}$ the derivative of $U^*(0,t)$ with respect to ω is still positive and welfare is still rising. An intriguing property of the expression above is that it includes only items that in principle are observable so that one can do quick, back-of-the-envelope welfare calcuations.

5.4   The reallocation

One is interested in knowing how the outcomes above relate to the economy's reallocation of labor across sectors. Using (16) we have

$$\frac{L_M}{L} = y \frac{\epsilon - 1}{\epsilon} S_X^M \left(1 - S_M^R\right).$$

It is then easy to show that (see the Appendix)

$$\frac{d}{d\omega} \left(\frac{L_M}{L}\right) > 0 \quad \forall \omega,$$

so that, intuitively, resource abundance yields a reallocation of labor from manufacturing to primary production. This result implies that

$$\frac{L_X}{L} + \frac{L_Z + L_N}{L} = 1 - \frac{L_M}{L}$$

falls to its new steady state value when ω increases.

   This is an interesting property as it says that there is a reallocation of labor from production of manufacturing goods to production of materials, but that this reallocation is *not necessarily* associated to a TFP slowdown. This is a fundamental difference between this model and models that generate the curse of natural resources by tying productivity growth to manufacturing employment through learning by doing mechanisms.

   Since the inter-sectoral reallocation is instantaneous, the dynamics that drive the time path of TFP take place within manufacturing. Equations (4), (14) and the formulation of the entry cost yield that the R&D share of

employment is

$$\frac{L_Z + L_N}{L} = \frac{L_Z}{L} + \frac{\dot{N}}{L} \cdot \beta \frac{Y}{N}$$

$$= \left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] y - \phi n.$$

Recall that y jumps on impact to its new steady state value y^* while n is predetermined and does not jump. Then, using (24) at any time t we have

$$\frac{L_Z + L_N}{L} = \left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] y^* - \frac{\phi n^*}{1 + e^{-\nu t}\Delta^*}.$$

The new steady state value is

$$\left(\frac{L_Z + L_N}{L}\right)^* = y^* \left[\left[\frac{1}{\epsilon} - \beta\left(\rho - \lambda + \delta\right)\right] - \phi\frac{n^*}{y^*}\right],$$

where we know from the analysis of section 3 that $\frac{n^*}{y^*}$ is independent of ω. Thus, when the economy experiences a growth acceleration (deceleration) because y^* rises (falls), the R&D share of employment jumps up (down) and converges from above (below) to a permanently higher (lower) value. Obviously, then, the ratio $\frac{L_X}{L}$ jumps down (up) and converges from below (above) to a permanently lower (higher) value.

5.5   Other interesting considerations

It is also interesting to study in some detail the role of measures of abundance and specialization. I consider the following two measures of resource abundance that are close analogs to measures that have been studied in the empirical literature: the value of natural wealth as a share of GDP and the value of natural wealth per capita. Notice that since $A = \beta Y$ and y is constant, in this economy (nominal) GDP is

$$GDP = Y + \dot{A} = Y + \beta \dot{Y} = Y\left(1 + \beta\lambda\right).$$

Then, my two measures of abundance are:

$$\frac{p\Omega}{L} = p\omega;$$

$$\frac{p\Omega}{GDP} = \frac{p\omega}{\left(1 + \beta\lambda\right) y}.$$

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Differentiation yields:

$$\frac{d\left(p\omega\right)}{d\omega} = \left(\omega + 1\right)p\omega\Gamma\frac{dp}{d\omega};$$

$$\frac{d}{d\omega}\left(\frac{p\Omega}{GDP}\right) = \frac{1}{\left(1 + \beta\lambda\right)y}\left[\frac{d\left(p\omega\right)}{d\omega} - \frac{p\omega}{y}\frac{dy}{d\omega}\right] = \frac{p\omega}{\left(1 + \beta\lambda\right)y}\Gamma\frac{dp}{d\omega}.$$

Notice how for both measures the effect of a change in ω *necessarily* has the same sign as the effect on expenditure,

$$\Delta^* = \frac{1}{y^*\left(\omega\right)}\frac{dy^*\left(\omega\right)}{d\omega} = \omega\Gamma\frac{dp}{d\omega}.$$

The model, in other words, predicts a positive correlation between natural wealth (per unit of GDP or per capita) and growth whether the "curse" occurs or not. It thus suggests that the positive correlation reported in the papers mentioned in the introduction does not provide sufficient information to assess the underlying economic mechanism — which here is driven by an income effect in which the adjustment of the resource price plays a crucial role — without specifying further what restrictions one imposes on the endogenous variables that show up in these measures.

For example, Brunneschewiler and Bulte (2007) note that there is a crucial difference between the two measures of abundance in that the latter has an endogenous variable at the denominator while the former does not. They argue that institutional factors drive down both the level and the growth rate of GDP per capita, so that it is quite possible that the ratio $\frac{p\Omega}{GDP}$ rises and regressions that do not control for its endogeneity reveal a negative correlation between growth and this measure of abundance. They thus suggest that one should use the former measure because it is not subject to this problem, and they show that doing so produces a positive correlation with growth. From this result they infer that the "curse" does not exist. The premise of this line of reasoning — that the endogeneity of the level of GDP at the denominator of the second measure of abundance must be taken into account — is absolutely correct. My analysis, however, shows that the first measure is not immune from endogeneity issues either, since it contains the price of the natural resource. Thus, the inference that the "curse" does not exist is correct if we know that the price effect is zero so that an increase in ω surely yields higher resource income. To my knowledge, the question of whether the data on natural capital used in this type of regressions supports such an assumption has not been investigated.[15]

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[15] This strikes me as a first-order question since much of the debate hinges on the per-

It is also interesting to study the value of primary production as a share of GDP since it is a relatively close analog of the export-based measures of specialization considered in the literature. Using (12) and (15), I have

$$\frac{P_M M}{GDP} = \frac{\frac{\epsilon-1}{\epsilon}}{1 + \beta\lambda} S_X^M$$

so that

$$\frac{d}{d\omega}\left(\frac{P_M M}{GDP}\right) = \frac{\frac{\epsilon-1}{\epsilon}}{1 + \beta\lambda}\frac{dS_X^M}{dP_M}\frac{dP_M}{dp}\frac{dp}{d\omega} = \frac{\frac{\epsilon-1}{\epsilon}}{1 + \beta\lambda}\left(1 - \epsilon_X^M\right)\frac{dp}{d\omega}.$$

