Contagion of Liquidity Crisis between Firms

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In this paper, I present a model in which contagion of the liquidity crisis between non-financial institutions occurs due to the co-creditors' learning behavior. The fact that different firms share the same group of creditors leads to the transmission of information on the type of other creditors from one firm to the other. That is, after observing what happens in one firm, creditors get to conjecture the type of other creditors. And when they decide their actions in another firm, creditors reflect this information. Without common knowledge of each firm's fundamentals, each firm's switching equilibrium - the threshold for the liquidity crisis there - is uniquely determined and particularly the latter firm's equilibrium is affected by creditors' updated beliefs on the other creditors' type. Analysis of these switching equilibria shows that the contagion of the liquidity crisis from the firm having a small possibility to fail is more propagable than otherwise. Comparative statics analysis provides policy proposals to mitigate the severity of contagion on the liquidity crisis in the latter firm. (JEL G33, G38, D82, D83)

Key Words: Contagion; Liquidity crisis; Global games; Learning; Collateral; Government's bailout; Information structure; Financial disclosure

1. INTRODUCTION

Contagion is the propogation of solvency problems encountered by a single institution to other institutions. It is one of the most striking features of the financial crisis in that the crisis spreads across countries and institutions. In late 1990's, most of east Asian countries suffered the severe financial crises via contagion among countries. It was called "Asian *Flu*." When South Korea suffered the Asian *Flu*, we observed that the liquidity crisis spread from one firm to other firms even though their businesses were not closely related. For example, in January 1997, Hanbo Steel Group - the country's fourteenth-largest conglomerate - went bankrupt, and within a few months, Jinro - the largest liquor group in Korea - failed. How can we explain this contagion phenomenon that propogates the financial distress problem from one firm to the unrelated firms? Can we just say that the entire economy's systemic risk caused that phenomenon?

[1] I started to write this paper when I took Haluk Ergin and David Levine's *Advanced Economic Theory I* (Fall 2008) at Washington University in St. Louis. I sincerely thank them for encouraging me to embark on writing this topic. I also deeply thank Hyun Song Shin. He guided me with great kindness to study global games and apply the concepts to finance. I am grateful to Phil Dybvig, Steve Fazzari, Armando Gomes, Radha Gopalan, Hong Liu, Stephen Morris, Bruce Petersen and Guofu Zhou for their helpful comments on this work. I am truly appreciative of Todd Gormley's earnest advice on this paper having regular discussions with me. All errors and omissions are my own. This paper is very preliminary as of now. Please do not distribute it without the direct permission of the author.

In this paper, I present a model in which contagion of the liquidity crisis between non-financial institutions whose businesses are not even closely connected occurs due to the co-creditors' learning behavior: learning about others' type. Some fairly extensive studies deal with the contagion of the financial crisis between financial institutions and/or international financial markets based on their interlinkages and changes in asset prices. However, studies on the contagion of the liquidity crisis between non-financial businesses have received only scant attention. My aim is to make a contribution to understanding the contagion phenomenon between non-financial institutions based on co-creditors' learning behavior and to provide policy proposals to reduce the severity of contagion.

I focus on the self-fulfilling crisis - a crisis that occurs just because creditors believe it is going to occur. This is an important feature since the liquidity crisis in a firm is often viewed as the result of a coordination failure among creditors. However, this self-fulfilling approach produces the multiple equilibria and thus it is hard to demonstrate the contagion effect in the self-fulfilling context. To get the unique equilibrium, I employ the global games method introduced by Carlsson and van Damme (1993). Global games technique has recently been applied to explanations for financial crises in a number of papers.[2] This method allows me to get the unique equilibrium in each firm, and therefore I can capture the contagion effect in which the liquidity crisis in one firm affects the likelihood of a crisis in another.

Like the contagion setting of Goldstein and Pauzner [G-P] (2004), I examine a sequential framework in which the events in firm A take place before in firm B. Creditors in my model hold loans of two firms' investment projects. In each firm, they can either roll over their loans until maturity, in which case they can get the full repayment from the firm if the investment project succeeds, or recall their loans in the interim stage, in which case they can get some prematured liquidation value (collateral debt) but less than the full repayment amount. The success of the investment project depends on the fundamentals of the firm and the number of creditors who keep rolling over their loans until maturity in that firm. That is, creditors' coordination on whether or not to roll over the loan determines whether there will be the liquidity crisis in the firm or not. There are two types of creditors. One is "pessimistic" and the other is "optimistic." The terminology "pessimistic creditors" means that they worry about the failure of the firm's investment project more than "optimistic creditors."[3]

Following the global games method, I assume that creditors do not have common knowledge of the fundamentals of each firm A and B. However, creditors get noisy signals about the firm's fundamentals after they are realized. Different type of creditors get different noisy signals. In this setting, creditors uniquely determine their beliefs on the fundamentals of firm A and their actions whether or not to roll over the loans until maturity in firm A. Before creditors determine their actions in firm B, they observe the aggregate outcomes of firm A which depend on fundamentals and the behavior of creditors there. Observing what occurs in firm A, creditors update their beliefs about other creditors' type and determine their actions in firm

[2] Morris and Shin (2003) showed many applied examples of global games to economic models.

[3] In practice, this different type reflects the strength of balance sheet of each creditor and the degree of information advantage on firm-related issues including the economic situation. That is, a creditor with a weak balance sheet and/or information disadvantage on firm-related issues is more "pessimitic" than the other who has strong balance sheet and/or information advantage on firm-related issues.

B. That is, by learning the type of other creditors in the process of firm A's case, creditors uniquely determine their beliefs on the fundamentals of firm B and their actions in firm B. If there is the liquidity crisis in firm A and firm B also suffers the liquidity crisis by the creditors' learning process, then I refer to this as "contagion" of the liquidity crisis from firm A to firm B. I show that in each firm creditors do not roll over their loans until maturity if the fundamentals of each firm are below some threshold; and most importantly, the contagion effect from firm A to firm B by creditors' learning process changes the threshold values of firm B. I refer to the increased probability of the liquidity crisis in firm B due to the contagion as the "severity of contagion" on the liquidity crisis.

Having shown the severity of contagion on the liquidity crisis from firm A to firm B, I numerically analyze the phenomenon that the severity of contagion is more serious when the originating firm's failure point – the probability that the firm's investment project fails - is lower. In other words, the contagion of the liquidity crisis from the firm having a small possibility to fail is more propagable than otherwise. This is a very striking result compared with other contagion-related papers which deal with the contagion between the international financial markets and/or financial institutions through capital linkages and asset price changes. In these papers, the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected.

Also, I analyze the policy implications addressing how to reduce the severity of contagion on the liquidity crisis from firm A to firm B by doing comparative statics. Firm B can minimize the severity of contagion from firm A by setting the small value of its collateral debt. The government can also play a role to reduce the severe contagion damage of the liquidity crisis by making the pessimistic creditors more optimistic on the firm's fundamentals (e.g., providing bailouts to the firm which suffers a transitory liquidity problem) and by reducing the degree of incomplete information on the creditors' types in the market (e.g., implementing financial disclosure policy which discloses the type of creditors). Regarding creditor's information structure, I find that even though increasing the accuracy of the creditors' information on the firm's fundamentals lowers the failure point of the individual firm, it increases the severity of contagion.

For the global games setting of the firm and creditors, I basically refer to Morris and Shin [M-S] (2004). After showing the unique equilibrium in currency attacks paper (1998), M-S (2004) analyzed the coordination game in debt market by using tools of global games. They showed that the creditors of a distressed borrower face a coordination problem, and without common knowledge of fundamentals of the distressed borrower, the incidence of failure is uniquely determined given that private information is precise enough. Bruche (2003) developed a continuous time version of M-S (2004)'s model, and Takeda and Takeda (2008) investigated the role of large creditors in determining the price of corporate bonds based on M-S (2004). However, these papers are not concerned with the contagion, which is the central issue of this paper.

