Need I remind you? Monitoring with collective memory*

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Abstract

We consider a team setting where forgetful players with limited memories have costly but socially efficient tasks to complete. Each teammate promises to complete some subset of the tasks, and strategically memorizes her own promises as well as a subset of her teammates' promises. She can be contractually punished for an unfulfilled promise only if another player remembers it. Hence the team's collective memory serves as a costly monitoring device.

We show that linear contracts are the optimal way to ensure that a player completes as many promises as she remembers, and characterize the optimal linear contract when players' memories differ in size and quality. Linear contracts are indeed optimal if players are not very forgetful. However, when players are more forgetful, an optimal equilibrium has *empty promises*; these are promises a player might not complete even if she remembers them. The corresponding optimal non-linear contract will "forgive" some failures. As players become more forgetful, they make more empty promises and devote more of their memories to monitoring.

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"To be wronged is nothing unless you continue to remember it"—Confucius

1 Introduction

Many tasks are too complicated to be fully specified in written form. For example, a construction contractor could not reasonably write down the entire battery of supplies, procedures, and safety checks needed to properly add a wing to a home. Any task details not incorporated into the formal contract must be enforced informally in equilibrium. Furthermore, the agent who is to perform the task must rely on his memory to fill in these details. To detect whether he has "botched" the task by either forgetting or ignoring these details, another agent must remember them herself. Unfortunately, a strong body of evidence suggests that memory is both bounded and imperfect. As a consequence, several tradeoffs arise. First, bounded memory introduces a tradeoff between devoting memory to performing tasks and devoting memory to monitoring tasks. Furthermore, forgetful agents cannot avoid punishments on the equilibrium path, leading to a tradeoff between the cost of punishments and their effectiveness as incentives.

This paper departs from the common assumption in contract theory, and much of the economic literature at large, that an agent's memory has unbounded capacity and perfect recall.[1] The literature in cognitive psychology has established that individual memories are imperfect, and, most importantly for models of interaction, that the *collective memory* of a group has very different properties than individual memory.[2] In particular, collective memory can be generated and maintained by collaborative recall processes such as *cross-cueing*, by which one individual's recall triggers a forgotten memory in another (Weldon and Bellinger 1997). We study team production among players with imperfect memories, a question that falls into the intersection of the literatures on teams (e.g., Holmström 1982), contracting with costly monitoring (e.g., Williamson 1987, Border and Sobel 1987, Mookherhjee and Png 1989), public goods (e.g., Palfrey and Rosenthal 1984) and bounded rationality (e.g., Rubinstein 1998).[3] We find that in this setting it is often optimal for players to make "empty promises" and to be "forgiven" for having done so.

[1]Notable exceptions, typically in the decision-theoretic literature, include Dow (1991), Piccione and Rubinstein (1997), Hirshleifer and Welch (2001), Mullainathan (2002), Benabou and Tirole (2002) and Wilson (2004). There is also a literature on repeated games with finite automata which can be interpreted in terms of memory constraints (e.g., Piccione and Rubinstein 1993, Cole and Kocherlakota 2005, Compte and Postlewaite 2008), as well as work on self-delusion in groups (e.g., Benabou 2008).

[2]A seminal paper by Miller (1956) suggests that the capacity of working memory is approximately 7 ± 2 "chunks." A chunk is a set of strongly associated information—e.g., information about a task. More recently, Cowan (2000) suggests a grimmer view of 4 ± 1 chunks for more complex chunks. Other studies on information processing and memory include Cloitre, Cancienne, Brodsky, Dulit and Perry (1996), Tafarodi, Tam and Milne (2001), Franken, Rosso and van Honk (2003) and Tafarodi, Marshall and Milne (2003).

[3]A variety of related issues arise in the principal-agent literature. At the most basic level, we build on the seminal results on optimal contracts, such as Mirrlees (1999) and Holmström (1979). More specifically, our results have some of the flavor of the stochastic auditing literature (e.g., Border and Sobel 1987, Mookherhjee and Png 1989).

We assume that each agent can memorize only a limited number of tasks, and recalls each memorized task with i.i.d. probability less than one. In Section 2, we propose a model in which a team of players has access to a set of socially efficient but privately costly tasks to be completed. Players make promises to each other regarding the set of tasks they will complete, and the team's collective memory serves as a costly monitoring device to enforce promise-keeping. Specifically, each player fills her memory strategically with some combination of her own and her teammates' promises. A player can choose to complete only those tasks that she has not forgotten. She can be punished by the team only when someone reminds (or cross-cues) the team that she has failed to fulfill a promise. Because their memories are bounded, the players can monitor each other only at the expense of tasks they can accomplish themselves. The punishment for an unfulfilled promise takes the form of embarrassment, loss of status, or other penalty that does not enrich her teammates. The team commits to the schedule of punishments ahead of time; we call this schedule a *contract*.

Our model applies to tasks that are sufficiently difficult to describe that only a few of them can be stored in memory. A task contains detailed information, such as a decision tree, that is necessary to complete it properly.[4] If a player "forgets" a task she had stored in memory, she actually forgets relevant details and is unable to complete the task properly. Even if she remembers the details, by ignoring them she can "botch" the task at no cost to herself. Another player can discover that she has botched the task only if he himself remembers the relevant details.[5] Throughout this paper we use "completing a task" as shorthand for "completing a task properly."

We are interested in settings where performance is not formally contractible and tasks must be divided up among team members. Consider the following examples:

- *A medical team in a busy hospital ward.* Each doctor takes primary responsibility for carrying out the treatment plan for some subset of the patients on the ward. To properly treat a patient, the doctor should select appropriate questions, tests, and procedures based on medical best practices, which are too vast to specify contractually. The doctors do not monitor each other directly, but convene as a team at the end of each day to discuss their activities.

- *A team of detectives investigating a crime.* For a detective to thoroughly interview a witness, she needs to be able to notice any details that contradict or corroborate previously collected evidence. Upon noticing such a connection, the detective can expend additional effort to follow up.

[4] Al-Najjar, Anderlini and Felli (2006) characterize finite contracts regarding "undescribable" events, which can be fully understood only using countably infinite statements. In this interpretation, to carry out an undescribable task properly, a player must memorize and recall an infinite statement.

[5] We assume that the benefit of a task is in expectation, and that players cannot contract on their ex-post payoffs.

- *A legal team working on a case.* There may be many legal precedents related to a case that a team of lawyers will review while preparing. Remembering these details is important during the proceedings, for example, to argue in court in order to prevent opposing counsel from striking helpful evidence.

- *Coauthors on a research paper.* Each coauthor promises to make improvements to the paper—such as proving a conjecture, rewriting a section, or developing connections to related literature—which require remembering potentially complex details and applying methods that are mutually understood but not specified ahead of time.

We study *counting contracts*, in which each player's punishment depends on the number of her unfulfilled promises that are reported by her teammates. In Sections 3–5, we focus on two-player teams; in Section 6 we show that the results extend naturally to larger teams. In Section 3, we first consider the benchmark case of linear contracts, which treat each task independently. We fully characterize optimal symmetric linear contracts when punishments are bounded. Under a linear contract, each team member completes as many of his promised tasks as he can recall. We show that when players are very forgetful, they optimally make zero promises; but if they are not too forgetful, they optimally devote an increasing fraction of their memories to their own promises and a decreasing fraction to their teammate's promises.

We then take up the problem of optimal non-linear contracts, in Section 4. Linear contracts are optimal in this class when the probability that players forget each promise is either very low or very high, but for intermediate forgetting probabilities it is optimal to implement a non-linear contract. In particular, optimal contracts are generally *forgiving*: a player who fails to fulfill a small number of promises is punished only mildly, if at all, and not enough to make her willing to fulfill all her promises if she indeed remembers them. That is, players make *empty promises*—promises that they do not intend to fulfill.

There are several tradeoffs in constructing an optimal contract. First, since memory is limited, memory that a player devotes to monitoring her teammate's promises cannot be devoted to her own promises, and therefore reduces the expected number of her promises that she will remember. Second, since players are forgetful, they incur punishments with positive probability, so using punishments to induce task completion is costly.[6] More subtly, although finding a large number of unfulfilled promises is an informative signal of moral hazard, it may not arise with positive probability if a player completes all but a few of her promised tasks. Promise keeping is thus costly

[6] The model would be uninteresting if the players could transfer utility. For example, with three or more players it would be possible to implement costless punishments, by rewarding a third player when one player's unfulfilled promise is discovered by a second player. With costless punishments, the players would put all but a minimal portion of their memory resources toward promise making. Indeed, if they could randomize, then there would be no well-defined optimum.

to implement. This opens the door for empty promises, which also help players to remember more promises.

For any memory size, there is a range of parameters for which the optimal contract involves making empty promises. When players can memorize no more than five tasks, every optimal contract induces *cutoff strategies*, in which each team member fulfills as many promised tasks as she remembers up to some cutoff (which in many cases is less than the number of tasks she promised). Moreover, both the cutoff and the total number of promises are increasing in the quality of memory; and a player optimally devotes approximately the same number of memory slots to monitoring as to empty promises. Based on the neuropsychology literature, a small memory bound is realistic. Nonetheless, we expect these results to extend to larger memory sizes.

In Section 5, we also study asymmetric teams, in which players can differ in both the size of their memory and their ability to recall. The asymmetric monitoring that results can be viewed as selecting an endogenous supervisor. To focus on the allocation of supervisory responsibility, we examine linear contracts, under which empty promises do not arise. (This restriction is without loss of generality when players are not too forgetful.) We show that greater supervisory responsibility is optimally assigned to the player with the weaker memory. Moreover, an increase in the strength of one player's memory reduces the number of tasks that her teammate optimally promises.