Thus, if $1 < \epsilon_X^M$ this derivative is positive and the increase of the share is associated to either a growth acceleration or a growth deceleration depending on what happens to y, i.e., on whether $\Gamma < 0$ or $\Gamma > 0$. Similarly, if $1 > \epsilon_X^M$ this derivative is negative and the decrease of the share is associated to either a growth acceleration or a growth deceleration depending on whether $\Gamma > 0$ or $\Gamma < 0$. The model, in other words, says that the sign of the correlation between growth and the share of the primary sector in GDP depends on the properties of the overall pattern of substitution in manufacturing and resources processing and one cannot interpret the results of regression exercises without further information on the function $\Gamma(p)$ and its individual components, in particular, $\epsilon_X^M(p)$. The important point, therefore, is that the correlation between growth and the share of primary production in GDP does not provide sufficient information to assess the underlying mechanism.

Notice, however, that Γ and its individual components are observable so that the model provides specific guidelines for the derivation of testable predictions. The most obvious is that one should sort economies according to ϵ_X^M and Γ as the sign of the regression coefficient depends on these values. For example, if we stipulate (and document empirically) that demand for processed resources is elastic in all economies in the sample, then the discriminating factor is whether the elasticity of demand for raw resources in the primary sector, ϵ_M^R, and the share of resources in primary producers' costs, S_M^R, yield $\Gamma > 0$ or $\Gamma < 0$.

---

formance of oil, gas or mineral exporting countries. Is it a fact that the discovery of new reserves of some resource in some country does *not* affect the world price of the resource? Even under price-taking behavior by the country's suppliers, the country's overall effect on the world price can be non-negligible.

6   Conclusion

The debate on whether natural resource abundance is good or bad for the long-run fortunes of the economy is almost as old as economics itself. The modern incarnation, spurred by the work of Sachs and Warner (1995, 2001), hinges on the sign of the coefficient of measures of resource abundance in growth regressions. Differently from the past, thus, it is much more driven by solid and systematic econometric work. This is the clearest benefit of the comprehensive data sets now available. As this evidence accumulates and adds to the already existing large body of historical information and analyses, a definitive answer might finally emerge. We are not there yet, and the debate rages on.

One striking feature of this debate — and of the associated theoretical literature — is that at its heart is a tacit assumption $growth = welfare$ that is rarely, if ever, taken to task. Yet, the notion that growth and welfare are different is the cornerstone of modern growth theory. Moreover, natural resource abundance is positively related to initial income in the very data that researchers use to debate whether it is positively or negatively related to subsequent growth. Accordingly, it is not obvious at all that the "curse of natural resources" is, in fact, a curse.

In this paper, I developed a model of endogenous growth that allowed me to focus on the role of the intertemporal trade-off in determining whether the effects of natural resource abundance on initial income and growth yield an increase or a decrease in welfare. It is only in the latter case that the "curse" is indeed a curse. I found that resource abundance has non-monotonic effects on growth and welfare. More precisely, growth and welfare are hump-shaped functions of resource abundance. The property that the peak of growth occurs earlier than the peak of welfare captures the crucial role of initial consumption, which rises with resource abundance, and is an important reminder that the welfare effect of resource abundance depends on the whole path of consumption, not on a summary statistic of its slope. This reminder applies to all explanations of the "curse" — institution-based, reallocation-based or any other type that future researchers will develop.

If resource abundance means that we sacrifice growth to boost current consumption, it stands to reason that to figure out whether we are better or worse off requires us to resolve the intertemporal trade-off. Consequently, growth regressions that ignore the effect of resource abundance on initial income provide only one piece of the puzzle, not the solution. This is not a simple restatement of the fact that correlation is not causation. We cannot infer from the negative correlation between resource abundance and growth

that resource abundant economies are worse off even if we can establish beyond reasonable doubt that the relation is indeed negative and causal.

7 Appendix

7.1 The typical firm's behavior

To characterize the typical firm's behavior, consider the Current Value Hamiltonian

$$CVH_i = [P_i - C_X(1, P_M)Z_i^{-\theta}]X_i - \phi - L_{Z_i} + z_i\alpha K L_{Z_i},$$

where the costate variable, z_i, is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i, is the state variable; R&D investment, L_{Z_i}, and the product's price, P_i, are the control variables. Firms take the public knowledge stock, K, as given.

Since the Hamiltonian is linear, one has three cases. The case $1 > z_i\alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest. The case $1 < z_i\alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in R&D, this case violates the general equilibrium conditions and is ruled out. The first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $1 = z_i\alpha K$, the constraint on the state variable, (8), the terminal condition,

$$\lim_{s\to\infty} e^{-\int_t^s [r(v)+\delta]dv} z_i(s)Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(1, P_M)Z_i^{-\theta-1}\frac{X_i}{z_i},$$

that defines the rate of return to R&D as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge. The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(1, P_M)Z_i^{-\theta}\frac{\epsilon}{\epsilon - 1}. \tag{33}$$

Peretto (1998, Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $1 = z_i \alpha K$ holds, and equilibrium is symmetric.

The cost function (7) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

Then, the price strategy (33), symmetry and aggregation across firms yield (11) and (12).

Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where L_Z is aggregate R&D. Taking logs and time derivatives of $1 = z_i \alpha K$ and using the demand curve (5), the R&D technology (8) and the price strategy (33), one reduces the first-order conditions to (13).

Taking logs and time-derivatives of V_i yields

$$r + \delta = \frac{\Pi_{Xi}}{V_i} + \frac{\dot{V_i}}{V_i},$$

which is a perfect-foresight, no-arbitrage condition for the equilibrium of the capital market. It requires that the rate of return to firm ownership equal the rate of return to a loan of size V_i. The rate of return to firm ownership is the ratio between profits and the firm's stock market value plus the capital gain (loss) from the stock appreciation (depreciation).