There are several leading explanations about the contagion phenomenon.[4] A first widely mentioned mechanism of the contagion is the existence of capital links between financial institutions. Allen and Gale (2000) explained the systemic risk in the banking industry with the introduction of interbank markets, and Cifuentes, Ferrucci and Shin (2005) dealt with the contagion phenomenon between intercon-

[4] I mainly refer to Rochet (2004)'s arguments here.

nected financial institutions focusing on changes in their asset prices. A second mechanism of the contagion is the transmission of information by depositors. Chen (1999) showed that the systemic risk may occur in the absence of any interbank relations due to the first-come, first-served rule and information externalities on the negative payoffs. A few papers including Kyle and Xiong (2001) and G-P (2004) explained the contagion based on the wealth effects. Particularly, G-P (2004) used global games method to explain the contagion phenomenon between two countries. They looked at two countries that have independent fundamentals but share the same group of investors. In their model, a crisis in one country reduces agents' wealth and which makes them more risk averse to the strategic risk associated with the unknown behavior of other agents in the second country, increasing their incentive to withdraw their investments. Unlike the previous literatures, I focus on the creditors' learning behavior as the contagion mechanism. Learning process is very important since it can directly explain the creditors' strategic behaviors in a coordination game.[5]

The remainder of this paper is as follows. I present the model in section 2, and solve for the equilibria in section 3 where I explain equilibria from firm A to firm B in sequence and show what the contagion is. In section 4, I define the severity of contagion on the liquidity crisis and discuss some policy implications to reduce the severity of contagion by doing comparative statics. In section 5, I discuss the applicability of the model to real-world phenomena focusing on the case of 1997 Korea's financial crisis. Section 6 concludes.

2. MODEL

Basically, here I extend M-S (2004)'s model to the case of two firms / different type creditors.[6]

There are two firms: A and B. Both firms own no capital and firms' investment projects are only financed by debts. The state of firm i's fundamentals is θ_i, where $i = A, B$. θ_i can be interpreted as a measure of the ability of firm i to meet short term claims from creditors. The firm remains in operation given that θ_i is large enough to meet the claims from creditors. Otherwise, it is pushed into default. The higher value of θ_i refers to the better fundamentals. θ_i is randomly drawn from the real line. And I assume that θ_A and θ_B are independent.

A group of creditors are financing investment projects of two firms. In other words, two firms share the same creditors. For the simplicity of my analysis, I assume that those are in unit interval $[0, 1]$. Each creditor is small so that an individual creditor's stake is negligible as a proportion of the whole.[7] There are two

[5] Taketa (2004a) analyzed the contagion phenomenon of currency crises between two countries using global games method and learning process of speculators. However, he did not numerically analyze the contagion effect and policy proposals. In my work, focusing on non-financial institutions, I specifically analyze the contagion effects and suggest policy proposals to reduce the severity of contagion on the liquidity crisis from one firm to the other.

[6] By doing so, I explain the contagion phenomenon between two firms, which is dealt in the next section.

[7] Corsetti, Dasgupta, Morris and Shin [C-D-M-S] (2004) used the global games approach to consider implications of the existence of a large speculator like George Soros in a currency crisis in the dynamic setting. But they did not cover contagion effect there. Based on C-D-M-S (2004), Taketa (2004b) analyzed the implication of the presence of a large speculator in contagious currency crises: making countries more vulnerable to crises but mitigating the contagion of crises across countries. In my paper, for simplicity, I just assume all small players and the static / simultaneous game setting.

sub-groups of creditors: group 1 and group 2. The size of group 1 is λ and that of group 2 is $(1-\lambda)$, where $0<\lambda<1$. Group 1's type is its private information and it is "pessimistic" with probability q and "optimistic" with probability $(1-q)$. Group 2's type is "pessimistic" and this is public information. I assume that the type of creditors remains the same without big exogenous shocks such as government's intervention or whole market breakdown. "Pessimistic" creditors worry about the failure of the firm's investment project more than "optimistic" creditors. In other words, "pessimistic" creditors are more negative on the success of firm's investment project than "optimistic" creditors. Hence, I can assume that "pessimistic" creditors use δ_P as their discount factor which is less than δ_O – the discount factor of "optimistic" creditors (i.e., $0<\delta_P<\delta_O<1$).[8]

The investment project of each firm is completed in period 2 and yields the return v_i $(i=A,B)$ which is uncertain in period 0 because it depends on creditors' actions in period 1. Financings are undertaken by a standard debt contract. For simplicity, I assume that both firms have the same debt contract. That is, the face value of the repayment is L, and each creditor receives this full amount in period 2 if the realized value of v_i is large enough to cover the repayment of debt. At period 1, before the final realization of v_i, the creditors have an opportunity to review their investment. Hence, in this period, creditors have to decide whether or not to roll over their loan until period 2. If creditors collect and liquidate the collateral after they do not roll over the loan, the liquidation value of the seized collateral is $K^* \in (0,L)$. However, if the creditors collect and liquidate the collateral because they cannot get the full repayment after they roll over the loan, the liquidation value of the seized collateral is K_*, which is less than K^* (i.e., $K_*<K^*<L$). As M-S (2004) did, for the simplicity of my discussion, I normalize the payoffs so that $L=1$ and $K_*=0$. Then, creditors who do not roll over the loan at period 1 get K which is in $(0,1)$.[9] In summary, the present values of the payoffs at the interim stage (period 1) to a creditor are given by the following matrix:

	Project succeeds	Project fails
Rollover	$\delta_m \cdot 1 = \delta_m$	0
Not rollover	K	K

where m is P for a "pessimistic" creditor or O for an "optimistic" creditor. I assume $K<\delta_P<\delta_O<1$.

I say "there is the *liquidity crisis* in a firm" when the firm's investment project fails. If I denote the proportion of creditors who do not roll over the loan of firm i by l_i, where $i=A,B$, then the firm's investment project fails if and only if $l_i \geq \theta_i$.

If the creditors know the value of θ_i perfectly before deciding on whether or not to roll over the loan, their optimal strategies are as follows like Obstfeld (1996)'s self-fulfilling story: If $\theta_i>1$, then creditors will roll over their loans irrespective of other creditors' actions because the project survives even if every other creditor recalls. Conversely if $\theta_i \leq 0$, then it is optimal for creditors not to roll over the loan since the state of the fundamentals of the firm is so bad and thus the project

[8] δ_m $(m=P$ or $O)$ can be interpreted as the probability of the success of firm's investment project. That is, "pessimistic" creditors put less probability (δ_P) on the success of the project than "optimisitic" creditors (δ_O). If the future value of firm's investment project is L, then the present value of this project is $\delta_P L$ and $\delta_O L$ for "pessimistic" creditors and "optimistic" creditors, respectively.

[9] The exact value of K is $\frac{K^*-K_*}{L-K_*}$ by normalizing the payoffs, and it is in $(0,1)$ since $K_*<K^*<L$.

fails even if all other creditors roll over their loans. When $\theta_i \in (0, 1]$, here comes the coordination problem among the creditors. If all other creditors roll over their loans, then the payoff to rolling over the loan is 1 at maturity (period 2)[10], so that rolling over the loan yields more than the premature liquidation value K. Meanwhile, if everyone else recalls the loan, then the payoff is 0 which is less than K, so that early liquidation is optimal. Hence, the common knowledge assumption of creditors on θ_i leads to multiple equilibria.[11]

To get the unique equilibrium, I apply the global games method here: θ_i is not the common knowledge. Rather, at the interim stage when creditors decide on whether or not to roll over the loan, they receive private information concerning θ_i, but it is not perfect. In other words, each creditor in group j ($j = 1, 2$) gets the private signal: $x_{ij} = \theta_i + \varepsilon_{ij}$, where ε_{ij} is uniformly distributed in the interval $[-\varepsilon, \varepsilon]$. Note that the creditor's present value of the expected utility of rolling over the loan conditional on his private signal is $U = \Pr[\theta_i \geq l_i \mid x_{ij}] \cdot \delta_m$, where $m = P$ or O. And that of recalling the loan is K. A *strategy* for the creditor is a decision rule which maps each realization of x_{ij} to an action – rolling over the loan or not rolling over the loan. An *equilibrium strategy* consists of (1) *firm's switching fundamentals* ($\bar{\theta}_i$) below which the project fails (i.e., liquidity crisis occurs in the firm) and (2) *creditors' switching private signals* ($\bar{x}_{ij}$) such that every creditor who receives a signal lower than $\bar{x}_{ij}$ does not roll over the loan.