The canonical model we propose to capture these tradeoffs can be extended to study new questions that arise in settings with memory constraints and incomplete contracts. In Section 7 we discuss several possibilities that we leave for future work.

Our model bears interesting relations to theories in cognitive psychology and organizational behavior. Remembering a promise (i.e., remembering one's intention to complete a task at a later point) is termed *prospective memory* in the theory of cognitive psychology; Dismukes and Nowinski (2007) study prospective memory lapses in the airline industry, noting that they are "particularly striking" because that industry has "erected elaborate safeguards... including written standard operating procedures, checklists, and requirements... to cross check each other's actions." In view of such difficulties, various theories of how to optimally store, recall, and share information have been proposed in the literature on organizational behavior; for example, consider Mohammed and Dumville (2001), Xiao, Moss, Mackenzie, Seagull and Faraj (2002) and Haseman, Nazareth and Paul (2005), which draw on the seminal work of Wegner (1987). Wegner develops the notion of *transactive knowledge*, the idea that while we cannot remember everything, we know who remembers what we need to know. That is, "memory is a social phenomenon, and individuals in continuing relationships often utilize each other as external memory aids to supplement their own limited and unreliable memories" (Mohammed and Dumville 2001). In our model, players know who is responsible for each task as well as who is responsible for monitoring the promiser. This bears a formal relationship to *transactive responsibility*, a concept that Xiao et al. (2002) introduce to study the division of responsibilities and cross-monitoring by trauma teams in hospitals.

We view a contract as an informal agreement that is enforced by selecting among equilibria in some unspecified continuation game. In such a context, any common knowledge event at the end of the game is "contractible." We assume that cross-cueing generates common knowledge. One justification for this is based on an underlying conceptual model that separates working memory, which is tightly bounded, from long-term memory, which is effectively unbounded. (Baddeley 2003 reviews the relevant psychological and neurological literature.) Information held in working memory (including cues to retrieve information from long-term memory) can be acted on, while information held in long-term memory can be used to verify claims about the past. A player who has forgotten one of his promises from his working memory still holds it in his long-term memory. If another player holds his promise in her working memory, she can cross-cue him, reminding him of his promise and restoring common knowledge.[7]

2 The model

We first provide a loose overview of the model. Before the game starts, a contract is in place that governs the punishment each player will receive as a function of the messages sent at the end of the game. There are three stages:

1. *Promise-making.* Each player promises to complete certain tasks, and then memorizes some subset of the team's promises. Promises are public, but memorization is private.
2. *Task-completion.* Any given promise that was stored in memory has been forgotten with some probability, independently across promises. Based on her remaining memory, each player chooses some subset of her promised tasks to complete. Task completion is private.
3. *Review.* Each player sends a public report about the tasks she completed and the promises she remembers other players made. Based on these reports, each player is punished according to the contract.

For most of the paper we focus on the case of a two-player team, $\mathcal{I} = \{1, 2\}$. In Section 6, we extend the analysis to larger teams. A countably infinite set of *tasks* $\mathcal{X}$ is available to the team. Each task can be completed by one team member, who must memorize and recall detailed information about the task in order to complete it. Each player i has a bounded memory with

[7]Ericsson and Kintsch (1995) note, "the primary bottleneck for retrieval from LTM [long-term memory] is the scarcity of retrieval cues that are related by association to the desired item, stored in LTM." Here the review stage of the game provides the necessary retrieval cues. Smith (2003) shows that intending to perform a task later requires using working memory to monitor for a cue that the time or situation for performing the task has arrived.

M_i slots, each of which may be used to store a *promise* $(x, j) \in \mathcal{X} \times \mathcal{I}$ encoding a task x and the player j who promises to complete it. The same promise cannot be stored in multiple memory slots, so a player's memory state is an element of $\mathcal{M}_i = \{m_i \subseteq (\mathcal{X} \times \mathcal{I}) \big| |m_i| \leq M_i\}$. A player reaps a benefit b from each task that is completed by the team, but incurs cost c for each task he completes himself. Completing any given task is efficient but a player would rather not do it; i.e., $b < c < 2b$.

With a contract in place at the outset of the game (we formalize contracts in Section 2.2, below), the players enter the promise-making stage. Each player i publicly announces promises $\pi_i \subset \mathcal{X} \times \{i\}$. Given the collection of all promises, $\pi = \bigcup_{j \in \mathcal{I}} \pi_j$, each player privately decides which of these promises to memorize. Player i's memorization strategy is $\mu_i : 2^{\mathcal{X} \times \mathcal{I}} \to \Delta \mathcal{M}_i$. We assume that players cannot delude themselves; i.e., the support of $\mu_i(\pi)$ must be contained within π.[8]

By the task-completion stage, each promise that player i had memorized is recalled with probability $\lambda_i \in [0, 1]$, independently across promises. Her resulting memory state is $m_i \in \mathcal{M}_i$. A player cannot fulfill a promise for which she has forgotten the necessary details. Consequently, player i's decision strategy $d_i : \mathcal{M}_i \to \Delta 2^{\mathcal{X}}$ for which promises to fulfill can put positive probability only on promises contained in m_i.

At the review stage, the players observe the tasks that have been completed, and each player publicly reports the promises she recalls that her teammate made. Let $A_i \subset \mathcal{X} \times \{i\}$ be the set of promises that player i fulfilled, and let $\hat{m}_i \subseteq m_i \cap \pi_{-i}$ be the set of her teammate's promises that she reports. The collective memory, then, contains both the union of all completed tasks and the union of all reported promises. We assume that messages are verifiable, and that only verified reports are incorporated into the collective memory. This is in line with the literature on *cross-cueing* (e.g., Weldon and Bellinger 1997): a player triggers the memory of his teammate when he reports on the details of a task.

2.1 Simple memory strategies

To determine whether she would like to fulfill some subset of her recalled promises, a player must be able to compute—at the task-completion stage—the conditional distribution over which subsets of her recalled promises will be monitored. To avoid forcing players to remember potentially complicated memorization strategies in a setting in which they have bounded memory and imperfect recall, we focus on a class of simple memory strategies that are a straightforward generalization of pure strategies, where any randomization (if necessary) is trivial.[9] Such strategies can be viewed as satisfying a technological constraint of memory.

[8] Hence the memory process differs significantly from Benabou (2008), which is interested in distortions of reality.

[9] Indeed, if remembering a complicated strategy is a matter of choice, the contract may need to incentivize doing so, which raises a variety of circular problems relating to how to incentivize remembering a memorization strategy.

Player i's memory strategy μ_i is *simple* if (i) the allocation of memory between own promises and monitoring is deterministic and (ii) she randomizes uniformly which promises to monitor, if the space allocated for monitoring is smaller than the number of promises made. Outside of the class of simple strategies, each player i would have to memorize (and possibly forget) a potentially complicated distribution over subsets of π_i. Under simple memory strategies, player i's task-completion strategy need depend only on the number of promises she recalls, the contract, how many promises she made, and how many of those are being monitored. We assume she recalls these bare outlines of the promise-making stage perfectly, even if she cannot recall the promises made in greater detail. This formalizes the sentiment in Wegner (1987) that "we have all had the experience of feeling we had encoded something... but found it impossible to retrieve."

2.2 Counting contracts

A contract, fixed at the outset of the game, determines a vector of punishments that will be applied at the end of the game. First, the contract can enforce any number of equilibrium promises using the threat of harsh punishments.[10] Second, if nobody deviated in the promise-making stage, then the contract yields a vector of punishments as a function of the collective memory at the end of the review stage, $V : 2^{\mathcal{X}\times\mathcal{I}} \times 2^{\mathcal{X}\times\mathcal{I}} \to \mathbb{R}_-^{|\mathcal{I}|}$. The ex-post payoff of player i is

$$U_i = b\sum_{j\in\mathcal{I}}\big|A_j\big| - c\big|A_i\big| + V_i\Big(\bigcup_{j\in\mathcal{I}} A_j, \bigcup_{j\in\mathcal{I}} \hat{m}_j\Big). \tag{1}$$

We study symmetric *counting contracts*, a straightforward and intuitive class of contracts, in which each player's punishment depends only on the number of her unfulfilled promises that are reported by her teammate. She can compute the distribution of this number using only the number of promises she recalls, how many promises she made, and how many of those are being monitored. Hence a counting contract is compatible with simple memory strategies.

Assumption 1 (Counting contracts)**.** *Let $\hat{m}_{-i} \equiv \big(\mathcal{X}\times\{i\}\big)\cap\bigcup_{j\neq i}\hat{m}_j$, and let $f_i \equiv \big|\hat{m}_{-i}\backslash A_i\big|$. A contract must be a* counting contract *of the form $V_i\big(\bigcup_j A_j, \bigcup_j \hat{m}_j\big) = v(f_i)$, where $v : \mathbb{I}_+ \to \mathbb{R}_-$.*

Since a counting contract cannot punish a player for her report (which is verifiable), it follows that she is willing to fully disclose what she recalls of her teammate's promises. Without loss of generality, we focus on equilibria with full disclosure.

[10] Alternatively, any number of promises can be part of a perfect Bayesian equilibrium under the following deviation response: if anyone promises a deviant set of tasks, nobody commits any promises to memory, yielding zero payoffs. Since players are indifferent to monitoring or not, this off-equilibrium play is sequentially rational.

Definition 1. *A contract and a full-disclosure perfect Bayesian equilibrium in simple memory strategies in the game it induces are (together)* optimal *if they yield expected payoffs that are Pareto optimal in the set of all such expected payoffs. Such a contract is also (itself)* optimal.