In symmetric equilibrium the demand curve (5) yields that the cost of entry is $\beta \frac{Y}{N}$. The corresponding dmand for labor in entry is $L_N = \dot{N}\beta\frac{Y}{N}$. The case $V > \beta\frac{Y}{N}$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta\frac{Y}{N}$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $\dot{N} = 0$. Free-entry requires $V = \beta\frac{Y}{N}$. Using the price strategy (33), the rate of return to entry becomes (14).

7.2 The economy's resources constraint

I now show that the household's budget constraint reduces to the economy's labor market clearing condition. Starting from (2), recall that $A = NV$ and $(r + \delta)V = \Pi_X + \dot{V}$. Substituting into (2) yields

$$\dot{N}V = N\Pi_X + L + p\Omega + \Pi_M - Y.$$

Observing that $N\Pi_X = NPX - L_X - L_Z - P_M M$, $NPX = Y$, $\Pi_M = P_M M - L_M - pR$, $R = \Omega$, and that the free entry condition yields that total employment in entrepreneurial activity is $L_N = \dot{N}V$, this becomes

$$L = L_N + L_X + L_Z + L_M.$$

7.3   Proof of Proposition 1

Taking logs of (25) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s)\, ds + \frac{1}{\epsilon - 1} \log N(t).$$

Using the definition $n \equiv N e^{-\lambda t}$ and adding and subtracting $\hat{Z}^*$ from $\hat{Z}(t)$ yields

$$\log T(t) = \theta \log Z_0 + gt + \theta \int_0^t \left[\hat{Z}(s) - \hat{Z}^* \right] ds + \frac{1}{\epsilon - 1} \log n(t).$$

Using (19) the integral becomes

$$\begin{aligned}
\theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^* \right) ds &= \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \int_0^t \left(\frac{y^*}{n(s)} - \frac{y^*}{n^*} \right) ds \\
&= \gamma \int_0^t \left(\frac{n^*}{n(s)} - 1 \right) ds,
\end{aligned}$$

where

$$\gamma \equiv \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{y^*}{n^*} = \theta \frac{\alpha\theta(\epsilon - 1)}{\epsilon} \frac{\phi - \frac{\rho + \delta}{\alpha}}{\frac{1 - \theta(\epsilon - 1)}{\epsilon} - \rho\beta}.$$

Finally, (24) and the definition of Δ^* yield

$$\begin{aligned}
\theta \int_0^t \left(\hat{Z}(s) - \hat{Z}^* \right) ds &= \gamma \int_0^t e^{-\nu s} \left(\frac{n^*}{n_0} - 1 \right) ds \\
&= \frac{\gamma}{\nu} \Delta^* \left(1 - e^{-\nu t} \right).
\end{aligned}$$

Using this result and (24) again, and noting that by construction $T_0 = Z_0^\theta n_0^{\frac{1}{\epsilon - 1}}$ since $n_0 = N_0 e^{-\lambda \cdot 0} = N_0$, yields

$$\log T(t) = \log T_0 + g^* t + \frac{\gamma \Delta^*}{\nu} \left(1 - e^{-\nu t} \right) + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t}\Delta^*}.$$

Next consider

$$\log u^* (t) = \log \frac{\epsilon - 1}{\epsilon} + \log \left(\frac{y^*}{c^*} \right) + \log T (t)$$

and use the expression just derived to write

$$\log u^* (t) = \log \frac{y^*}{c^*} + g^* t + \frac{\gamma \Delta^*}{\nu} \left(1 - e^{-\nu t} \right) + \frac{1}{\epsilon - 1} \log \frac{1 + \Delta^*}{1 + e^{-\nu t} \Delta^*}.$$

Substituting this expression into (1) yields

$$
\begin{aligned}
U^* &= \int_0^\infty e^{-(\rho - \lambda) t} \log u^* (t) \, dt \\
&= \int_0^\infty e^{-(\rho - \lambda) t} \left[\log \left(\frac{y^*}{c^*} \right) + g^* t \right] dt \\
&\quad + \frac{\gamma}{\nu} \Delta^* \int_0^\infty e^{-(\rho - \lambda) t} \left(1 - e^{-\nu t} \right) dt \\
&\quad + \frac{1}{\epsilon - 1} \int_0^\infty e^{-(\rho - \lambda) t} \log \frac{1 + \Delta^*}{1 + e^{-\nu t} \Delta^*} dt.
\end{aligned}
$$

The first and second integrals have straightforward closed form solutions; the third has a complicated solution involving the hypergeometric function. For my purposes, it is useful to introduce the following approximation that allows me to simplify the expression for welfare. Since in general $\log (1 + x) \simeq x$, I can rewrite

$$\log \frac{1 + \Delta^*}{1 + e^{-\nu t} \Delta^*} = \log \left(1 + \Delta^* \right) - \log \left(1 + e^{-\nu t} \Delta^* \right) = \Delta^* \left(1 - e^{-\nu t} \right),$$

which yields (26), (27) and

$$
\begin{aligned}
U^* &= \int_0^\infty e^{-(\rho - \lambda) t} \left[\log \left(\frac{y^*}{c^*} \right) + g^* t \right] dt \\
&\quad + \left(\frac{\gamma}{\nu} + \frac{1}{\epsilon - 1} \right) \Delta^* \int_0^\infty e^{-(\rho - \lambda) t} \left(1 - e^{-\nu t} \right) dt,
\end{aligned}
$$

which upon integration yields (28).

7.4 Proof of Lemma 2

Differentiating and manipulating terms yields:

$$
\begin{aligned}
\frac{\partial\left(S_X^M S_M^R\right)}{\partial p} &= \frac{\partial S_X^M}{\partial p} S_M^R + \frac{\partial S_M^R}{\partial p} S_X^M \\
&= \frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M}\cdot\frac{\partial P_M}{\partial p}\frac{p}{P_M}\cdot\frac{S_X^M S_M^R}{p} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}\cdot\frac{S_M^R S_X^M}{p} \\
&= \left[\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M}\cdot\frac{\partial P_M}{\partial p}\frac{p}{P_M} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}\right]\frac{S_M^R S_X^M}{p} \\
&= \left[\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M}\cdot\frac{\partial C_M}{\partial p}\frac{p}{C_M} + \frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R}\right]\frac{S_M^R S_X^M}{p}.
\end{aligned}
$$

Recalling that

$$
\frac{\partial S_X^M}{\partial P_M}\frac{P_M}{S_X^M} = 1 - \epsilon_X^M,
$$

$$
\frac{\partial S_M^R}{\partial p}\frac{p}{S_M^R} = 1 - \epsilon_M^R,
$$

$$
\frac{\partial C_M}{\partial p}\frac{p}{C_M} = S_M^R
$$

and substituting into the expression above yields (29).