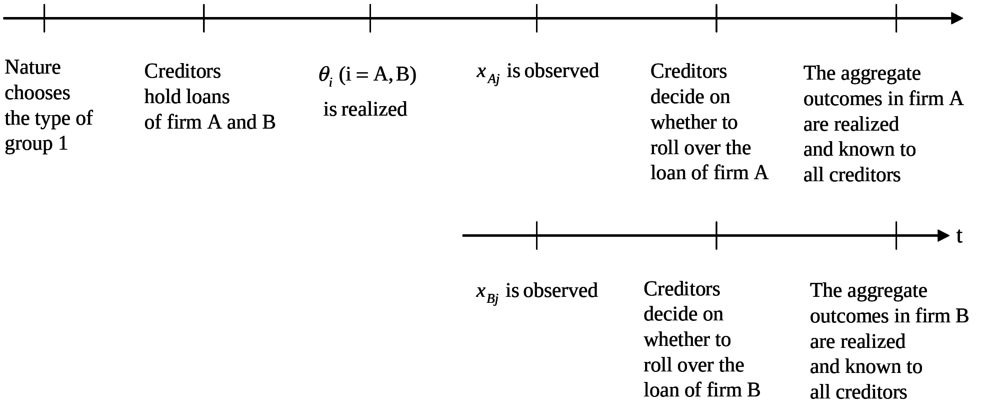

Figure 1: Timeline

The timeline (see figure 1) is as follows:[12] In the first stage, nature chooses a type of group 1 creditors. And the state of the fundamentals (θ_i, $i = A, B$) of each firm is realized. Each creditor in each group ($j = 1, 2$) receives private signals (x_{Aj}) on the fundamentals of firm A and decides whether or not to roll over the loan. In the second stage, each creditor in each group ($j = 1, 2$) observes signals (x_{Bj}) on the fundamentals of firm B and decides his actions in firm B. The exact realization of the fundamentals of firm A and the result of creditors' actions (i.e.,

[10] At the interim stage (period 1), the present value of 1 is δ_P or δ_O for "pessimistic" and "optimistic" creditors, respectively.

[11] As M-S (2004) mentioned in their paper, this type of coordination problem among creditors is analogous to Diamond and Dybvig (1983)'s bank run problem.

[12] I generally follow G-P (2004)'s sequence.

firm A's project failure or success) are known to creditors before they decide their actions in firm B.[13]

3. SOLVING THE MODEL

In this section, I explain what happens in firm A and then what happens in firm B sequentially. By doing so, I show how and why the liquidity crisis in firm A triggers the liquidity crisis in firm B. I derive the equilibrium strategies ($\bar{\theta}_i$ and $\bar{x}_{ij}$) in each firm i ($i = A, B$) and for each group j of creditors ($j = 1, 2$), and define the contagion.

3.1. Equilibrium in Firm A

The global games approach has shown that the equilibrium strategy consists of (1) *firm's switching fundamentals* ($\bar{\theta}_A$) below which the project fails (i.e., liquidity crisis occurs in firm A) and (2) *creditors' switching private signal* ($\bar{x}_{Aj}$) such that every creditor who receives a signal lower than $\bar{x}_{Aj}$ does not roll over the loan. Here, the equilibrium values $\bar{\theta}_A$ and $\bar{x}_{Aj}$ are as follows:

$$\bar{\theta}_A = \begin{cases} \theta^*_{AP} & \text{if the type of group 1 creditors is "pessimistic";} \\ \theta^*_{AO} & \text{if the type of group 1 creditors is "optimistic."} \end{cases}$$

$$\bar{x}_{A1} = \begin{cases} x^*_{A1P} & \text{if group 1 creditors are "pessimistic";} \\ x^*_{A1O} & \text{if group 1 creditors are "optimistic."} \end{cases}$$

$$\bar{x}_{A2} = x^*_{A2}.$$

After getting a private signal in period 1, each creditor has to decide whether or not to roll over the loan. The indifference condition gives the following equation:

$$\underbrace{K}_{\text{payoff from recalling}} = \underbrace{\Pr\left[\text{rollover is successful} \mid \bar{x}_{Aj}\right] \cdot \delta_m \cdot 1}_{\text{PV of the payoff from successful rollover}} \tag{1}$$

Now, let's think about the decisions of group 1 creditors. They privately know their type ("pessimistic" or "optimistic") and also know group 2 creditors' type ("pessimistic"). Hence, they know the value of $\bar{\theta}_A : \theta^*_{AP}$ or θ^*_{AO}. Note that $\varepsilon_{Aj} := x_{Aj} - \theta_A$ is uniformly distributed (i.e., $\varepsilon_{Aj} \sim U\left[-\varepsilon, \varepsilon\right]$). So, the equation (1) becomes:

$$\begin{aligned} K &= \Pr\left[\text{rollover is successful} \mid \bar{x}_{A1}, \bar{\theta}_A\right] \cdot \delta_m \\ &= \Pr\left[\theta_A \geq \bar{\theta}_A \mid \bar{x}_{A1}, \bar{\theta}_A\right] \cdot \delta_m \\ &= \Pr\left[\bar{x}_{A1} - \varepsilon_{A1} \geq \bar{\theta}_A \mid \bar{x}_{A1}, \bar{\theta}_A\right] \cdot \delta_m \\ &= \Pr\left[\varepsilon_{A1} \leq \bar{x}_{A1} - \bar{\theta}_A \mid \bar{x}_{A1}, \bar{\theta}_A\right] \cdot \delta_m \\ &= \frac{\bar{x}_{A1} - \bar{\theta}_A + \varepsilon}{2\varepsilon} \cdot \delta_m \end{aligned} \tag{2}$$

[13] That is, before creditors decide their actions, they did not know the exact value of firm's fundamentals. However, I assume that after the rollover game ends, creditors get to know the true value of firm's fundamentals. As G-P (2004) mentioned in their paper, in equilibrium, it is sufficient that creditors receive information regarding either the fundamentals or aggregate behaviors of creditors since one can be inferred from the other.

From (2), I get the following two equations:

$$\frac{x^*_{A1P} - \theta^*_{AP} + \varepsilon}{2\varepsilon} = \frac{K}{\delta_P}, \tag{3}$$

$$\frac{x^*_{A1O} - \theta^*_{AO} + \varepsilon}{2\varepsilon} = \frac{K}{\delta_O}. \tag{4}$$

Next, let's think about the decisions of group 2 creditors. They only know their own type ("pessimistic") and do not know the type of group 1 creditors. They can just conjecture the probability that the type of group 1 creditors is "pessimistic" as q. They do not know the value of $\bar{\theta}_A$: θ^*_{AP} or θ^*_{AO}, either. Then the equation (1) becomes:

$$\begin{aligned}
K &= \Pr\left[\text{rollover is successful} \mid x^*_{A2}\right] \cdot \delta_P \\
&= \Pr\left[\begin{array}{l}\text{rollover is successful}\\ \text{when 1's are pessimistic}\end{array} |x^*_{A2}\right] \cdot \delta_P + \Pr\left[\begin{array}{l}\text{rollover is successful}\\ \text{when 1's are optimistic}\end{array} |x^*_{A2}\right] \cdot \delta_P \\
&= q \times \Pr\left[\theta_A \geq \theta^*_{AP} \mid x^*_{A2}\right] \cdot \delta_P + (1-q) \times \Pr\left[\theta_A \geq \theta^*_{AO} \mid x^*_{A2}\right] \cdot \delta_P \\
&= q \times \frac{x^*_{A2} - \theta^*_{AP} + \varepsilon}{2\varepsilon}\delta_P + (1-q) \times \frac{x^*_{A2} - \theta^*_{AO} + \varepsilon}{2\varepsilon}\delta_P
\end{aligned} \tag{5}$$

Lastly, let's think about the critical threshold value of firm A's fundamentals (i.e., switching fundamentals). The proportion of creditors who do not roll over the loan is expressed as follows:

$$\begin{aligned}
l_A(\theta_A) &= \lambda \Pr\left[x_{A1} \leq \bar{x}_{A1} \mid \theta_A\right] + (1-\lambda)\Pr\left[x_{A2} \leq x^*_{A2} \mid \theta_A\right] \\
&= \lambda \Pr\left[\theta_A + \varepsilon_{A1} \leq \bar{x}_{A1} \mid \theta_A\right] + (1-\lambda)\Pr\left[\theta_A + \varepsilon_{A2} \leq x^*_{A2} \mid \theta_A\right] \\
&= \lambda \Pr\left[\varepsilon_{A1} \leq \bar{x}_{A1} - \theta_A \mid \theta_A\right] + (1-\lambda)\Pr\left[\varepsilon_{A2} \leq x^*_{A2} - \theta_A \mid \theta_A\right] \\
&= \lambda\frac{\bar{x}_{A1} - \theta_A + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{A2} - \theta_A + \varepsilon}{2\varepsilon}
\end{aligned}$$

The critical threshold value is determined by:

$$\bar{\theta}_A = l_A\left(\bar{\theta}_A\right) = \lambda\frac{\bar{x}_{A1} - \bar{\theta}_A + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{A2} - \bar{\theta}_A + \varepsilon}{2\varepsilon}. \tag{6}$$

From equation (6), I get the following two equations:

$$\theta^*_{AP} = \lambda\frac{x^*_{A1P} - \theta^*_{AP} + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{A2} - \theta^*_{AP} + \varepsilon}{2\varepsilon}, \tag{7}$$

$$\theta^*_{AO} = \lambda\frac{x^*_{A1O} - \theta^*_{AO} + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{A2} - \theta^*_{AO} + \varepsilon}{2\varepsilon}. \tag{8}$$