3 Linear contracts

We begin by studying the benchmark case of symmetric linear contracts with a per-task punishment bound of $\underline{v} < 0$. That is, contracts of the form $v(f_i) = vf_i$, where $v \in [\underline{v}, 0]$. The main result of this section is the following theorem, which characterizes optimal symmetric linear contracts when M is even.[11]

Theorem 1. *Suppose M is even. Then there exist p^* and v^* (given below) such that $v(f_i) = v^* f_i$ is an optimal symmetric linear contract in the symmetric environment, and in its associated optimal equilibrium each player i makes $|\pi_i| = p^*$ promises; memorizes π_i with probability 1; monitors $M - p^*$ of player $-i$'s promises, randomizing uniformly over memorizing each $(M - p^*)$-element subset of π_{-i}; completes each promise in π_i that she recalls; and reports what she recalls of player $-i$'s promises truthfully. Furthermore, if $\lambda \geq \max\{\frac{c-b}{b}, \frac{b-c}{\underline{v}}\}$, then*

$$p^* = \left\lfloor \frac{\lambda \underline{v} M}{b - c + \lambda \underline{v}} \right\rfloor \text{ and } v^* = \frac{p^*(b-c)}{\lambda(M - p^*)},$$

where $\lfloor \cdot \rfloor$ is the "floor" function $\lfloor y \rfloor \equiv \max\{\hat{y} \in \mathbb{I} : \hat{y} \leq y\}$; otherwise $p^* = v^* = 0$ is optimal.

Under the optimal linear contract, each player fully utilizes all her memory slots, either for storing her own promises or for monitoring her teammate, and fulfills as many promises as she remembers. The optimal number of promises is depicted in Figure 1 as a function of the recall parameter λ. When λ is very low, the players should make no promises in order to avoid virtually inevitable punishments. As λ rises, it reaches a threshold at which it becomes optimal to make some promises. At this threshold, monitoring is still not very effective, so each player must devote half of her memory to monitoring in order to maintain the other player's incentives. As λ rises further, the amount of memory devoted to monitoring decreases—and hence the optimal number of promises increases.

Proof. First we show by backward induction that every element of the strategies is sequentially rational given beliefs. First, since this is a counting contract, each player is willing to report her

[11] There may be superior asymmetric linear contracts, but they will not differ from the optimal symmetric contract by more than a task per player. Similarly, for M odd all optimal linear contracts, symmetric or otherwise, will be close to the optimal symmetric linear contracts for $M - 1$ and $M + 1$. See footnote 12, below.

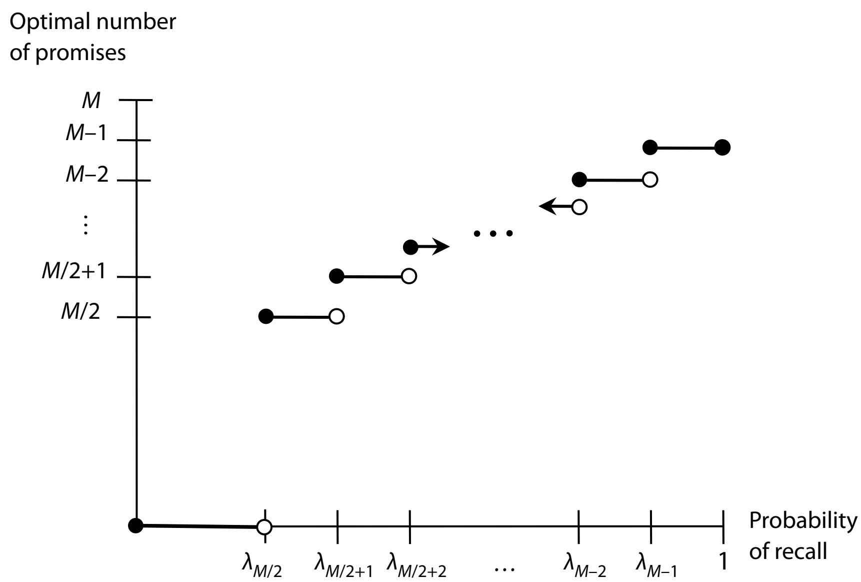

FIGURE 1: OPTIMAL LINEAR CONTRACT REGIMES. Here, $\lambda_{M/2} = \max\{\frac{c-b}{b}, \frac{b-c}{\underline{v}}\}$. All λ-ranges shown are nonempty if $-b \leq \underline{v} \leq (M-1)(b-c)$.

teammate's promises truthfully in the review stage. Since player i would be harshly punished for making the wrong promises, and cannot be punished for reporting on her teammate's promises, her promising and memorization strategies in the promise-making stage are incentive compatible as well. Hence under consistent beliefs in the task-completion stage the incentive constraint for player i to complete promise $(x,i) \in \pi_i \cap m_i$ is

$$b - c \geq \lambda\, \mu_{-i}\big((x,i);\pi\big)v = \lambda \min\left\{\frac{M-p}{p}, 1\right\} v, \tag{2}$$

where $\mu_{-i}\big((x,j);\pi\big)$ denotes the marginal probability that $\mu_{-i}(\pi)$ assigns to (x,j). This constraint is guaranteed by the condition $\frac{1}{2}M \leq p \leq \frac{\lambda v}{b-c+\lambda v}M$, which in turn is implied by the conditions on λ and p^* in the theorem.

Next we demonstrate that either $b - c = \frac{M-p^*}{p^*}\lambda v^*$ or $p^* = 0$. If $0 < p^* < \frac{1}{2}M$ and the incentive constraints are satisfied, then in the promise-making stage each player can memorize all of his teammate's tasks with probability 1 and still have at least two empty slots left over, so each player can promise an additional task for which the incentive constraint is also satisfied.[12] Hence in any optimal equilibrium in which $p^* > 0$, we must have $p^* \geq \frac{1}{2}M$. Therefore, assuming $p^* > 0$, we can simplify each incentive constraint to $b - c \geq \frac{M-p^*}{p^*}\lambda v^*$, or, equivalently, $p^* \leq \frac{\lambda v^*}{b-c+\lambda v^*}M$. However,

[12]Here we use the assumption that M is even. If M were odd, an optimal symmetric contract might leave the one leftover slot empty, but there would be a superior asymmetric contract in which one player uses the leftover slot to make an extra promise and the other player uses it for monitoring.

if this constraint is slack, then it would improve matters to marginally increase v^*, reducing the severity of punishments (which occur with positive probability) without disrupting any incentive constraints. Hence either $b - c = \frac{M-p^*}{p^*}\lambda v^*$ or $p^* = 0$.

Now we consider the problem of choosing p^* and v^* optimally. Clearly, if $p^* = 0$ then it is optimal to set $v^* = 0$, attaining zero utility for both players. So suppose that $p^* > 0$; then an optimal contract solves

$$\begin{aligned} \max_{p\in\mathbb{I}, v\in[\underline{v},0]} \; & 2p\left(\lambda(2b-c) + (1-\lambda)\lambda\frac{M-p}{p}v\right) \\ \text{s.t.} \quad & \frac{1}{2}M \le p \le \frac{\lambda v}{b-c+\lambda v}M. \end{aligned} \tag{3}$$

Since the incentive constraints bind, it suffices to solve

$$\begin{aligned} \max_{p\in\mathbb{I}} \; & 2p\big(\lambda(2b-c) + (1-\lambda)(b-c)\big) \\ \text{s.t.} \quad & \frac{1}{2}M \le p \le \frac{\lambda \underline{v}}{b-c+\lambda \underline{v}}M. \end{aligned} \tag{4}$$

Clearly $\lambda \ge \frac{b-c}{\underline{v}}$ is a necessary condition for this problem to have a solution. Since the objective and the constraints are linear in p, it is easy to see that for $\lambda \ge \max\left\{\frac{b-c}{\underline{v}}, \frac{c-b}{b}\right\}$ it is optimal to maximize p subject to the constraints; i.e., set $p^* = \left\lfloor \frac{\lambda\underline{v}}{b-c+\lambda\underline{v}}M \right\rfloor$ and $v^* = \frac{p^*(b-c)}{\lambda(M-p^*)}$. In contrast, for $\lambda < \max\left\{\frac{b-c}{\underline{v}}, \frac{c-b}{b}\right\}$ the players cannot earn positive utility from this problem (if it has a solution), so it is optimal to set $p^* = v^* = 0$. □

Note that even in the special case of $\lambda = 1$, the optimal contract still must devote resources to monitoring, in order to maintain incentive compatibility. In particular, when $\underline{v}$ is close to zero, incentive compatibility requires that close to half of the players' memories should be devoted to monitoring.

Because the optimal linear contract treats each task separately and symmetrically, a player is willing to complete every task she remembers so long as she is willing to complete any single task. Note that if punishment per task were unbounded ($\underline{v} = -\infty$) it would be possible to punish severely enough to optimally devote only one slot to monitoring and implement the maximal number of promises ($M - 1$). In the following section, in which we consider nonlinear counting contracts, we show that even if punishment can be unboundedly severe, it will not always be optimal to implement the maximal number of promises, or even to complete all promises that are recalled.

4 General counting contracts

The linearity assumption made in the previous section simplified the analysis, since a linear contract treats each task separately. However, under a linear contract there is a significant likelihood that the players will not recall all of their promises, which means they face a significant likelihood of being punished. Intuitively, a linear contract might be improved on by "forgiving" a player who completes all but the last few of her promised tasks. Of course, she will not fulfill any promises for which she will be forgiven, so some of her promises will be "empty." The drawback of such a contract is that, in the unlikely event in which she recalls all of her promises, she will not fulfill all of them. The benefit is that in the very likely event that she recalls less than all of her promises, she will not be punished too severely.