7.5 Proof of Proposition 3

Refer to Figure 1. In the case of complementarity, depicted in the upper panel, we have the following pattern. For $\omega \to 0$ the expenditure line (17) is almost, but not quite, flat and intersects the income relation (18) for $p \to \infty$ and $y \to y^*(\infty)$. As ω grows, the expenditure line rotates counterclockwise and the intersection shifts left, tracing the income relation. We thus obtain that both p^* and y^* fall. As $\omega \to \infty$, the expenditure line becomes vertical and the intersection occurs at $p \to 0$ and $y \to y^*(0)$.

In the case of substitution in the lower panel, we have a similar pattern with the difference that the income relation now has negative slope so that as the expenditure line rotates counterclockwise y^* increases. Specifically, for $\omega \to 0$ the expenditure line is almost, but not quite, flat and intersects the income relation for $p \to \infty$ and $y \to y^*(\infty)$. As ω grows, the expenditure line rotates counterclockwise and the intersection shifts left yielding that

p^* falls while y^* rises. As $\omega \to \infty$, the expenditure line becomes vertical and the intersection occurs at $p \to 0$ and $y \to y^*(0)$.[16]

7.6 Proof of Proposition 4

Refer to Figure 2. The proof is essentially the same as above. The only difference is that as the expenditure line rotates it traces the hump-shaped income relation yielding that p^* always falls while y^* rises for $0 < \omega < \bar{\omega}$ and falls for $\bar{\omega} < \omega < \infty$.

7.7 Proof of Proposition 5

Observe first that

$$\frac{d\log\left(\frac{y}{c}\right)}{d\omega} = \frac{d\log y}{d\omega} - \frac{d\log c}{d\omega}.$$

Now,

$$\frac{d\log y}{d\omega} = \frac{d\log y}{dp}\frac{dp}{d\omega} = \omega\Gamma(p)\frac{dp}{d\omega}$$

and

$$
\begin{aligned}
\frac{d\log c}{d\omega} &= \frac{1}{C_X}\frac{dC_X}{d\omega} = \frac{1}{C_X}\frac{dC_X}{dP_M}\frac{dP_M}{dp}\frac{dp}{d\omega}\\
&= \frac{P_M}{C_X}\frac{dC_X}{dP_M}\frac{dP_M}{dp}\frac{p}{P_M}\frac{1}{p}\frac{dp}{d\omega}\\
&= \frac{S_X^M S_M^R}{p}\frac{dp}{d\omega}.
\end{aligned}
$$

---

[16] An interesting question is whether the model admits an equilibrium for $\omega = 0$. To investigate whether it does, it is useful to focus on the CES case considered in the text. The answer depends on wheteher resources are essential. Under $\sigma_M < 0$, $\sigma_X < 0$ they are and thus an equlibrium with $\omega = 0$ (due to $\Omega = 0$) cannot exist. To see this, notice that in this case $S_X^M(0)S_M^R(0) = 0$ and $S_X^M(\infty)S_M^R(\infty) = 1$ so that the representation in the upper panel of Figure 1 applies with the difference that the expenditure and income relations have the intercept $y^*(0)$ in common. (This point, obviously, is not an equilibrium.) We then have that for $\omega = 0$ the expenditure line is exactly flat and there is no intersection with the income relation. If instead we posit $\sigma_M > 0$, $\sigma_X > 0$ we have $S_X^M(0)S_M^R(0) > 0$ and $S_X^M(\infty)S_M^R(\infty) = 0$ and the representation in the lower panel of Figure 1 applies with the crucial difference that the expenditure line starts out at $y^*(\infty)$. Hence, for $\omega = 0$ the line is exactly tangent to the horizontal asymptote of the income relation implying that $p = \infty$ and $y^*(\infty)$ *is* a feasible equilibrium. This is because we now allow for substitution so that processed natural resources are *not* essential in manufacturing and the economy *can* exist with $\Omega = 0$.

Hence,
$$\frac{d\log\left(\frac{y}{c}\right)}{d\omega} = \left(\Gamma - \frac{S_X^M S_M^R}{p\omega}\right)\omega\frac{dp}{d\omega}.$$

which using (12) becomes (30).

7.8 Proof of Proposition 6

Observe that by construction
$$\frac{d\Delta^*}{d\omega} = \frac{1}{y^*}\frac{dy^*}{d\omega}$$

Differentiation of (28) then yields

$$
\begin{aligned}
\frac{dU^*}{d\omega} &= \frac{1}{\rho-\lambda}\left[\frac{d\log\left(\frac{y^*}{c^*}\right)}{d\omega} + \mu\frac{1}{y^*}\frac{dy^*}{d\omega}\right]\\
&= \frac{1}{\rho-\lambda}\left[(1+\mu)\frac{d\log y^*}{d\omega} - \frac{d\log c^*}{d\omega}\right].
\end{aligned}
$$

Using the expressions calculated above and rearranging terms yields (31).

7.9 The CES economy with $\sigma_X > 0$ and $\sigma_M < 0$

The $\Gamma(\omega)$ curve. Dropping stars to simplify notation, the equation is:

$$\Gamma(\omega) = \left[-\frac{\sigma_X}{1-\sigma_X}\left(1 - S_X^M(p(\omega))\right) + \frac{\sigma_M}{1-\sigma_M}\right]S_M^R(p(\omega)) - \frac{\sigma_M}{1-\sigma_M}$$

Proposition 4 yields that $p(0) = \infty$ and $p(\infty) = 0$. Therefore, for $\omega \to 0$:

$$S_M^R(0) = \lim_{p\to\infty}\frac{1}{1 + \left(\frac{\psi_M}{1-\psi_M}\right)^{\frac{1}{1-\sigma_M}}p^{\frac{\sigma_M}{1-\sigma_M}}} = 1.$$