Solving equations (3), (4), (5), (7) and (8), I get x^*_{A1P}, x^*_{A1O}, x^*_{A2}, θ^*_{AP}, and θ^*_{AO}. The unique equilibrium values of the switching fundamentals of firm A (θ^*_{AP} and θ^*_{AO}) and the switching private signals (x^*_{A1P}, x^*_{A1O} and x^*_{A2}) are as follows:

$$\theta^*_{AP} = \frac{K}{\delta_P}\left(1 - \Sigma_1\right), \tag{9}$$

$$\theta^*_{AO} = \frac{K}{\delta_P}\left(1 - \Sigma_1 - \Sigma_2\right), \tag{10}$$

$$x^*_{A1P} = \frac{K}{\delta_P}\left(1 - \Sigma_1 + \Sigma_3\right), \tag{11}$$

$$x^*_{A1O} = \frac{K}{\delta_P}\left(1 - \Sigma_1 - \Sigma_2 + \frac{\delta_P}{\delta_O}\Sigma_3\right), \tag{12}$$

$$x^*_{A2} = \frac{K}{\delta_P}\left(1 - \Sigma_1 - (1-q)\,\Sigma_2 + \Sigma_3\right). \tag{13}$$

where

$$\Sigma_1 = \frac{\lambda(1-\lambda)(1-q)(\delta_O - \delta_P)}{\delta_O(1+2\varepsilon-\lambda)},\ \Sigma_2 = \frac{2\lambda\varepsilon(\delta_O-\delta_P)}{\delta_O(1+2\varepsilon-\lambda)} \text{ and } \Sigma_3 = \left(\frac{2K-\delta_P}{K}\right)\varepsilon.$$

Note that $\theta^*_{AP} > \theta^*_{AO}$ and $x^*_{A1P} > x^*_{A2} > x^*_{A1O}$ hold since λ, q, and ε are in $(0,1)$ and $0 < \delta_P < \delta_O < 1$ (see figure 2 and figure 3). The intuition of the inequalities is the following: x^*_{A1P} is greater than x^*_{A1O} because the pessimistic creditors are more likely not to roll over the loan than optimistic creditors. By the same logic, θ^*_{AP} is greater than θ^*_{AO} because firm A's project will be more likely to fail (i.e., be liquidated early) if creditors in group 1 are pessimistic.

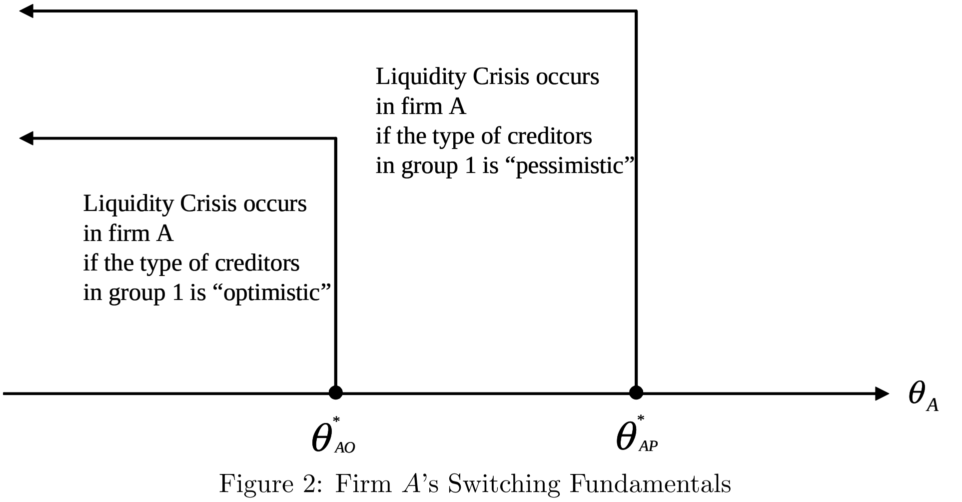

Figure 2: Firm A's Switching Fundamentals

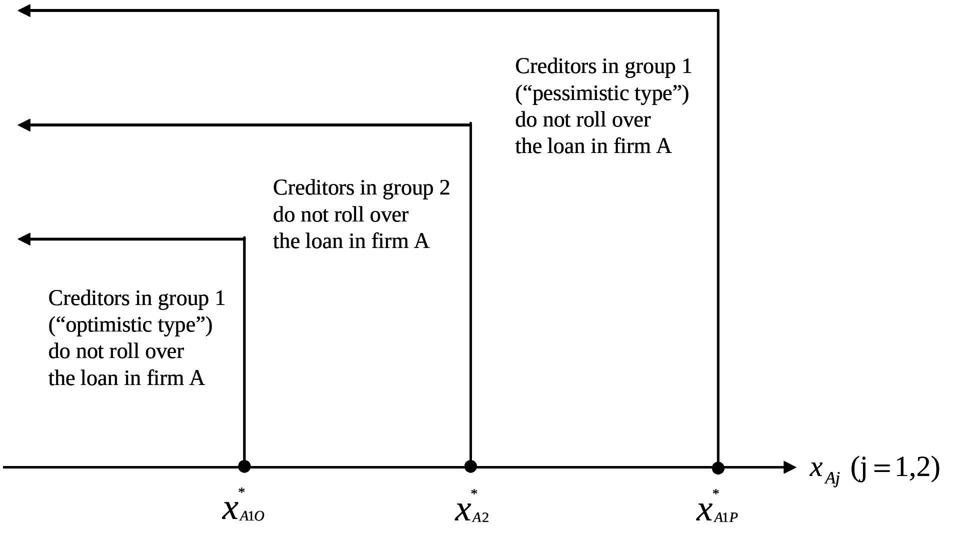

Figure 3: Creditors' Switching Private Signals in Firm A

Firm A's equilibrium is summarized in the following proposition.[14]

PROPOSITION 1. *Conditional on the type of group* 1 *creditors, there exists a unique equilibrium in firm A. In this equilibrium, each creditor in each group (*$j = 1, 2$*) does not roll over the loan if his signal* x_{Aj} *is below* $\bar{x}_{Aj}$ *and rolls over the loan if the signal is above.*

Proof. I need to show that every creditor in each group strictly prefers not to roll over the loan (prefers to roll over the loan) if his private signal is less than (greater than) the switching private signals conditional on θ^*_{AP} *and* θ^*_{AO}. *Suppose that every other creditor follows the switching strategy. Then, an individual creditor in each group takes* θ^*_{AP} *and* θ^*_{AO} *as given. From equations* (2) *and* (5)*, the present value of the expected payoff of rollover is strictly increasing in the private signals (*x^*_{A1P}, x^*_{A1O} *and* x^*_{A2}*) given* θ^*_{AP} *and* θ^*_{AO}. *Therefore, for any private signal greater than the switching signal, the expected payoff of rolling over is strictly greater than that of not rolling over. Thus, it is optimal for a creditor to follow the switching strategy given that every other creditor follows the switching strategy.* ∎

3.2. Equilibrium in Firm B

Now every creditor knows what happened in firm A, including the exact value of θ_A. This conveys information about the type of creditors in group 1 to the market because different types use different switching signals, resulting in different outcome in firm A under certain conditions.

I can think about two scenarioes of θ_A. First, if $\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$, then the type of creditors in group 1 is not revealed. Why? If $\theta_A \leq \theta^*_{AO}$, then the liquidity crisis certainly occurs in firm A regardless of the type of creditors in group 1. Meanwhile, if $\theta_A \geq \theta^*_{AP}$, then the liquidity crisis never occurs in firm A regardless of the type

[14] Note that for sufficiently small ε, there exists one threshold equilibrium in firm A since $\theta^*_{AP} = \theta^*_{AO} = x^*_{A1P} = x^*_{A1O} = x^*_{A2}$ as $\varepsilon \longrightarrow 0$.

of creditors in group 1. Hence, if $\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$, creditors in group 2 do not get to know the type of creditors in group 1 and face the same game, which was played in firm A, in determining whether or not to roll over the loan in firm B.[15]

Next, however, if $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$, then the type of creditors in group 1 is revealed to the market. Conditional on such θ_A, the liquidity crisis occurs in firm A if and only if creditors in group 1 are pessimistic. Likewise, conditional on such θ_A which is between θ^*_{AO} and θ^*_{AP}, the liquidity crisis does not occur in firm A if and only if group 1 creditors are optimistic. Hence, if $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$, then the new game is played by creditors whether or not to roll over the loan in firm B.

In the following, I explain the two scenarioes: $\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$ and $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$. In each scenario, I derive the equilibrium strategy (i.e., $\bar{\theta}_B$ and $\bar{x}_{Bj}$, $j = 1, 2$).