In this section we analyze non-separable contracts, in which a player's punishment can depend in an arbitrary way on the number of her unfulfilled promises that are recalled by her teammate. The main tradeoff in designing optimal non-separable contracts is between using information efficiently and ensuring that a player recalls sufficiently many promises. To provide incentives for a player to complete any given number of recalled tasks, it is most cost-effective to use the most informative signal for punishment. Mirrlees (1974, 1999) proposed this basic intuition, but our model raises the complication that a player may be able to move the support of the monitoring distribution by fulfilling enough promises. If a player recalls a small number of promises, then being punished only for the worst outcome (the maximal number of unfulfilled promises are discovered) provides the most efficient incentives. However, if a player happens to recall a large number of promises, she may have incentive to fulfill only enough of them that the worst outcome cannot arise. Thus she may leave some promises unfulfilled; these are *empty promises.* A memory slot devoted to an empty promise is a memory aid: it helps the player recall more promises, yielding a first-order stochastic improvement in the number of promises she recalls. At the same time, an empty promise uses up a memory slot that could be used towards obtaining a more informative monitoring signal. The better the players' memories, the more slots they devote to "earnest promises" and the fewer slots they need devote to monitoring and empty promises.

We show that the optimal symmetric contract generally takes a specific, simple form. Let p be the number of promises each player makes, and let F be the number of memory slots she devotes to monitoring her teammate.[13] Properties 1–4 correspond to Theorems 2–5, which hold for all M, while properties 5–8 are proven for $M \leq 5$ in Theorem 6.

1. If λ is sufficiently high, then the optimal contract is linear with $p^* = p = M - 1$ and $F = 1$ (;

[13] Except in special cases in which the optimal contract has $p = F$, and thus full rank. In such cases (which arise only for a narrow range of λ) the players still use cutoff strategies, but the contract may be more complicated than described here. In particular, properties 6–8 may not hold.

2. If λ is sufficiently low, then it is optimal to do nothing;

3. For a range of parameters, it is optimal for players to make empty promises;

4. Players make empty promises if and only if they make less than the maximum number of promises $(M-1)$;

5. Each player performs as many tasks as she recalls up to a cutoff p^*;

6. Each player is punished if and only if her teammate discovers the maximum number (F) of her unfulfilled promises;

7. Promises (p) and cutoffs (p^*) increase in λ, while empty promises $(p-p^*)$ and monitoring (F) decrease in λ;

8. The number of monitoring slots (F) is the same as or one more than the number of empty promises;

We begin by developing the problem of designing an optimal symmetric contract. Let $s: \{0,\ldots,p\} \to \{0,\ldots,p\}$ be a player's strategy that maps the number of her promises that she recalls to the number of tasks she performs; i.e., s expresses the task completion strategy d_i in a simpler form.[14] Naturally, the strategy must satisfy $s(k) \le k$. To determine whether a strategy s is incentive compatible, we need to consider the probability distribution over $f = \left|\hat{m}_{-i} \setminus A_i\right|$ conditional on $s(k)$ for each $k = 0,\ldots,p$. Given F and p, if a player fulfills a of her promises, the probability that her teammate will find f of her unfulfilled promises is given by the compound hypergeometric-binomial distribution (this distribution is studied in Johnson and Kotz 1985):

$$g(f,a) = \sum_{k=f}^{F} \frac{\binom{p-a}{k}\binom{a}{F-k}}{\binom{p}{F}} \binom{k}{f} \lambda^f (1-\lambda)^{k-f}. \tag{5}$$

To interpret Eq. 5, observe that in order to discover f unfulfilled promises of player i, player $-i$ must have drawn $k \ge f$ promises from the $p-a$ promises player i failed to fulfill, and $F-k$ promises from the a promises player i fulfilled; this is described by a hypergeometric distribution. Of these k promises, player $-i$ must then recall exactly f; this is described by a binomial distribution.

Given a strategy s, the probability of performing a tasks is

$$t_s(a) = \sum_{a'=a}^{p} \mathbb{I}\big(s(a') = a\big) \binom{p}{a'} \lambda^{a'} (1-\lambda)^{p-a'}. \tag{6}$$

[14] With simple strategies, she need not differentiate promises according to their identities. By restricting $s(k)$ to be a number rather than a random variable, we use the fact that to randomize, the player must be indifferent, but then it would be optimal for her to put probability 1 on the highest number in the support of her randomization.

The incentive constraints for strategy s are

$$\sum_{f=0}^{F} v(f)\Big(g\big(f,s(k)\big)-g(f,\ell)\Big) \geq \big(s(k)-\ell\big)(c-b) \quad \text{for all } \ell \leq k \text{, and all } k. \tag{7}$$

We call these "downward" constraints when $\ell < s(k)$, and "upward" constraints when $s(k) < \ell \leq k$. The problem of *optimally implementing* strategy s at minimum cost is

$$\max_{v} \sum_{a=0}^{p} t_s(a) \sum_{f=0}^{F} v(f) g(f,a) \quad \text{s.t. } v(f) \leq 0 \text{ for all } f \text{, and Eq. 7.} \tag{8}$$

Let $h_v(a) \equiv \sum_{f=0}^{F} v(f)g(f,a)$ be the expected punishment for fulfilling a promises. An optimal contract maximizes expected benefits net of punishments, subject to incentive compatibility:

$$\max_{p,F,s,v} \sum_{a=0}^{p} \binom{p}{a} \lambda^a (1-\lambda)^{p-a} \Big(s(a)(2b-c) + h_v(s(a))\Big) \quad \text{s.t. } v(f) \leq 0 \text{ for all } f \text{, and Eq. 7.} \tag{9}$$

Next we characterize some elementary properties of optimal contracts.

Lemma 1. *Suppose λ is sufficiently high that it is optimal for the players to perform at least some tasks. Then there exists an optimal contract satisfying the following:*

1. *Uniform task completion: Each player randomizes uniformly over which $s(k)$ tasks to complete when she recalls k of her promises;*
2. *Memorize all your own promises: $\mu_i(\pi_i) = 1$;*
3. *Increasing strategies: If $k \geq \ell$, then $s(k) \geq s(\ell)$;*
4. *Jump to the maximum: If $s(k) > s(k-1)$ then $s(k) = k$;*
5. *Upward constraints do not bind.*

Proof. Proof of this and all succeeding results are given in the appendix. □

We say that a strategy is *promise keeping* if $s(a) = a$ for all $a \leq p$, and has *empty promises* otherwise. Let $p^* \equiv \max_a s(a)$ be the largest number of promises that are ever fulfilled under strategy s. We call s a *cutoff strategy* if $s(a) = a$ for $a \leq p^*$ and $s(a) = p^*$ for all $a > p^*$. The following lemma shows that promise keeping is optimally implemented by a linear contract, and that promise keeping is optimal among strategies satisfying $s(p) = p$.

Lemma 2 (Promise-keeping with linear contracts)**.** *For any M and any p, promise keeping is optimally implemented by a linear contract with $v(f) = f\frac{p}{\lambda F}(b-c)$, delivering expected social utility $2p(b - c + b\lambda)$.*

This result is used to prove the next two theorems, the first of which shows that linear contracts are optimal when λ is sufficiently high. Intuitively, when λ is very high the players expect to recall most or even all of their promises. Since each player must devote at least one memory slot to monitoring, setting $p = M - 1$ maximizes the number of promises. At the same time, monitoring is very effective, so even with only one monitoring slot the punishment need not be too large to induce a cutoff of $p^* = p$. Finally, with one monitoring slot, every task is treated identically, so the contract is linear.

Theorem 2. *There exists $\overline{\lambda} < 1$ such that, for all $\lambda \geq \overline{\lambda}$, $p^* = p = M - 1$ under the optimal contract. Furthermore, $F = 1$, $v(0) = 0$, $v(1) = (M-1)\frac{b-c}{\lambda}$, and all incentive constraints are satisfied with equality.*

Our next theorem shows that it is optimal to do nothing when λ is sufficiently low. Intuitively, when λ is very low the players expect to recall few or none of their promises. At the same time, monitoring is not very effective, so large punishments would be needed to induce the players to perform what few tasks they might recall. Rather than risk incurring these punishments, it is better not to do any tasks at all.

Theorem 3. *There exists $\underline{\lambda} > 0$ such that, for all $\lambda \leq \underline{\lambda}$, $p^* = 0$ under the optimal contract. Furthermore, $v(f) = 0$ for all f.*

Between these extremes, however, it may be optimal for players to make empty promises, as demonstrated in the next theorem.

Theorem 4 (Empty promises)**.** *For any M there exists $\alpha(M) \in (1,2)$ such that if $b < c < \alpha(M)b$, then there is $\bar{\lambda} > \frac{c-b}{b}$ such that for all $\lambda \in (\frac{c-b}{b}, \bar{\lambda})$, the optimal contract involves empty promises.*[15]

The result follows from Lemma 5, in the appendix, which shows that although promise-keeping in the range of parameter values above gives positive social utility, it is dominated by making roughly half as many promises and fulfilling at most one one of them.

The following theorem shows that empty promises can be optimal only when it is optimal to make less than the maximal number of promises. In other words, if the players make as many promises as possible, they should intend to follow through on them.