Also,

$$P_M(0) = \lim_{p\to\infty}\left[\psi_M^{\frac{1}{1-\sigma_M}} + (1-\psi_M)^{\frac{1}{1-\sigma_M}}p^{\frac{\sigma_M}{\sigma_M-1}}\right]^{\frac{\sigma_M-1}{\sigma_M}} = \infty,$$

so that
$$S_X^M(0) = \lim_{P_M\to\infty}\frac{1}{1 + \left(\frac{\psi_X}{1-\psi_X}\right)^{\frac{1}{1-\sigma_X}}P_M^{\frac{\sigma_X}{1-\sigma_X}}} = 0.$$

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Consequently,

$$\Gamma(0) = \frac{-\sigma_X}{1 - \sigma_X} < 0.$$

In contrast, for $\omega \to \infty$:

$$S_M^R(\infty) = \lim_{p \to 0} \frac{1}{1 + \left(\frac{\psi_M}{1 - \psi_M}\right)^{\frac{1}{1 - \sigma_M}} p^{\frac{\sigma_M}{1 - \sigma_M}}} = 0.$$

Also,

$$P_M(\infty) = \lim_{p \to 0} \left[\psi_M^{\frac{1}{1 - \sigma_M}} + (1 - \psi_M)^{\frac{1}{1 - \sigma_M}} p^{\frac{\sigma_M}{\sigma_M - 1}}\right]^{\frac{\sigma_M - 1}{\sigma_M}} = \psi_M^{-\frac{1}{\sigma_M}},$$

so that

$$S_X^M(\infty) = \frac{1}{1 + \left(\frac{\psi_X}{1 - \psi_X}\right)^{\frac{1}{1 - \sigma_X}} \psi_M^{-\frac{1}{\sigma_M} \frac{\sigma_X}{1 - \sigma_X}}}.$$

Consequently,

$$\Gamma(\infty) = \frac{-\sigma_M}{1 - \sigma_M} > 0.$$

The next step is to show that the curve is monotonically increasing:

$$\Gamma'(\omega) = \underbrace{\frac{\sigma_X}{1 - \sigma_X}}_{+} \cdot \underbrace{\frac{dS_X^M}{dP_M}}_{-} \cdot \underbrace{\frac{dP_M}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-} \cdot S_M^R$$

$$+ \underbrace{\left[-\frac{\sigma_X}{1 - \sigma_X}\left(1 - S_X^M\right) + \frac{\sigma_M}{1 - \sigma_M}\right]}_{-} \cdot \underbrace{\frac{dS_M^R}{dp}}_{+} \cdot \underbrace{\frac{dp}{d\omega}}_{-}$$

$$> 0.$$

Finally, notice that by continuity there exists a value $\bar{\omega}$ where $\Gamma(\bar{\omega}) = 0$.

The $\Psi(\omega)$ curve. Again dropping stars, the equation is:

$$\Psi(\omega) = \kappa - S_M^R(p(\omega)) S_X^M(p(\omega)).$$

The limiting behavior at 0 and ∞ is straightforward. The previous calculations yield:

$$\Psi(0) = \kappa - S_M^R(0) S_X^M(0) = \kappa;$$

$$\Psi(\infty) = \kappa - S_M^R(\infty) S_X^M(\infty) = \kappa.$$

Next observe that

$$
\begin{aligned}
\Psi'(\omega) &= \frac{d\left(S_M^R\left(p\left(\omega\right)\right)S_X^M\left(p\left(\omega\right)\right)\right)}{dp\left(\omega\right)}\frac{dp\left(\omega\right)}{d\omega} \\
&= \Gamma\left(\omega\right)\frac{S_M^R\left(p\left(\omega\right)\right)S_X^M\left(p\left(\omega\right)\right)}{p\left(\omega\right)}\frac{dp\left(\omega\right)}{d\omega},
\end{aligned}
$$

so that the curve is hump-shaped with its maximum exactly at the value $\bar\omega$ where $\Gamma(\omega)$ changes sign.

The threshold values. Observe that $\kappa > 1 > \frac{-\sigma_M}{1-\sigma_M}$. It is evident from Figure 2 then that $\Gamma(\omega) < \Psi(\omega)\ \forall\omega$. It follows that there is only one relevant intersection, of $\Psi(\omega)$ with $(1+\mu)\,\Gamma(\omega)$, that yields the threshold value $\tilde\omega$ such that $\bar\omega < \tilde\omega$.

7.10   The reallocation

The expression for the share of employment in the primary sector and the expressions derived in the proofs above yield

$$
\begin{aligned}
\frac{d}{d\omega}\left(\frac{L_M}{L}\right) &= \frac{\epsilon-1}{\epsilon}\left[\frac{dy}{d\omega}S_X^M\left(1-S_M^R\right)+y\frac{dS_X^M}{d\omega}-\frac{d\left(S_X^M S_M^R\right)}{d\omega}\right] \\
&= \frac{\epsilon-1}{\epsilon}\left[\frac{\Gamma}{\Psi}S_X^M\left(1-S_M^R\right)+1-\epsilon_X^M-\Gamma\right]\omega\frac{dp}{d\omega}.
\end{aligned}
$$

Recall that $\frac{dp}{d\omega} < 0$. Then

$$
\frac{d}{d\omega}\left(\frac{L_M}{L}\right) > 0 \quad \forall\omega
$$

because the term in brackets is

$$
\frac{\Gamma}{\Psi}\left(S_X^M-\kappa\right)+1-\epsilon_X^M < 0
$$

since $\kappa > 1$ and $1-\epsilon_X^M < 0$ under the assumption that manufacturing exhibits substitution between labor and materials.

References

[1] Alexeev M. and Conrad R., 2007, The Elusive Curse of Oil, Department of Economics, Indiana University, manuscript (available at http://mypage.iu.edu/~malexeev/alexeev_conrad_rev_5-07.pdf).

[2] Banks R.B., 1994, *Growth and Diffusion Phenomena*, Berlin Heidelberg, Springer-Verlag.

[3] Boyce J.R. and Emery J.C.H, What Can Exhaustible Resource Theory Tell Us About Per Capita Income Growth and Levels in Resource Abundant Economies?, Department of Economics, University of Calgary, manuscript (available at http://econ.ucalgary.ca/profiles/john-boyce).