3.2.1. *Scenario 1: $\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$*

In this scenario, the type of group 1 creditors is not revealed. Hence, the equilibrium values of the switching fundamentals of firm B and the switching private signals are exactly the same as those of firm A. This is the benchmark case of firm B, and particularly benchmark switching fundamentals of firm B are (1) θ^*_{AO}, if the type of group 1 creditors is optimistic, and (2) θ^*_{AP}, if the type of group 1 creditors is pessimistic.

3.2.2. *Scenario $2-1$: Liquidity crisis in firm A when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$*

This scenario implies that the type of group 1 is "pessimistic." Then, both creditors in group 1 and 2 have the same switching strategy signal (say x^*_B). Hence, the equilibrium strategy consists of (1) *firm's switching fundamentals* (θ^*_{BP}) below which the project fails (i.e., liquidity crisis occurs in firm B) and (2) *creditors' switching private signal* (x^*_B) such that every creditor who receives a signal lower than x^*_B does not roll over the loan. Here, I get the following equilibrium strategy:[16]

$$\theta^*_{BP} = \frac{K}{\delta_P}, \; x^*_B = \frac{K}{\delta_P}(2\varepsilon + 1) - \varepsilon. \tag{14}$$

3.2.3. *Scenario $2-2$: No liquidity crisis in firm A when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$*

This scenario implies that the type of group 1 is "optimistic." Then, creditors in group 1 and 2 have different switching strategy signals (say x^*_{B1} for group 1 and x^*_{B2} for group 2). Hence, the equilibrium strategy consists of (1) *firm's switching fundamentals* (θ^*_{BO}) below which the project fails (i.e., firm B suffers the liquidity crisis) and (2) *creditors' switching private signal* (x^*_{B1} for group 1 and x^*_{B2} for group 2) such that every creditor in group 1 who receives a signal lower than x^*_{B1} does not roll over the loan and that every creditor in group 2 who receives a signal lower than

[15] Note that in this case ($\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$), even though the number of creditors who did not roll over their loans is known, the type of creditors in group 1 is not revealed since x_{A1} is in the ε-neighborhood of θ_A and x^*_{A1P} and x^*_{A1O} are very closely located in θ^*_{AP} and θ^*_{AO}, respectively.

[16] The derivation is in Appendix.

x^*_{B2} does not roll over the loan. Here, I get the following equilibrium strategy:[17]

$$\theta^*_{BO} = \frac{\lambda K}{\delta_O} + \frac{(1-\lambda)K}{\delta_P}, \tag{15}$$

$$x^*_{B1} = \frac{K(\lambda + 2\varepsilon)}{\delta_O} + \frac{(1-\lambda)K}{\delta_P} - \varepsilon, \tag{16}$$

$$x^*_{B2} = \frac{\lambda K}{\delta_O} + \frac{K(1-\lambda+2\varepsilon)}{\delta_P} - \varepsilon. \tag{17}$$

Note that $\theta^*_{BO} < \theta^*_{BP}$ and $x^*_{B1} < x^*_{B2} < x^*_B$ hold since λ and ε are in $(0,1)$ and $0 < \delta_P < \delta_O < 1$. The intuition of the inequalities is the following: x^*_B is greater than x^*_{B1} and x^*_{B2} because when all creditors are pessimistic, they are more likely not to roll over the loan than when there exist optimistic creditors. By the same logic, θ^*_{BP} is greater than θ^*_{BO} because firm B's project will be more likely to fail (i.e., be liquidated early) if creditors in group 1 are pessimistic.

Firm B's equilibrium is summarized as follows.

PROPOSITION 2. *Conditional on the realized underlying state of the fundamentals of firm A (θ_A) and whether the liquidity crisis occurs in firm A or not, there exists a unique equilibrium in firm B. When the realized underlying state of the fundamentals of firm A (θ_A) is not in the interval $[\theta^*_{AO}, \theta^*_{AP}]$, the same equilibrium values (switching fundamentals and switching private signals) as those of firm A are obtained irrespective of whether the liquidity crisis occurs in firm A or not. Meanwhile, when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$ and there is the liquidity crisis in firm A, every creditor in any group does not roll over the loan if his signal x_{Bj} ($j = 1, 2$) is below x^*_B and rolls over the loan if the signal is above; and when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$ but there is no liquidity crisis in firm A, each creditor in group 1 does not roll over the loan if his signal x_{B1} is below x^*_{B1} and each creditor in group 2 does not roll over the loan if his signal x_{B2} is below x^*_{B2}.*

Proof. *Follows from the same logic as proof of Proposition 1.* ∎

3.3. Contagion of the Liquidity Crisis from Firm A to Firm B

3.3.1. What is Contagion?

In this paper, contagion is defined as the propogation of solvency problems between two firms. And the contagion of the liquidity crisis from firm A to firm B is propagated by creditors who determine whether or not to roll over the loan. After observing what happened in firm A, creditors will update their beliefs about other creditors' types and reflect this information on their optimal decisions in firm B. If the realized fundamentals of firm A (θ_A) is so bad, which means $\theta_A \leq \theta^*_{AO}$, then firm A suffers the liquidity crisis regardless of creditors' recalls. In this case, the type of group 1 creditors is not revealed. So if $\theta_A \leq \theta^*_{AO}$, then this does not cause the contagion of the liquidity crisis from firm A to firm B because group 2 creditors' decisions in firm B are not affected by the situation in firm A. Only when θ_A is between θ^*_{AO} and θ^*_{AP} and there is the liquidity crisis in firm A, I can discuss whether there is the contagion of the liquidity crisis from firm A to firm B.

As I discussed in section 3.2, if $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$ and there is no liquidity crisis in firm A, this implies that the type of group 1 creditors is "optimistic." This information is reflected on group 2 creditors' decisions and the switching fundamentals θ^*_{BO}

[17] The derivation is in Appendix.

is determined. Likewise, if $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$ and there is the liquidity crisis in firm A, this implies that the type of group 1 creditors is "pessimistic." This information is reflected on group 2 creditors' decisions and the switching fundamentals θ^*_{BP} is determined. That is, only when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$, the behavior of creditors in firm A affects their behavior in firm B.

Now if the realized fundamentals of firm B (θ_B) is so bad, which means $\theta_B \leq \theta^*_{BO}$, then firm B suffers the liquidity crisis regardless of the occurrence of the liquidity crisis in firm A. Hence in this case, even though there are liquidity crises in both firms, I cannot say there is the contagion of solvency problems from firm A to firm B. Meanwhile, if θ_B is between θ^*_{BO} and θ^*_{BP} and there is the liquidity crisis in firm B, then this is the contagion of the liquidity crisis from firm A to firm B since there can be the liquidity crisis in firm B in $\theta_B \in [\theta^*_{BO}, \theta^*_{BP}]$ only when there was the liquidity crisis in firm A in $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$ (see figure 4).

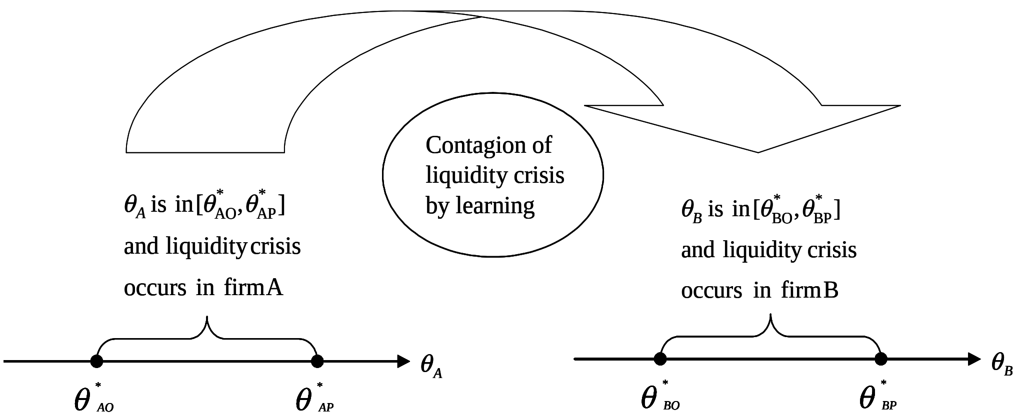

Figure 4: Contagion of Liquidity Crisis from Firm A to Firm B

DEFINITION 1. *The contagion of the liquidity crisis from firm A to firm B by creditors' learning behavior is that there is the liquidity crisis in firm B when $\theta_B \in [\theta^*_{BO}, \theta^*_{BP}]$; and there was the liquidity crisis in firm A when $\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$.*

3.3.2. Scenario 1 *versus scenario* 2

Now, let's compare the scenario 1 ($\theta_A \notin [\theta^*_{AO}, \theta^*_{AP}]$) with the scenario 2 ($\theta_A \in [\theta^*_{AO}, \theta^*_{AP}]$). From scenario 1, I get the benchmark switching fundamentals (θ^*_{AO} and θ^*_{AP}) in firm B. Meanwhile, from scenario 2, I get the new switching fundamentals (θ^*_{BO} and θ^*_{BP}) in firm B. By comparing these values of switching fundamentals, I get the following lemma (see figure 5).