[15] *Proof for M odd complete; proof for M even in progress.*

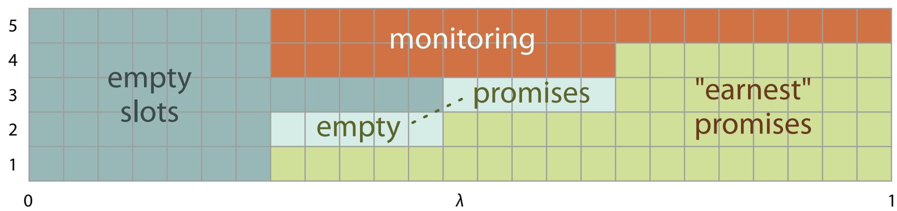

FIGURE 2: OPTIMAL MEMORY ALLOCATION FOR $M = 5$. Each column represents the optimal memory allocation for a particular value of λ, in steps of 0.04 from 0.02 to 0.98, where $b = 2$ and $c = 3$. In row a, $s(a) = a$ in the "earnest" promises region, while $s(a) < a$ in the empty promises region. Any strategy characterized by Lemma 1 can be represented in this format.

Theorem 5. *In an optimal contract that implements fulfilling a positive number of promises, $p^* < p$ if and only if $p < M - 1$.*

The intuition for this result is that empty promises serve as memory aids. By memorizing more promises than she plans to fulfill, a player attains a first order stochastic improvement in the number of tasks she will complete according to her plan. However, the corresponding increase in the number of promises she leaves unfulfilled will lead her to expect a more severe punishment unless the contract is *forgiving*: if it does not punish her when the other players find only a "small" number of her unfulfilled promises. But if she makes the maximal number of promises ($p = M - 1$), then the contract cannot be forgiving, since it must punish her when it finds one unfulfilled promise.

Our analysis enables us to prove the following theorem for $M \leq 5$, which states that the optimal contract implements cutoff strategies, that both the number of promises and the cutoff increase in λ, and that both monitoring and the number of empty promises decrease in λ. A specific example is visualized in Figure 2.

Theorem 6. *Suppose $M \leq 5$. Then for any λ the optimal contract implements cutoff strategies, with $p - p^* \leq F \leq p - p^* + 1$. Both p and p^* increase in λ, while both F and $p - p^*$ decrease in λ. Furthermore, the social welfare of the optimal contract is strictly increasing and concave in λ.*

This is proven using our previous results and two additional lemmas in the appendix. The first main ingredient is Lemma 6, which shows (for arbitrary M) that if F is not too large relative to the number of empty promises and a technical condition is satisfied, then a cutoff is optimal and the optimal social welfare from implementing the cutoff is given by

$$2\sum_{a=0}^{p}\binom{p}{a}\lambda^a(1-\lambda)^{p-a}\Big((2b-c)s(a)+\frac{(c-b)g(F,s(a))}{g(F,p^*)-g(F,p^*-1)}\Big). \tag{10}$$

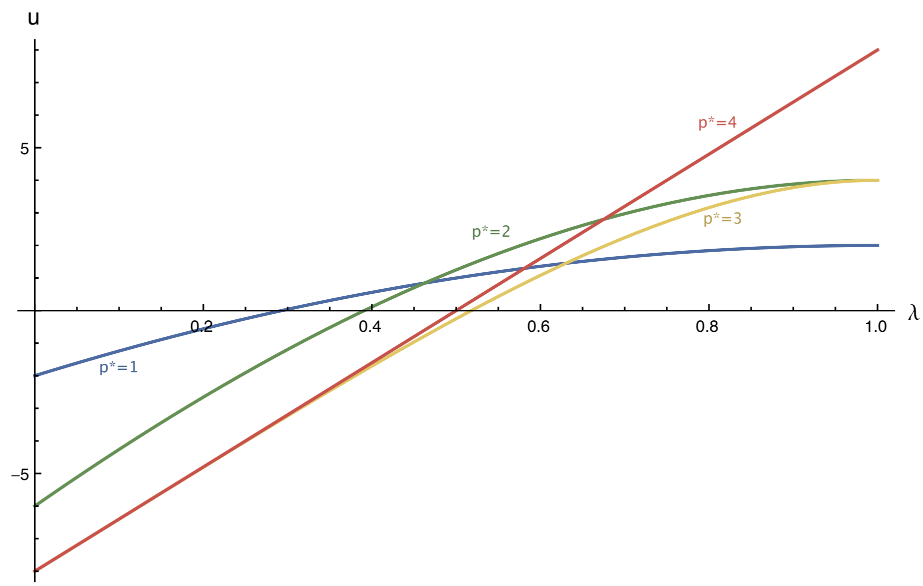

FIGURE 3: SOCIAL WELFARE ENVELOPE FOR $M = 5$. For each $p^* \in \{1, \ldots, M-1\}$, the value of the best strategy is plotted as a function of λ. In each case the best strategy is a cutoff strategy.

The second main ingredient is Lemma 8, which shows that Eq. 10 satisfies single-crossing and concavity properties. This is illustrated in Figure 3, in which the optimal social welfare for a specific example is given by Eq. 10 for $p^* \in \{1, 2, 3, 4\}$.

We are working to extend these ideas to larger M. Specifically, we conjecture that cutoff strategies are always optimal, and that the monotonicity results extend (except for certain special cases that may arise from dividing memory equally between own promises and monitoring). Reaching these more general conclusions requires ruling out two possibilities that could violate the assumptions of Lemma 6 (in a region of optimality) but which do not arise for $M \leq 5$. These results have held in all our numerical computations. Illustrative examples are shown in Figure 4.

5 Asymmetric memories

In this section we study optimal linear contracts when players can differ in both memory capacity and recall probability, and allow for asymmetric contracts. By focusing on linear contracts, we abstract away from empty promises in order to highlight the allocation of monitoring responsibilities and the role of individual rationality constraints (for a organizational setting in which this is without loss of generality, see Section 6.2). We show below in Corollary 1 that the player with the weaker memory is optimally given greater responsibility for monitoring.

Denote the memory capacity and recall probability of player i by M_i and $\lambda_i \in (0, 1)$, respectively.

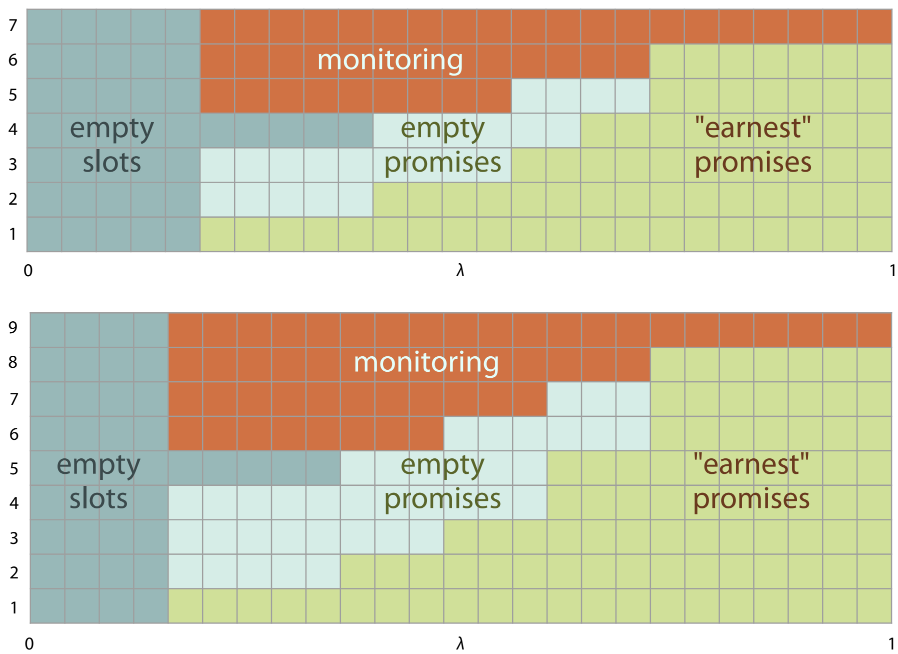

FIGURE 4: OPTIMAL MEMORY ALLOCATION FOR $M = 7$ (TOP) AND $M = 9$ (BOTTOM). See Figure 2 for explanation.

In an asymmetric setting, the optimal contract chooses v_i, p_i, for $i = 1, 2$ to maximize

$$(2b - c) \sum_{i=1,2} p_i \lambda_i + \sum_{i=1,2} (1 - \lambda_i) \lambda_{-i} v_i p_i \mu_{-i}\big((x, i); \pi\big) \tag{11}$$

$$\begin{aligned} \text{subject to} \quad & \text{Feasibility: } \underline{v} \leq v_i \leq 0 \text{ and } p_i \in \{0, 1, 2, \ldots, M_i\} \\ & \text{IC}_i\text{:} \quad b - c \geq \lambda_{-i} v_i \mu_{-i}\big((x, i); \pi\big) \text{ if } p_i > 0, \qquad (12) \\ & \text{IR}_i\text{:} \quad p_i \lambda_i (b - c) + p_{-i} \lambda_{-i} b + (1 - \lambda_i) \lambda_{-i} p_i v_i \mu_{-i}\big((x, i); \pi\big) \geq 0, \end{aligned}$$

where IR_i is the constraint that player i prefers the contract to autarky.