[4] Bravo-Ortega C. and de Gregorio J., 2007, The Relative Richness of the Poor? Natural Resources, Human Capital, and Economic Growth, in *Natural Resources: Neither Curse Nor Destiny*, edited by D. Lederman and W.F. Maloney, Stanford University Press.

[5] Brunneschweiler C.N. and Bulte E.H., 2007, The Resource Curse Revisited and Revised: A tale of Paradoxes and Red Herrings, *Journal of Environmental Economics and Management* (forthcoming).

[6] Etro F., 2004, Innovation by Leaders, *Economic Journal*, 114, 281-303.

[7] Gylfason T., 2001a, Lessons from the Dutch Disease: Causes, Treatment, and Cures, manuscript (available at http://www.hi.is/~gylfason/pdf/statoil22.pdf).

[8] Gylfason T., 2001b, Natural Resources, Education, and Economic Development, *European Economic Review*, 45, 847-859.

[9] Lederman D. and Maloney W.F. (eds.), 2007a, *Natural Resources: Neither Curse Nor Destiny*, Stanford University Press.

[10] Lederman D. and Maloney W.F., 2007b, Trade Structure and Growth, in *Natural Resources: Neither Curse Nor Destiny*, edited by D. Lederman and W.F. Maloney, Stanford University Press.

[11] Papyrakis E. and Gerlagh R., 2003, Natural Resources: A Blessing or a Curse?, Fondazione Eni Enrico Mattei, Nota di Lavoro 8.2003 (available at http://www.feem.it/web/activ/_wp.html).

[12] Papyrakis E. and Gerlagh R., 2007, Resource Abundance and Economic Growth in the United States, *European Economic Review*, 51, 1011-1039.

[13] Peretto, P.F., 1998,. Technological Change and Population Growth, *Journal of Economic Growth*, 3, 283-311.

[14] Peretto P.F. and Connolly M., 2007, The Manhattan Metaphor, *Journal of Economic Growth*, 12, 329-350.

[15] Peretto P.F. and Smulders S., 2002, Technological Distance, Growth and Scale Effects, *Economic Journal*, 112, 603-624.

[16] Rodriguez F. and Sachs J.D., 1999, Why Do Resource-Abundant Economies Grow More Slowly?, *Journal of Economic Growth*, 4, 277-303.

[17] Sachs J.D. and Warner A.M., 1995, Natural Resource Abundance and Economic Growth, NBER WP 5398.

[18] Sachs J.D. and Warner A.M., 2001, The Curse of Natural Resources, *European Economic Review*, 45, 827-838.

[19] Sala-i-Martin X., 1997, I Just Run 2 Million Regressions, *American Economic Review Papers and Proceedings*, 87, 178-83

[20] Sala-i-Martin X., Doppelhofer G. and Miller R.I., 2004, Determinants of Long-Run Growth: A Bayesian Averaging of Classical Estimates (BACE) Approach, *American Economic Review*, 94, 813-835.

[21] Smulders S., 2005, Endogenous Technological Change, Natural resources, and Growth, in *Scarcity and Growth Revisited: Natural Resources and the Environment in the New Millennium*, edited by H. Folmer and L. Gabel L., The Johns Hopkins University Press for Resources for the Future.