LEMMA 1. $\theta^*_{BO} < \theta^*_{AO} < \theta^*_{AP} < \theta^*_{BP}$

Proof. From the values of θ^*_{AO}, θ^*_{AP}, θ^*_{BO}, and θ^*_{BP} (equations (9), (10), (14), (15)), I get

$$\theta^*_{BP} - \theta^*_{AP} = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left(\frac{(1-\lambda)(1-q)}{1+2\varepsilon-\lambda} \right) > 0,$$

$$\theta^*_{AO} - \theta^*_{BO} = \frac{\lambda K (\delta_O - \delta_P)}{\delta_O \delta_P} \left(\frac{q(1-\lambda)}{1+2\varepsilon-\lambda} \right) > 0.$$

From the fact that $\theta^*_{AP} - \theta^*_{AO} > 0$, $\theta^*_{BO} < \theta^*_{AO} < \theta^*_{AP} < \theta^*_{BP}$ *hold.* ∎

Figure 5: Values of Switching Fundametels (Firm B)

The intuition of the inequalities is as follows: If the type of group 1 creditors is revealed and it is "optimistic," then the liquidity crisis less likely occurs in firm B compared to the case where the type is not revealed (i.e., $\theta^*_{BO} < \theta^*_{AO}$). Meanwhile, if the type of group 1 creditors is revealed and it is "pessimistic," then the liquidity crisis more likely occurs in firm B compared to the case where the type is not revealed (i.e., $\theta^*_{BP} > \theta^*_{AP}$). That is, if the type of creditors is known, the accident of whether the liquidity crisis occurs or not becomes more clear.

4. COMPARATIVE STATICS AND POLICY IMPLICATIONS

In this section, after defining the severity of contagion on the liquidity crisis, I show that the impact of contagion originating from the firm considered less likely to fail is bigger than otherwise. Then, I do comparative statics on the severity of contagion with respect to variables which comprise the severity of contagion. By doing so, I suggest some policy implications to reduce the severity of contagion.

4.1. Severity of Contagion on the Liquidity Crisis

In section 3.3.2, I showed that $\theta^*_{BO} < \theta^*_{AO} < \theta^*_{AP} < \theta^*_{BP}$ hold. What does this imply? This means that if the type of group 1 creditors is revealed as being "pessimistic," then the probability of the liquidity crisis in firm B is increased by the difference between θ^*_{BP} and θ^*_{AP}. This is the negative effect of contagion on the liquidity crisis in firm B. It is sure that if the type of group 1 creditors is revealed as being "optimistic," then the probability of the liquidity crisis in firm B is decreased by the difference between θ^*_{AO} and θ^*_{BO}. This can be interpreted as the positive effect on reducing the probability of the liquidity crisis in firm B via revelation of the optimistic type of group 1 creditors. I focus on the negative effect of contagion on the liquidity crisis in firm B as the severity of contagion. That is, I define the severity of contagion as the difference between the new switching fundamentals for the pessimistic-type creditors (θ^*_{BP}) and the benchmark switching fundamentals for the pessimistic-type creditors (θ^*_{AP}).

Definition 2. *The severity of contagion on the liquidity crisis in firm B is the increased probability of the liquidity crisis in firm B due to the negative effect of contagion: the difference between the new switching fundametels θ^*_{BP} and the benchmark switching fundamentals θ^*_{AP}. Specifically, it is expressed by*

$$\mathbf{SC} := \theta^*_{BP} - \theta^*_{AP} = \frac{\lambda(1-\lambda)(\delta_O - \delta_P)(1-q)K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)},$$

which is greater than 0 *since* λ, ε, q, K, δ_O, *and* δ_P *are in* $(0,1)$.[18]

[18] How can I express the positive effect on reducing the probability of the liquidity crisis in

Now, I get the following proposition.

PROPOSITION 3. *The liquidity crisis in the firm having a small possibility to fail is more contagious than otherwise.*

Proof. I need to show that the severity of contagion ($\theta^*_{BP} - \theta^*_{AP}$) *is decreasing with* θ^*_{AP}.[19] *This is trivial since the decrease of* θ^*_{AP} *will increase the difference between* θ^*_{BP} *and* θ^*_{AP}. *Specifically I can express* θ^*_{AP} *as*

$$\begin{aligned}\theta^*_{AP} &= \frac{K}{\delta_P}\left(1 + \frac{\lambda(1-\lambda)(\delta_O - \delta_P)(1-q)}{\delta_O(-1-2\varepsilon+\lambda)}\right) \\ &= \frac{K}{\delta_P} - \underbrace{\frac{\lambda(1-\lambda)(\delta_O-\delta_P)(1-q)K}{\delta_O\delta_P(1+2\varepsilon-\lambda)}}_{=\mathbf{SC}}.\end{aligned}$$

By arranging the above equation, I get

$$\mathbf{SC} = -\theta^*_{AP} + \frac{K}{\delta_P},$$

which implies that the severity of contagion (**SC**) *is decreasing with* θ^*_{AP}. ∎

This proposition says that the severity of contagion decreases with the level of firm A's failure point (i.e., firm A's switching fundamentals). It implies that the occurrence of the liquidity crisis in the firm considered less likely to fail (i.e., the firm having a lower failure point) would lead to a huge surprise in the market and hence the liquidity crisis can become more contagious than otherwise. The recent huge negative spillover effect of the financial crisis from the U.S. to the other whole countries can be understood in the same thread. That is, even though the failure point of the U.S. economic fundamentals is very low, there is an economic downturn in the U.S these days. This leads to a big surprise in the world economy, and thus the propogation of the financial crisis from the U.S. is so severe to other countries. Meanwhile in 2002, we observed that the Argentine financial crisis was not severely propagable to the other countries which were considered to have better economic fundamentals than Argentina.

In summary, the contagion impact of the liquidity crisis in the firm considered less likely to fail is much bigger than that in the firm considered to be not good enough to endure the liquidity crisis. This is very striking since other previous contagion-related papers which deal with the contagion between the international financial markets and/or financial institutions through capital linkages and asset price changes showed that the larger the negative impact originating from worse fundamentals, the more severely other financial institutions or countries are affected. However in my work, I find that the severity of contagion is more serious when the originating firm's failure point is low if I focus on co-creditors' learning

firm B due to the revelation of the optimistic type of group 1 creditors? It is $\theta^*_{AO} - \theta^*_{BO} = \frac{\lambda(1-\lambda)qK(\delta_O-\delta_P)}{\delta_O\delta_P(1+2\varepsilon-\lambda)}$, which is only different in the term of q rather than $(1-q)$ for **SC**. That is, the sign of the comparative statics of $\theta^*_{AO} - \theta^*_{BO}$ with respect to the variables which comprise it is totally the same as **SC**'s case except for q. This brings the trade-off relation of the policy proposals for reducing the severity of contagion, which I tackle in latter sections (e.g., the initial policies regarding K, ε and λ).

[19] By the definition of firms' switching fundamentals, the low value of the switching fundamentals means that the firm fails with a small probability. That is, Firm's switching fundamentals can be interpreted as its failure point.

behaviors between non-financial institutions whose businesses are not related with each other (i.e., independent fundamentals) and assume that the exact realization of the fundamentals of the originating firm and the result of creditors' actions (i.e., firm's project failure or success) are known to creditors before they determine their actions in the other firm.

4.2. Changes in the Value of Collateralized Debt (K)

As M-S (2003, 2004) mentioned in their papers, increasing the value of collateral (K) has two contrasting effects: first, it increases the value of debt (loan) in the event of default (i.e., the direct effect); second, it increases the range of θ at which default occurs (i.e., the strategic effect[20]). In the contagion context, I find that the strategic effect outweighs the direct effect, which means that decreasing the value of collateral (K) is helpful to reduce the severity of contagion on the side of firm B.