Due to the dimensionality of the problem (there are seven parameters: $M_1, M_2, \lambda_1, \lambda_2, b, c$ and $\underline{v}$), we relax the problem by ignoring integer constraints (e.g., $p_i \in \{0, 1, \ldots, M_i\}$). Since the players randomize uniformly over which promises to monitor, $\mu_{-i}\big((x, i); \pi\big) = \min\big\{\frac{M_{-i} - p_{-i}}{p_i}, 1\big\}$. Then substituting each binding IC_i into both IR_i's and the objective yields a reduced form problem:

$$\max_{p_1,p_2}\big\{(p_1+p_2)(b-c)+(p_1\lambda_1+p_2\lambda_2)b\big\} \tag{13}$$

$$\begin{aligned} \text{s.t.}\quad & \text{Feasibility: } 0\le p_i\le M_i \\ & \overline{\text{IC}}_i\text{:}\quad b-c\ge \lambda_{-i}\underline{v}\min\left\{\tfrac{M_{-i}-p_{-i}}{p_i},1\right\}\ \text{ if } p_i>0 \qquad (14)\\ & \overline{\text{IR}}_i\text{:}\quad p_i(b-c)+p_{-i}\lambda_{-i}b\ge 0. \end{aligned}$$

The constraint $\overline{\text{IC}}_i$ incorporates the bound on punishments into IC_i. The solution to the reduced form problem is characterized by four parameters,

$$\frac{M_1}{M_2},\quad \sigma_1\equiv\frac{b-c}{\lambda_1\underline{v}},\quad \sigma_2\equiv\frac{b-c}{\lambda_2\underline{v}},\quad \gamma\equiv-\frac{b}{\underline{v}}>0, \tag{15}$$

where $\sigma_i\in(0,1)$ captures the ratio of net benefit from completing a task to the expected punishment for player $-i$, and $\gamma>0$ is the ratio of the task benefit to the maximal punishment.

Theorem 7. *Suppose that* $\lambda_i\ge\max\{\frac{b-c}{\underline{v}},\frac{c-b}{b}\}$ *for* $i=1,2$. *Then the optimal linear contract is characterized by four binding constraints: the original* IC_1 *and* IC_2*, and two additional binding constraints determined by* $\frac{M_1}{M_2}$*,* σ_1*,* σ_2*, and* γ *according to*

<table>
<tr><td></td><td></td><td colspan="3">M_1/M_2</td></tr>
<tr><td></td><td></td><td>$(0,\frac{\sigma_1\sigma_2+\gamma}{\sigma_1(1+\gamma)})$</td><td>$(\frac{\sigma_1\sigma_2+\gamma}{\sigma_1(1+\gamma)},\frac{\sigma_2(1+\gamma)}{\sigma_1\sigma_2+\gamma})$</td><td>$(\frac{\sigma_2(1+\gamma)}{\sigma_1\sigma_2+\gamma},\infty)$</td></tr>
<tr><td rowspan="3">$\frac{\sigma_2}{\sigma_1}\frac{\sigma_1-\gamma}{\sigma_2-\gamma}$</td><td>$(\frac{1}{\sigma_1},\infty)$</td><td rowspan="2">$\overline{IR}_1$ and $\overline{IC}_1$</td><td colspan="2">$\overline{IR}_1$ and $\overline{IC}_2$</td></tr>
<tr><td>$(\sigma_2,\frac{1}{\sigma_1})$</td><td>$\overline{IC}_1$ and $\overline{IC}_2$</td><td rowspan="2">$\overline{IR}_2$ and $\overline{IC}_2$</td></tr>
<tr><td>$(0,\sigma_2)$</td><td colspan="2">$\overline{IR}_2$ and $\overline{IC}_1$</td></tr>
</table>

For each case, the number of promises is given by

$$\begin{aligned}
\overline{IC}_1 \text{ and } \overline{IC}_2: &\qquad p_1=\frac{M_1-\sigma_1M_2}{1-\sigma_1\sigma_2}\ \text{ and } p_2=\frac{M_2-\sigma_2M_1}{1-\sigma_1\sigma_2},\\
\overline{IR}_1 \text{ and } \overline{IC}_1: &\qquad p_1=\frac{M_2}{1+\gamma}\frac{\gamma}{\sigma_2}\ \text{ and } p_2=\frac{M_2}{1+\gamma},\\
\overline{IR}_1 \text{ and } \overline{IC}_2: &\qquad p_1=\frac{\gamma M_1}{\sigma_1\sigma_2+\gamma}\ \text{ and } p_2=\frac{\sigma_2M_1}{\sigma_1\sigma_2+\gamma},\\
\overline{IR}_2 \text{ and } \overline{IC}_1: &\qquad p_1=\frac{\sigma_1M_2}{\sigma_1\sigma_2+\gamma}\ \text{ and } p_2=\frac{\gamma M_2}{\sigma_1\sigma_2+\gamma},\\
\overline{IR}_2 \text{ and } \overline{IC}_2: &\qquad p_1=\frac{M_1}{1+\gamma}\ \text{ and } p_2=\frac{M_1\gamma\sigma_1}{1+\gamma}.
\end{aligned}$$

If $\lambda_i<\max\{\frac{b-c}{\underline{v}},\frac{c-b}{b}\}$ *for some* i *then the optimal contract has* $p_1=p_2=0$ *and* $v_1=v_2=0$.

Note that if $M_1 = M_2$ and $\lambda_1 = \lambda_2$, the optimal contract has both $\overline{\text{IC}}_1$ and $\overline{\text{IC}}_2$ binding, showing that the optimal symmetric linear contract we found in Section 3 is also the optimal contract (modulo an integer problem). Whenever $\overline{\text{IC}}_i$ binds, maximal punishments ($v_i = \underline{v}$) are delivered to player i whenever an unfulfilled promise is discovered. If $\overline{\text{IC}}_i$ is slack in Theorem 7, punishments are less severe because the IR_i constraint would otherwise be violated.

To understand the role of the IR constraints, suppose that player i has a larger memory than another. Without the IR_i constraint, if the difference in memory size is sufficiently large, player i should optimally make all the promises and player $-i$ should perform all of the monitoring. However, ensuring that the contract is individually rational for both players requires that the player $-i$ should still take on some responsibility for accomplishing tasks. The following corollary clarifies how the optimal contract and number of promises vary with the qualities of the players' memories.

Corollary 1. *Suppose $\lambda_i \geq \max\{\frac{b-c}{\underline{v}}, \frac{c-b}{b}\}$ for $i = 1, 2$. Starting from symmetry ($M_1 = M_2$ and $\lambda_1 = \lambda_2$), a marginal improvement in the memory of player 2 (either M_2 or λ_2) increases the number of promises player 2 makes (reducing her utility) and increases the utility of player 1. In particular, the optimal number of promises player 1 is supposed to make decreases.*

Relative to the symmetric setting, the player with the worse memory benefits not only from the greater number of promises her teammate optimally makes, but also from a reduction in the number of promises she will make. This is because in order to accomplish a greater number of tasks, she must increase her monitoring of the player with the better memory.

Let us develop graphical intuition for Theorem 7. Without loss of generality, suppose that $M_2 \geq M_1$. The problem is visualized in Figure 5, which depicts the promises of player 1 on the horizontal axis and those of player 2 on the vertical axis. These are bounded by the rectangle corresponding to their memory capacities. The requirement that $\lambda_i \geq \frac{c-b}{b}$ for $i = 1, 2$ guarantees that the set of non-zero IR promise pairs is nonempty. The requirement that $\lambda_i \geq \frac{b-c}{\underline{v}}$ ensures that each $\overline{\text{IC}}_i$ can be satisfied when player $-i$ monitors maximally with maximal punishments. In the case of Figure 5, the intersection of $\overline{\text{IC}}_1$ and $\overline{\text{IC}}_2$ occurs above the IR region. This implies that if $\lambda_1 = \lambda_2$, as in the figure, then the social indifference curves optimally select the promise levels $p_1 = 5$ and $p_2 = 7$. Hence the larger burden falls on the player with the larger memory. However, whether the intersection of $\overline{\text{IC}}_1$ and $\overline{\text{IC}}_2$ occurs above, below, or within the IR region depends on the parameters of the problem.

6 Larger teams

This section expands the analysis to larger teams, demonstrating that our results extend naturally to n-player contracts. Motivated by applications to organizational structure, we propose two extended

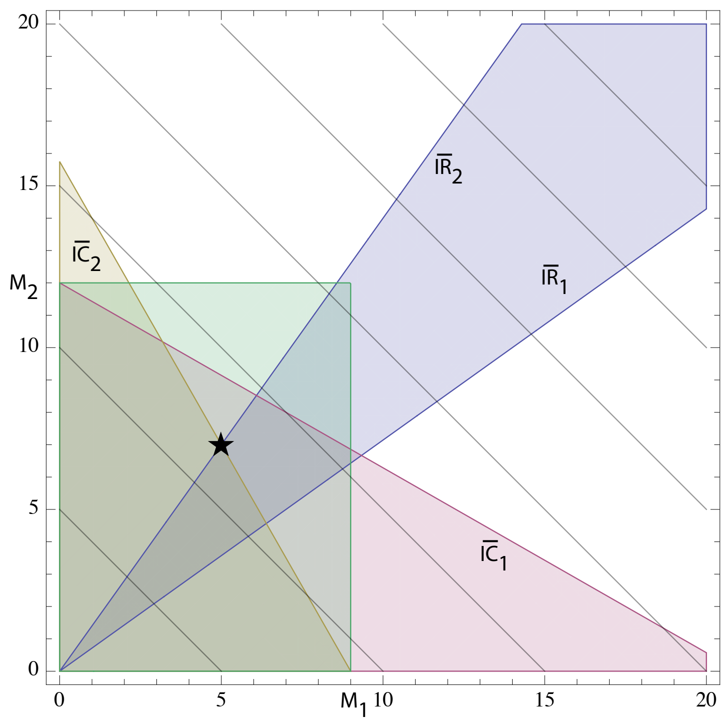

Figure 5: The optimal linear contract in an asymmetric example. The vertical axis measures promises of player 1, and the horizontal axis measures promises of player 2. Diagonal lines represent social indifference curves. The optimal contract (modulo an integer problem) implements the promise vector ★, which attains the highest social welfare in the intersection of the four regions bounded by $\overline{\mathrm{IR}}_1$, $\overline{\mathrm{IR}}_2$, $\overline{\mathrm{IC}}_1$, $\overline{\mathrm{IC}}_2$, M_1, and M_2. The parameters are $b = 2$, $c = 3$, $\lambda_1 = \lambda_2 = 0.7$, $\overline{v} = -2.5$, $M_1 = 9$, and $M_2 = 12$.

interpretations of our framework.