[22] Stevens P., 2003, Resource Impact – Curse or Blessing? A Literature Survey, *Journal of Energy Literature*, IX, 3-42.
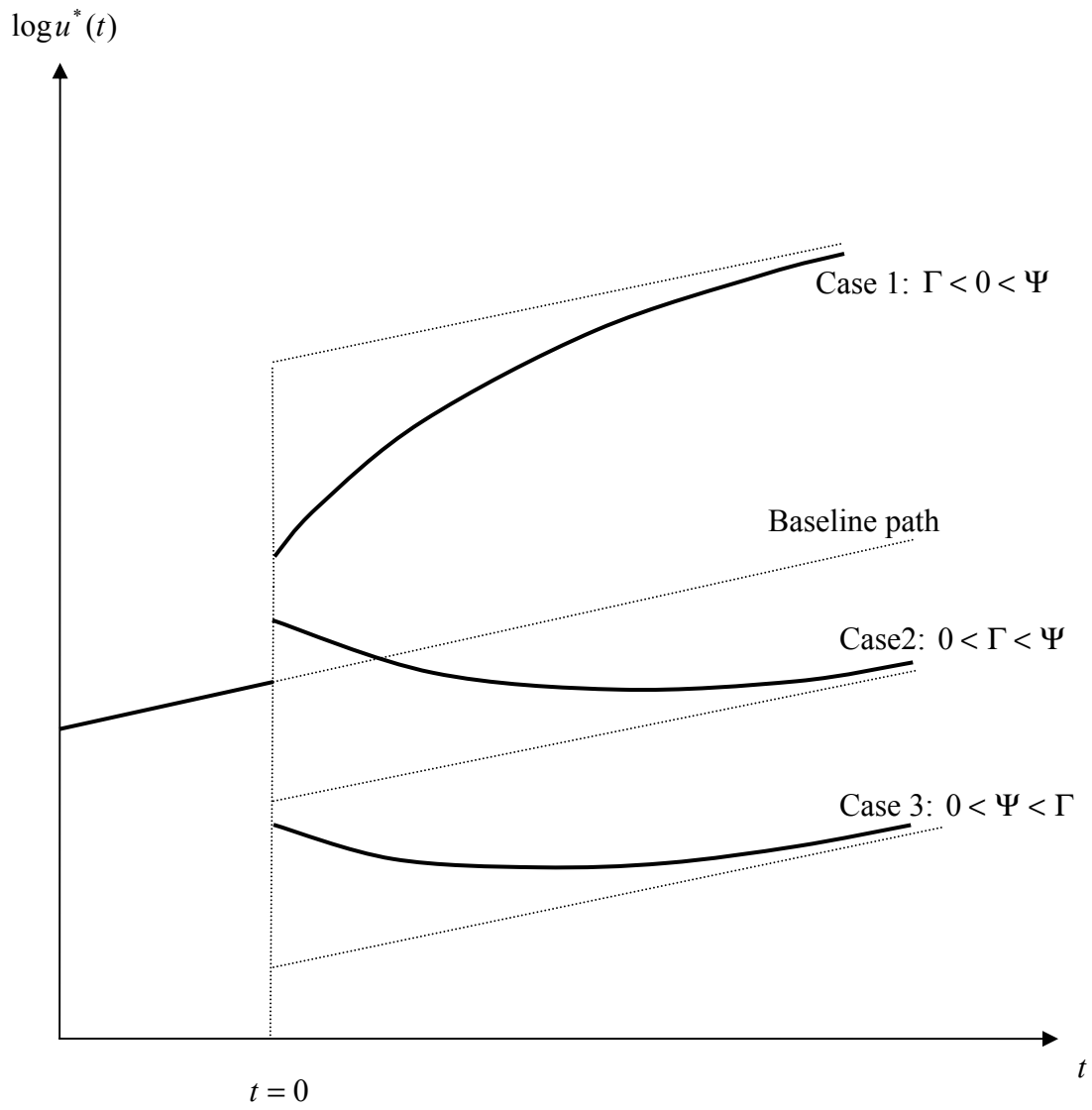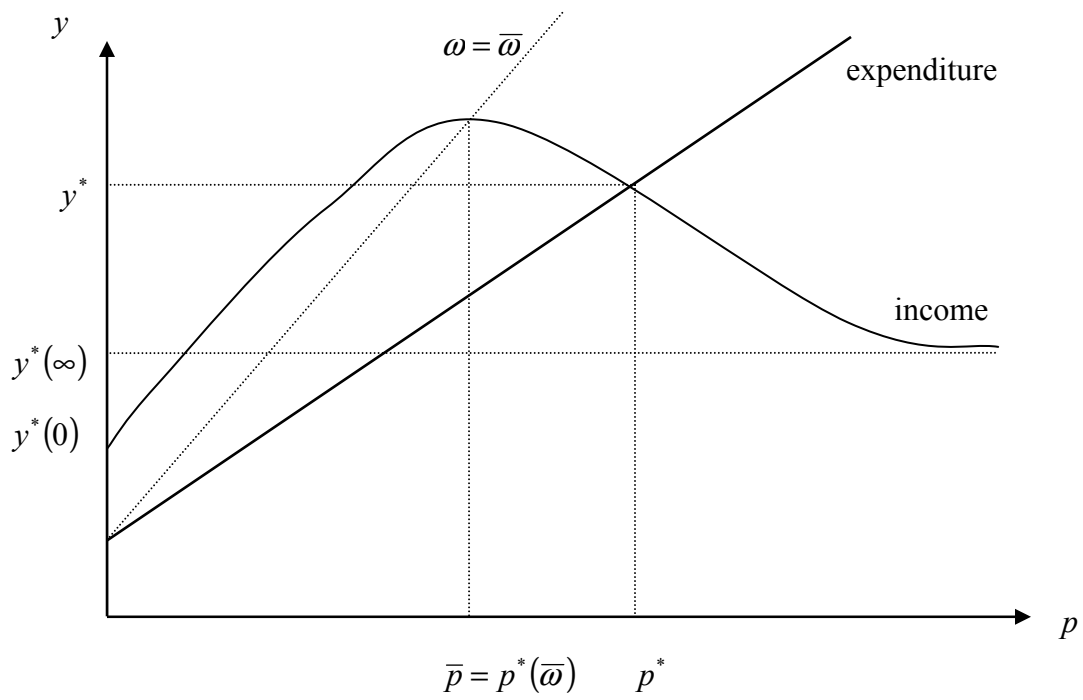
Figure 3: The path of consumption