PROPOSITION 4. *The severity of contagion on the liquidity crisis in firm B is reduced by the decrease in the value of its collateral (K).*

Proof.

$$\frac{\partial \mathbf{SC}}{\partial K} = \frac{\lambda (1-\lambda)(\delta_O - \delta_P)(1-q)}{\delta_O \delta_P (1+2\varepsilon-\lambda)} > 0,$$

which implies that if firm B decreases the value of K, then the severity of contagion on the liquidity crisis in firm B ($\mathbf{SC}$*) will be reduced.* ∎

What is the intuition of this proposition? The decreased value of collateral is the increased cost of not rolling over the loan from creditors' standpoint. Hence, creditors have more incentive to roll over their loans until maturity when the value of the collateral is small than otherwise. Then, firm B can reduce the severity of contagion on the liquidity crisis from firm A to itself. However, initially setting the value of collateral small makes the value of the loan decrease even though it helps reduce the severity of contagion.

4.3. Changes in the Gap of Discount Factors (δ_O and δ_P)

Diamond and Dybvig (1983) argued in their paper that the deposit insurance by the government can prevent the bank runs even though it might generate moral hazard problem. That is, patient agents know that the withdrawal by others is not going to harm their long-term return and will not withdraw their deposits. Likewise, let's think about the government's provision of bailouts to the firm which suffers a transitory liquidity problem. After observing the liquidity crisis in firm A and getting to know that the type of group 1 creditors is pessimistic, the government can expect the contagion of the liquidity crisis from firm A to firm B. If the government provides bailouts to firm B which is thought of suffering a transitory liquidity problem even though its state of the fundamentals is not too bad, then it is a good signal for the success of firm B's investment project in the market. In this case, pessimistic creditors become more optimistic toward the firm B's investment project (i.e., $\delta_P \longrightarrow \delta_O$). That is, the gap between δ_O and δ_P decreases, and hence

[20] In my model, I can verify this strategic effect result from θ^*_{AP} in equation (9) and θ^*_{BP} in equation (14).

it reduces the severity of contagion on the liquidity crisis in firm B.[21] I summarize this argument in the following proposition.

PROPOSITION 5. *The severity of contagion on the liquidity crisis in firm B is reduced by the decrease in $(\delta_O - \delta_P)$ which is obtained by the government's provision of bailouts to firm B.*

Proof.

$$\frac{\partial \mathbf{SC}}{\partial (\delta_O - \delta_P)} = \frac{\lambda (1-\lambda)(1-q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)} > 0,$$

which implies that if the government decreases $(\delta_O - \delta_P)$ by providing bailouts to firm B, then the severity of contagion on the liquidity crisis in firm B ($\mathbf{SC}$) will be reduced. ∎

4.4. Changes in the Information Structure (ε)

As creditors' information on firm's fundamentals becomes very precise (i.e., $\varepsilon \longrightarrow 0$), the value of firm's switching fundamentals is decreased.[22] In the similar context, M-S (2004) emphasized the role of transparent information in the crisis episode. Now, what is the effect of small noise (i.e., precise information on firm's fundamentals) on the severity of contagion? Will the precise information on firm's fundamentals reduce the severity of contagion? The result looks very surprising, that is, I find that it increases the severity of contagion.

PROPOSITION 6. *The severity of contagion on the liquidity crisis in firm B is increased in the accuracy of the information structure.*

Proof.

$$\frac{\partial \mathbf{SC}}{\partial \varepsilon} = -\frac{2\lambda (1-\lambda)(\delta_O - \delta_P)(1-q) K}{\delta_O \delta_P (1 + 2\varepsilon - \lambda)^2} < 0,$$

which implies that if creditors' (private) information on the firm's fundamentals becomes very precise (i.e., $\varepsilon \longrightarrow 0$), then the severity of contagion on the liquidity crisis in firm B ($\mathbf{SC}$) will be increased. ∎

If creditors' information on firm's fundamentals is very accurate, then the probability of firm A's liquidity crisis is reduced, i.e., the failure point of firm A (θ^*_{AP}) becomes lower. However, if there occurs the liquidity crisis in firm A even though ε is very small, then the contagion of the liquidity crisis to firm B is more severe. This can be interpreted as Proposition 3 addressed. That is, if the liquidity crisis occurs in the firm considered less likely to fail (i.e., small failure point (θ^*_{AP}) via small ε) then it leads to a big surprise in the market and thus the liquidity crisis can be more contagious.

4.5. Changes in the Size of Group 1 (λ)

The size of group 1 creditors, which is measured by λ, represents the incomplete information in the market. That is, even though the type of group 2 creditors

[21] In the very extreme case where the government "fully" guarantees firm B's investment project, then there occurs no contagion of the liquidity crisis from firm A to firm B.

[22] In my model, I can verify this result from θ^*_{AP} in equation (9) for instance. By the way, note that θ^*_{BP} in equation (14) and θ^*_{BO} in equation (15) do not have ε since the type of group 1 creditors is the known fact when those switching fundamentals are decided.

is "pessimistic," which is public information in the market, the type of group 1 creditors is not known in the market initially. What is the effect of this incomplete information on the severity of contagion? In other words, what is the impact of the degree of incomplete information on the contagion? I show the effect of λ on the severity of contagion when ε converges to zero in the following proposition.[23]

PROPOSITION 7. *The severity of contagion on the liquidity crisis in firm B is reduced by the decrease in the size of group 1.*

Proof.

$$\frac{\partial \mathbf{SC}}{\partial \lambda} = \frac{(\delta_O - \delta_P)(1-q)K}{\delta_O \delta_P} > 0 \text{ as } \varepsilon \longrightarrow 0,$$

which implies that as the size of group 1 is very small and creditors' (private) information on firm's fundamentals is very precise, then the severity of contagion on the liquidity crisis in firm B ($\mathbf{SC}$) will be decreased. ∎

What does this proposition imply? As I said above, the size of group 1 stands for the incomplete information in the market initially. If the size of this incomplete information becomes small, then the contagion of the liquidity crisis becomes less severe. Hence, the government can mitigate the severity of contagion by regulating the size of this incomplete information. For example, the government reinforces creditors to reveal their types via financial disclosure policy (i.e., to disclose their financial information in the market). In the extreme case where financial disclosure perfectly reveals the type of group 1 creditors in the market, then there occurs no learning process among creditors and thus there is no contagion of the liquidity crisis from firm A to firm B.

Related to the issue of the revelation of the type of group 1 creditors via financial disclosure policy, what is the effect of the type of group 1 creditors on the severity of contagion? The type of group 1 creditors is "pessimistic" with probability q. I find that the severity of contagion is decreasing with q[24], which implies that if group 2 creditors initially expect that group 1 creditors are more likely the same type as theirs, then the learning process of creditors' type does not have as much impact on the contagion of the liquidity crisis as otherwise.

5. DISCUSSION IN REAL-WORLD PHENOMENA

In order to assess the applicability of my model to real-world phenomena, let me revisit 1997 Korea's financial crisis in the middle of Asian *Flu*.[25] The bankruptcy of Hanbo Steel Group was a sobering experience for Korean banks. They started to strictly reexamine the profitability of their loans on other companies and call in most of short-term loans. This led to a "domino effect" as more and more companies suffered liquidity crises. For example, Kia Motors - Korea's eighth-largest conglomerate - failed even though its reputation in the market was fairly good.[26]

[23] When ε does not converge to zero, the effect of λ on the severity of contagion depends on the relative size of λ and ε. Hence, here I tackle the case where ε converges to zero, which means that the information of creditors on firm's fundamentals is very precise.

[24] θ^*_{AP} is increasing in q, but θ^*_{BP} is independent of q. That is, $\mathbf{SC}$ ($:=\theta^*_{BP} - \theta^*_{AP}$) is decreasing in q. Specifically, I get $\frac{\partial \mathbf{SC}}{\partial q} = -\frac{\lambda(1-\lambda)(\delta_O-\delta_P)K}{\delta_O\delta_P(1+2\varepsilon-\lambda)} < 0$.

[25] I mainly refer to Rhee (1998) here.

[26] In 1998, Kia was merged by Hyundai Motor.

The rush continued and as I mentioned in section 1, Jinro - Korea's nineteenth-largest conglomerate and also the largest liquor group - failed. By the end of 1997, over 15,000 companies, large and small, went bankrupt. In the process of serial firms' failures, foreign banks (especially, Japanese and U.S. banks[27]) pulled out their money en masse. And some Korean domestic banks (e.g., Korea First Bank (KFB)[28]) dramatically stopped rolling over their loans in firms.