6.1 Extending our results

Consider a team $\mathcal{I} = \{1, \ldots, n\}$, $n > 2$. In this more general setting, scale the benefits of each task so that each player reaps a benefit b from a task she performs herself but a benefit $\frac{b}{n-1}$ from a task performed by any other player. In any n-player symmetric equilibrium in which each task is monitored by a single player, it does not matter whose memory slot is used to monitor whom. Hence our results for two players extend naturally to a class of symmetric n-player equilibria, as formalized in Theorem 8, below.

A memory strategy profile μ is *simple and symmetric* if (i) each player's allocation of memory between own promises and monitoring is deterministic; (ii) there exists k such that the total number of slots allocated to monitoring each player is exactly k; (iii) each promise is monitored by at most one player; (iv) the probability that any particular k promises of player i, $\{(x_1, i), \ldots, (x_k, i)\} \subset \pi_i$, are monitored is identical. We say that a contract and a perfect Bayesian equilibrium in simple and symmetric memory strategies in the game it induces are (together) *optimal* if they yield expected payoffs that are Pareto optimal in the set of all such expected payoffs. Such a contract is also (itself) *optimal*.

Theorem 8. *Consider any symmetric counting contract v and an equilibrium in simple strategies with $n = 2$, number of promises p, number of monitoring slots F, and task completion strategy s. Then for $n > 2$ and the same contract v, there exists an equilibrium in simple and symmetric strategies with the same p, F, and s for each player. Moreover, if the contract v is optimal for $n = 2$ it remains optimal for $n > 2$. The converse also holds.*

Sketch of proof. Fix a symmetric counting contract for two players. Arrange the team of n players around a circle, and assign each player to monitor the teammate to her right, using the same contract as for the two-player setting. Since the IC constraints are unchanged, this yields an equilibrium in simple and symmetric strategies. Given the scaling of benefits for the larger team, the $n = 2$ and $n > 2$ optimization problems share the same objective function. Hence if the two-player contract and equilibrium are optimal, the n-player contract is clearly optimal in the class of counting contracts paired with simple symmetric strategies.

Conversely, fix a symmetric counting contract and an equilibrium in simple and symmetric strategies with $n > 2$. Select any two players and assign them to monitor only each other, but in the same amount as in the $n > 2$ equilibrium. Since the original equilibrium for $n > 2$ was simple and symmetric, once again the IC constraints are unchanged and the two problems share the same objective. □

In the class of linear contracts, conclusions about asymmetric settings also extend naturally. First, in a multiplayer setting with symmetric players, a symmetric contract is optimal (ignoring integer issues), as can easily be seen by generalizing Figure 5 to multiple dimensions. Starting from symmetry, if player 1's memory improves slightly (in terms of capacity or reliability), then player 1's task burden increases slightly, while the other players' burdens decrease slightly. Just as in the two-player case, individual rationality constraints can bind when the parameters are further away from symmetry. However, with three or more players, it can be individually rational for some players to specialize purely in monitoring, and for others to specialize purely in performing tasks; this may indeed be optimal for very asymmetric parameter values.

6.2 One person, one slot

Suppose that tasks are sufficiently complex that each player in the team can handle at most one task; effectively, each player has only one memory slot. Then, a reinterpretation of Section 5 suggests that the restriction to linear contracts therein is natural, and the results from that section suggest a natural division of players into sub-teams of similar "ability."

If there are several types of players with different recall abilities, we can view a player i in the model as representing a sub-team of M_i players, all with recall parameter λ_i. In this setting, only a linear contract can treat players of the same type equally. The $\overline{\text{IR}}$ constraints studied in Section 5 must be adapted slightly: those players assigned to perform tasks should either expect to benefit enough from the tasks of others to outweigh their costs of effort, or be paid a fixed amount ex ante.

In principle, this interpretation is flexible enough to allow people in the same sub-team to monitor each other. However, doing so would be equivalent to allowing two sub-teams of players of the same type. Hence this interpretation provides a justification for the linear contracts in Section 5 based on these primitives, if the individual rationality constraints are adjusted accordingly.

6.3 Organizational structure

The extension to n-player teams raises interesting questions about optimal organizational structure. Viewing our framework from a larger perspective, suppose that a principal can hire a team of n agents to perform tasks and monitor each other. The principal reaps the entire benefit B from each task, but cannot observe who performed it. He makes each agent a take-it-or-leave-it offer comprising:

- A fixed ex ante payment, no less than zero (agents have limited liability);
- A payment b for each task completed by the team;

- An informal contract of punishments v, which will be implemented by the team members in equilibrium (punishments destroy surplus; they are not transfers to the principal);

- A selected equilibrium in the game induced by the contract.

Each agent accepts the offer if and only if her expected utility from the offer is at least as high as her exogenous outside option. In this setting, the agents' incentive constraints are simply Eq. 7. The principal's optimization problem is similar to Eq. 9, except that $2b$ is replaced by $B - nb$ (the principal's net profit per task), and that b is a choice variable rather than a parameter.

Were the fixed ex ante payment not bounded below by zero, the optimal contract would set $b \geq c$ to make the agents willing to perform the tasks without any wasteful punishments, and then extract the surplus by asking them to pay the principal ex ante for access to the tasks. But because the principal must pay the team nb for each task, if $b \geq c$ the limited liability constraint will prevent the principal from extracting all the surplus whenever $B < nc$. The informal contract v thus serves as a costly mechanism for the principal to extract more of the surplus even when the agents have limited liability. In an optimal symmetric contract when $B < nc$, the principal pays the agents nothing ex ante (he would rather increase b than the ex ante payment), and chooses b and v to maximize net profit, subject to the constraint that the agents must be willing to accept the offer.

However, if the principal may make different offers to different agents, he may be able to improve over the symmetric contract by assigning one or more players to specialize purely in monitoring. These "supervisors" do not need to be paid for completed tasks. Instead, they can be compensated for their opportunity costs with ex ante payments. This allows the principal to pay the team just $\hat{n}b$ per task, where $\hat{n}$ is the number of non-supervisory (promise-making) agents. Since increasing b is less costly when some agents are supervisors, the non-supervisory agents can be induced to complete more tasks. When the agents are observably heterogeneous, the principal will select his supervisors endogenously based on their memory abilities. For the special case in which the principal is restricted to offer linear contracts, the results of Section 5 suggest that supervisors should be drawn from among those players with the weakest memories. General counting contracts in this environment are an object of continuing study.

7 Discussion

We study a team setting where forgetful players with limited memories have costly but socially efficient tasks to complete and characterize optimal contracts when the team's collective memory serves as a costly monitoring device. We show that promise keeping is optimally implemented by linear contracts, and that linear contracts are optimal only when players are not very forgetful.

Otherwise, optimal contracts induce players to make empty promises, and forgive some unfulfilled promises. As players become more forgetful, they make more empty promises and devote more of their memories to monitoring. Our model provides a simple formulation for studying some of the fundamental tradeoffs arising when bounded memories and incomplete contracts intersect, but can also accommodate extensions that can be used to study interesting new questions or cast new light on classic problems.

Our conclusions about asymmetric linear contracts can be viewed as endogenously allocating the responsibility for monitoring more to one player than the other. With three or more players, or gains from specialization in monitoring, such contracts could endogenously select the player with the worst memory as the "supervisor." Furthermore, our results for general counting contracts can be applied even when a vertical supervision structure is imposed exogenously, to show that empty promises and forgiving contracts can be optimal. Our framework could be extended to study how to select between horizontal and vertical supervision structures.

We have assumed that a player's own promises and her partner's promises occupy the same memory footprints. If it were less costly for her to store her partner's promises, or if it were easier to recall them, we expect the qualitative results to remain unchanged. It would also be interesting to extend the analysis to tasks that may be heterogeneous in their complexity, complementary in the utility function, or complementary to store or recall in memory.

We are currently working on understanding whether players can be induced to truthfully reveal private information about their memory capacities. In addition, the flavor of our results may extend to interesting applications in which "recalling" a promise is interpreted as having the opportunity to implement it, which is private information. For instance, if politicians privately learn whether they can implement their campaign promises, and both they and their constituencies have bounded and stochastic attention spans, then in an optimal "contract" politicians may make empty promises and constituents may forgive them for doing so.

We are also interested in allowing the memory bound to adjust endogenously to the complexity of the information being memorized. Cowan (2000), among others, suggests that the number of effective slots in memory decreases in the complexity of the information stored. If the agents can record some of the details of their tasks and then refer to their records when performing the tasks, they may have to less to memorize for each task. That is, the memory bound can be relaxed at the cost of creating and using physical records, such as less incomplete contracts. This tradeoff can be used to characterize the optimal level of contractual detail.

A Appendix: Proofs

Proof of Lemma 1.

1. *Uniform task completion*: Under simple strategies, player $-i$'s monitoring is uniform, and so player i is indifferent over which $s(k)$ promises to fulfill. Hence uniform task completion is without loss of generality.

2. *Memorize all your own promises:* By uniform task completion, making a promise that you do not intend to memorize only increases your expected punishment.