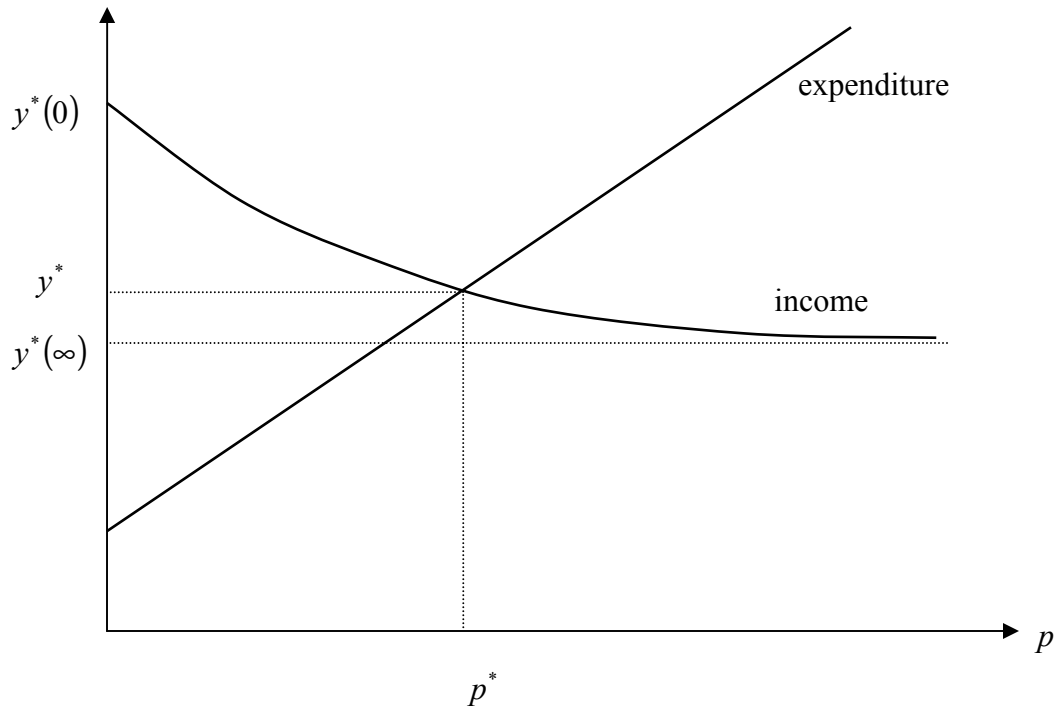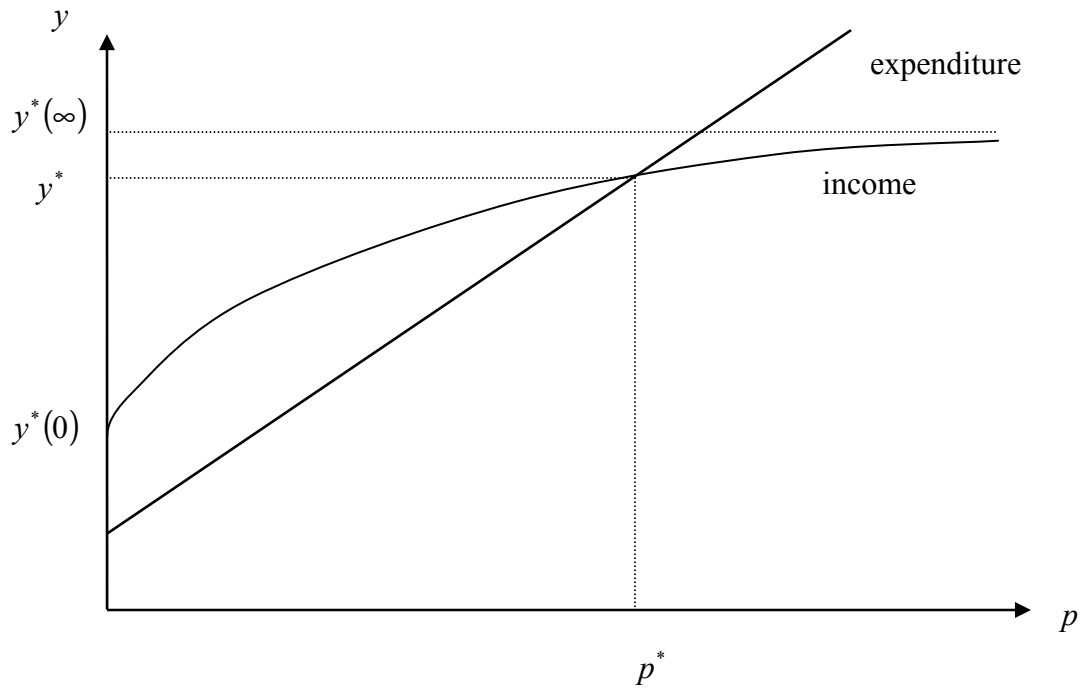Figure 2: General Equilibrium: Hump-shaped income relation
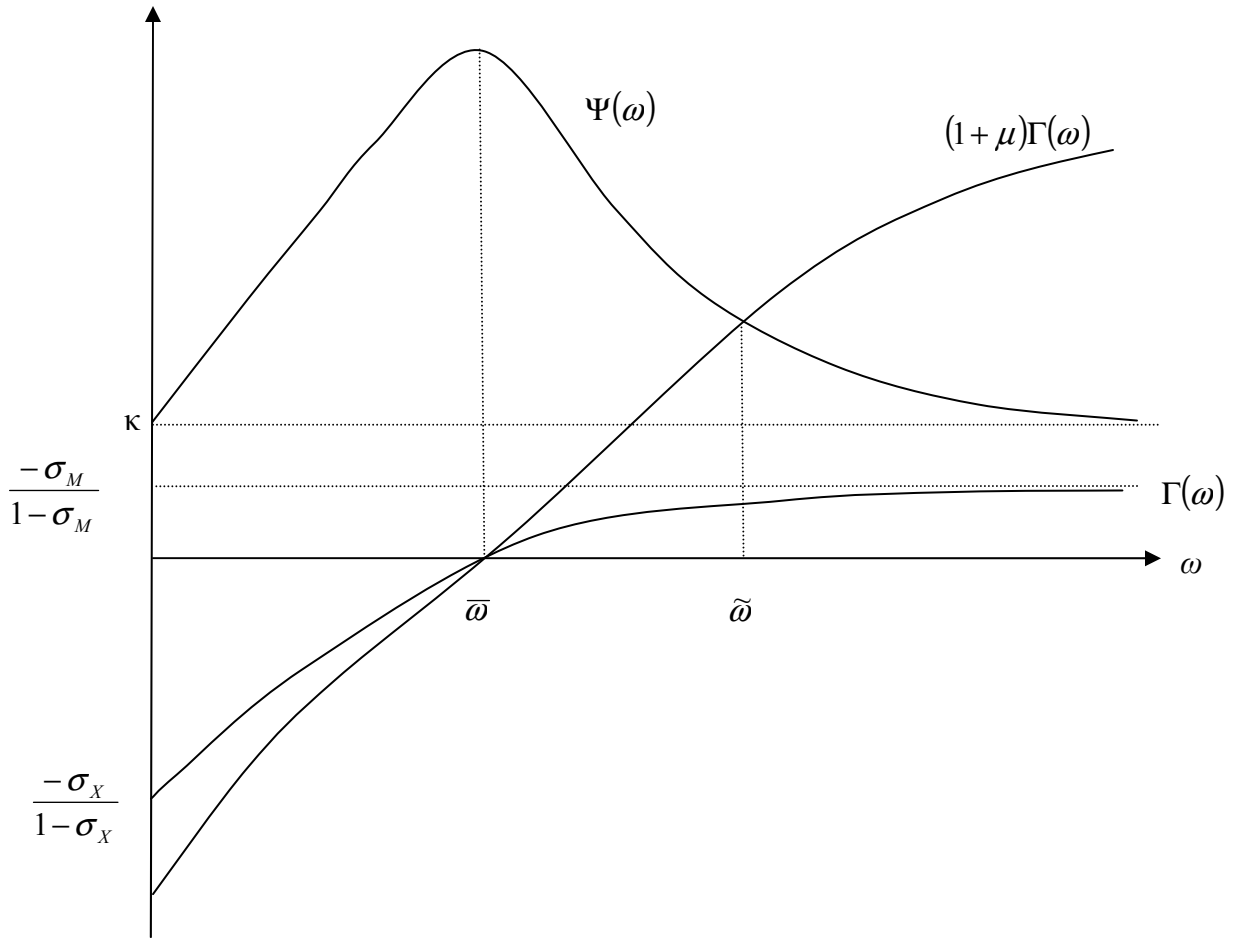
Figure 1: General Equilibrium: Monotonic income relation

Figure 4: The endowment ratio and equilibrium outcomes