This case of Korea's financial crisis in 1997 is consistent with my model. Note that Korea had a bank-centered financial system. Korean firms were highly leveraged by short-term loans of the domestic and foreign banks. By the end of 1996, the corporate debt relative to nominal GDP ratio was over 1.6, and the external debt to GDP ratio reached approximately 25% in which the share of short-term debt out of the total external debt peaked at 58%. Also note that as of the end of 1997, among 26 domestic commercial banks, 16 nationwide commercial banks[29] were actually common bank creditors of the top 30 conglomerates in Korea. Observing Hanbo Steel Group's liquidity crisis, common bank creditors could conjecture other creditors' types.[30] And reflecting this information, co-bank creditors decided their own actions - rolling over their loans or not - in other firms. Noting that there were no fundamental linkages among many firms which went bankrupt; and noting the leading roles of foreign banks and KFB on serial rushes in the market, the case of 1997 Korea's financial crisis provides the empirical evidence that supports my model of contagion mechanism: co-creditors' learning behavior.

6. CONCLUDING REMARKS

In this paper, focusing on liquidity crises in non-financial institutions, I studied that even though the states of two firms' fundamentals are not closely related with each other, what happened in one firm affects the optimal behavior of creditors in the other firm, and thus it affects what happens in that latter firm. The contagion mechanism between non-financial firms is based on co-creditors' learning behavior which has received little attention from existing literatures. Looking at creditors' learning behavior is very important because in a rollover coordination game, creditors' beliefs about others' type affect the probability of occurrence of the liquidity crisis in that firm, i.e., creditors' learning behavior can be very useful in explaining the creditors' strategic complementarities in a coordination game. Learning and updating their beliefs on others' type after observing what occurred in one firm, creditors determine their actions in the latter firm, which affects the probability of the liquidity crisis in the latter firm. I discussed the real-world example (i.e., Korea's financial crisis in 1997) which supports my model.

By analyzing the contagion process with creditors' learning, I found a very noticeable feature of the contagion which is different from the view of other previous contagion-related literatures: the contagion impact of the liquidity crisis originating from the firm having a low failure point is more severe than otherwise under the

[27] See Kaminsky, Lyons and Schmukler [K-L-S] (2001); and Kaminsky and Reinhart [K-R] (2000). K-R (2000) documented that on the eve of Korea's financial crisis, Japanese and U.S. banks' claims were held in Korea, and showed their withdrawals from Korea in the period of financial crisis. K-L-S (2001) analyzed the international mutual funds' withdrawals in Asian crises including when Korea suffered financial crisis.

[28] KFB went bankrupt right after Jinro's failure.

[29] The others were local commercial banks.

[30] Here, foreign banks and KFB, for example, can be thought of pessimistic creditors in my model due to the information disadvantage and a weak balance sheet, respectively.

assumptions that the exact realization of the fundamentals of the firm and the result of creditors' actions in that firm are known to creditors before they decide their actions in the other firm. Moreover, even though increasing the accuracy of creditors' information on the firm's fundamentals reduces the probability of the liquidity crisis in an individual firm, it increases the severity of contagion. I dealt with policy proposals addressing how to mitigate the severity of contagion including the government's provision of bailouts to the firm suffering a transitory liquidity problem and its financial disclosure policy. Also, the firm can initially reduce the severity of contagion by setting the value of its collateral small.

APPENDIX

Derivation of θ_{BP}^* and x_B^*

The proportion of creditors who do not roll over the loan conditional on θ_B is expressed as follows:

$$
\begin{aligned}
l_B(\theta_B) &= \Pr\left[x_B \leq x_B^* \mid \theta_B\right] \\
&= \Pr\left[\theta_B + \varepsilon_B \leq x_B^* \mid \theta_B\right] \\
&= \Pr\left[\varepsilon_B \leq x_B^* - \theta_B \mid \theta_B\right] \\
&= \frac{x_B^* - \theta_B + \varepsilon}{2\varepsilon}.
\end{aligned}
$$

The critical threshold value of firm B's fundamentals (i.e., switching fundamentals) is determined by:

$$
\theta_{BP}^* = l_B\left(\theta_{BP}^*\right) = \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon}. \tag{A1}
$$

From the fact that creditors' present value of the expected utility of rolling over the loan should be equal to the payoff from recalling the loan, I get:

$$
\begin{aligned}
K &= \Pr\left[\text{rollover is successful} \mid x_B^*\right] \cdot \delta_P \\
&= \Pr\left[\theta_B \geq \theta_{BP}^* \mid x_B^*\right] \cdot \delta_P \\
&= \Pr\left[x_B^* - \varepsilon_B \geq \theta_{BP}^* \mid x_B^*\right] \cdot \delta_P \\
&= \Pr\left[\varepsilon_B \leq x_B^* - \theta_{BP}^* \mid x_B^*\right] \cdot \delta_P \\
&= \frac{x_B^* - \theta_{BP}^* + \varepsilon}{2\varepsilon}\delta_P.
\end{aligned} \tag{A2}
$$

From equations (A1) and (A2), I get the equilibrium strategy:

$$
\theta_{BP}^* = \frac{K}{\delta_P},\ x_B^* = \frac{K}{\delta_P}(2\varepsilon + 1) - \varepsilon. \tag{A3}
$$

Derivation of θ_{BO}^*, x_{B1}^* and x_{B2}^*

The proportion of creditors who do not roll over the loan conditional on θ_B is expressed as follows:

$$\begin{aligned} l_B(\theta_B) &= \lambda \Pr\left[x_{B1} \leq x^*_{B1} \mid \theta_B\right] + (1-\lambda)\Pr\left[x_{B2} \leq x^*_{B2} \mid \theta_B\right] \\ &= \lambda \Pr\left[\theta_B + \varepsilon_{B1} \leq x^*_{B1} \mid \theta_B\right] + (1-\lambda)\Pr\left[\theta_B + \varepsilon_{B2} \leq x^*_{B2} \mid \theta_B\right] \\ &= \lambda \Pr\left[\varepsilon_{B1} \leq x^*_{B1} - \theta_B \mid \theta_B\right] + (1-\lambda)\Pr\left[\varepsilon_{B2} \leq x^*_{B2} - \theta_B \mid \theta_B\right] \\ &= \lambda \frac{x^*_{B1} - \theta_B + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{B2} - \theta_B + \varepsilon}{2\varepsilon}. \end{aligned}$$

The critical threshold value of firm B's fundamentals (i.e., switching fundamentals) is determined by:

$$\theta^*_{BO} = l_B\left(\theta^*_{BO}\right) = \lambda \frac{x^*_{B1} - \theta^*_{BO} + \varepsilon}{2\varepsilon} + (1-\lambda)\frac{x^*_{B2} - \theta^*_{BO} + \varepsilon}{2\varepsilon}. \tag{A4}$$

From the fact that creditors' present value of the expected utility of rolling over the loan should be equal to the payoff from recalling the loan, I get the following equations for "optimistic" group 1 creditors and "pessimistic" group 2 creditors:

$$\begin{aligned} K &= \Pr\left[\text{rollover is successful} \mid x^*_{B1}\right] \cdot \delta_O \\ &= \Pr\left[\theta_B \geq \theta^*_{BO} \mid x^*_{B1}\right] \cdot \delta_O \\ &= \Pr\left[x^*_{B1} - \varepsilon_{B1} \geq \theta^*_{BO} \mid x^*_{B1}\right] \cdot \delta_O \\ &= \Pr\left[\varepsilon_{B1} \leq x^*_{B1} - \theta^*_{BO} \mid x^*_{B1}\right] \cdot \delta_O \\ &= \frac{x^*_{B1} - \theta^*_{BO} + \varepsilon}{2\varepsilon}\delta_O, \end{aligned} \tag{A5}$$

and

$$\begin{aligned} K &= \Pr\left[\text{rollover is successful} \mid x^*_{B2}\right] \cdot \delta_P \\ &= \Pr\left[\theta_B \geq \theta^*_{BO} \mid x^*_{B2}\right] \cdot \delta_P \\ &= \Pr\left[x^*_{B2} - \varepsilon_{B2} \geq \theta^*_{BO} \mid x^*_{B2}\right] \cdot \delta_P \\ &= \Pr\left[\varepsilon_{B2} \leq x^*_{B2} - \theta^*_{BO} \mid x^*_{B2}\right] \cdot \delta_P \\ &= \frac{x^*_{B2} - \theta^*_{BO} + \varepsilon}{2\varepsilon}\delta_P. \end{aligned} \tag{A6}$$

From equations (A4), (A5) and (A6), I get the equilibrium strategy:

$$\theta^*_{BO} = \frac{\lambda K}{\delta_O} + \frac{(1-\lambda)K}{\delta_P}, \tag{A7}$$

$$x^*_{B1} = \frac{K(\lambda + 2\varepsilon)}{\delta_O} + \frac{(1-\lambda)K}{\delta_P} - \varepsilon, \tag{A8}$$

$$x^*_{B2} = \frac{\lambda K}{\delta_O} + \frac{K(1-\lambda+2\varepsilon)}{\delta_P} - \varepsilon. \tag{A9}$$

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