3. *Increasing strategies:* By revealed preference, if doing $s(\ell)$ is preferred to doing any $\ell' \leq \ell$ tasks when ℓ tasks are remembered, then $s(\ell)$ remains preferred to any $\ell' \leq \ell$ tasks when $k \geq \ell$ tasks are remembered.

4. *Jump to the maximum:* Similarly, if $s(k) > s(k-1)$ it cannot be that doing $s(k)$ tasks is possible if only $k-1$ tasks are remembered.

5. *Upward constraints do not bind:* Suppose to the contrary that the upward constraint for fulfilling $s(k)$ promises rather than ℓ promises binds, with $s(k) < \ell \leq k$. Then player i is indifferent between $s(k)$ and ℓ, and therefore the incentive constraint for choosing ℓ rather than ℓ' when she recalls k of her promises are satisfied for all $\ell' \leq k$. Hence it is incentive compatible for her to choose ℓ instead of $s(k)$, and doing so leads to a strict improvement in the objective (Eq. 9). □

Proof of Lemma 2. By incentive-compatibility, to ensure that a rather than $a-1$ promises are fulfilled when a are recalled, we need $h_v(a-1) \leq h_v(a) + b - c$. By induction, $h_v(a) \leq h_v(p) + (p-a)(b-c)$, with $h_v(p) = 0$ in the best case.

Letting $v(f) = f\frac{p}{\lambda F}(b-c)$,

$$h_v(a) = \sum_{f=0}^{F} v(f)g(f,a) = \frac{p}{\lambda F}(b-c)\sum_{f=0}^{F} fg(f,a) = (p-a)(b-c)$$

because the expectation of the compound hypergeometric-binomial is $(p-a)\lambda\frac{F}{p}$. Moreover, this contract gives expected social utility

$$
\begin{aligned}
&2\sum_{a=0}^{p}\binom{p}{a}\lambda^a(1-\lambda)^{p-a}[(2b-c)a+(p-a)(b-c)] \\
&= 2p(b-c)\sum_{a=0}^{p}\binom{p}{a}\lambda^a(1-\lambda)^{p-a} + 2b\sum_{a=0}^{p}a\binom{p}{a}\lambda^a(1-\lambda)^{p-a} \\
&= 2p(b-c+\lambda b).
\end{aligned}
$$

This is positive if $\lambda > \frac{c-b}{b}$ and largest for $p = M-1$. □

Lemma 3. *The value of an optimal contract in simple memory strategies (Eq. 9) is continuous in λ. The correspondence mapping λ to the set of optimal contracts in simple memory strategies, using strategies of the form $s(k)$ (as defined on page 12), is upper hemicontinous.*

Proof. By Berge's Theorem of the Maximum (e.g., Aliprantis and Border 2006, Theorem 17.31). □

Proof of Theorem 2. At $\lambda = 1$, in every optimal contract each player must promise $p = M-1$ tasks and fulfill all of them ($s(M-1) = M-1$); the contract must impose severe enough punishments to make it incentive compatible for them to do so, but the punishments may be arbitrarily severe since they are not realized on the equilibrium path. The value of any such contract is $2(M-1)(2b-c)$.

For $\lambda \to 1$, by Lemma 3 the value of the contract must converge to $2(M-1)(2b-c)$, and so must satisfy $p = M-1$ and $s(M-1) = M-1$ for λ sufficiently high. To minimize the cost of punishments, all the downward constraints $s(M-1)$ should bind, which is achieved by a linear contract. Finally, given a linear contract, $s(k) = k$ for all k is optimal. □

Proof of Theorem 3. At $\lambda = 0$, in any optimal contract either $s(k) = 0$ for all k or $v(f) = 0$ for all f. As $\lambda \to 0$, by Lemma 3 the optimal contracts must converge to either $s(k) = 0$ for all k or $v(f) = 0$ for all f. If punishments converge to zero, then it is incentive compatible only for the players to choose $s(k) = 0$ for all k, in which case it is optimal to set the punishments to exactly $v(f) = 0$ for all f. If the strategies converge to anything other than $s(k) = 0$ for all k, then for incentive compatibility the punishments must diverge ($v(f) \to -\infty$ for some f)—but the value of such contracts does not converge to zero, contrary to Lemma 3. Hence for λ sufficiently low, $s(k) = 0$ for all k and $v(f) = 0$ for all f. □

Lemma 4 (Only deserved punishments)**.** *In any optimal contract, $v(0) = 0$.*

Proof. In an optimal contract, the upward incentive constraints in Eq. 7 can be dropped as discussed earlier. Because $g(0, a)$ is decreasing in a, the downward incentive constraints can only be relaxed by imposing $v(0) = 0$. □

Lemma 5 (Scraping by)**.** *Let $M \geq 3$ and suppose M is odd for simplicity. There exists $\alpha(M) \in (1,2)$ such that if $0 < b < c < \alpha(M)b$, empty promises are optimal for a nonempty open interval of λ's. In particular, there exists $\bar{\lambda} > \frac{c-b}{b}$ such that for all $\lambda \in (\frac{c-b}{b}, \bar{\lambda})$, completing as many promises as one remembers (for any positive number of promises smaller than M) is feasible and gives positive social utility, but is dominated by making $\frac{M+1}{2}$ promises and completing only one promise whenever at least one is remembered.*

Proof. Let $p = \frac{M+1}{2}$, and $F = \frac{M-1}{2}$. Consider implementing the strategy where exactly one task is accomplished whenever at least one is remembered. Set $v(0) = v(1) = \cdots = v(F-1) = 0$. This implies $h(a) = 0$ for all $a > 1$.

For doing just one task to be incentive compatible, it must be that $h(1) - h(0) \geq c - b$ and $h(a) - h(1) \leq (c-b)(a-1)$ for all $a \in \{2, 3, \dots, p\}$. For the latter condition, it suffices that $h(1) \geq b - c$. For the latter condition, observe that $h(1) = v(F)g(F,1)$ and $h(0) = h(1)\frac{g(F,0)}{g(F,1)}$. Since

$$\frac{g(F,0)}{g(F,1)} = \frac{\binom{p}{F}}{\binom{p-1}{F}} = \frac{p}{p-F},$$

$h(0) = \frac{p}{p-F}h(1)$. Therefore, IC requires $h(1) \leq \frac{p-F}{F}(b-c)$. Let us set $h(1) = \frac{2}{M-1}(b-c)$ and $h(0) = \frac{M+1}{M-1}(b-c)$.

Therefore this contract is feasible and incentive compatible, and has expected social utility

$$2\big[(1-(1-\lambda)^{\frac{M+1}{2}})(\frac{2(b-c)}{M-1} + 2b - c) + (1-\lambda)^{\frac{M+1}{2}}(b-c)\frac{M+1}{M-1}\big].$$

After some algebra, this expression is larger than $2(M-1)(b-c+b\lambda)$ (the expected social utility from the optimal contract implementing $M-1$ promises and fulfilling all those remembered) if

$$\frac{c(M^2-3M) - b(M^2-4M+1)}{b(M-1)} > (1-\lambda)^{\frac{M+1}{2}} + (M-1)\lambda. \tag{16}$$

Define $\phi : [0,1] \to \mathbb{R}$ by $\phi(\lambda) = (1-\lambda)^{\frac{M+1}{2}} + (M-1)\lambda$, and note that ϕ is strictly increasing. Let

$$\bar{\lambda} = \phi^{-1}\Big(\frac{c(M^2-3M) - b(M^2-4M+1)}{b(M-1)}\Big).$$

To show that (16) holds for $\lambda \in (\frac{c-b}{b}, \bar{\lambda})$, it suffices to show that $\frac{c-b}{b} < \bar{\lambda}$, or that

$$\frac{c(M^2-3M) - b(M^2-4M+1)}{b(M-1)} > \phi(\frac{c-b}{b}).$$

After some algebra, this holds if

$$(2-\frac{c}{b})^{\frac{M+1}{2}} < \frac{2M}{M-1} - \frac{c}{b}\frac{M+1}{M-1}.$$

Define $\hat{\phi}:[1,2]\to\mathbb{R}$ by

$$\hat{\phi}(x) = \frac{2M}{M-1} - x\frac{M+1}{M-1} - (2-x)^{\frac{M+1}{2}}.$$

It can be seen that $\hat{\phi}$ is concave, first increasing and eventually negative, with a unique $\alpha(M)\in(1,2)$ such that $\hat{\phi}(\alpha(M))=0$. Hence the bound $0<b<c<b\alpha(M)$.

Consequently, this contract dominates the linear one for any $p\leq M-1$; this means there are empty promises in this range. □

Lemma 6. *Suppose that p^* satisfies $p-(p^*-1)\geq F$ and that*

$$\sum_{a=0}^{p} t_s(a)\left(g(f,a)-g(F,a)\frac{g(f,p^*)-g(f,p^*-1)}{g(F,p^*)-g(F,p^*-1)}\right)\geq 0 \text{ for all } f=1,\ldots,F-1. \quad (17)$$

Then the contract is suboptimal if it does not involve cutoff strategies. Moreover, the best-case punishments for implementing a cutoff strategy p^* are given by

$$\frac{c-b}{g(F,p^*)-g(F,p^*-1)}\Big(\sum_{a=0}^{p^*-1}\binom{p}{a}\lambda^a(1-\lambda)^{p-a}g(F,a)+g(F,p^*)\sum_{a=p^*}^{p}\binom{p}{a}\lambda^a(1-\lambda)^{p-a}\Big), \quad (18)$$

derived by setting $v(f)=0$ for $f<F$ and $v(F)$ high enough to make p^* indifferent to p^*-1.

Proof. If a contract is optimal, we can ignore the upward incentive constraints (if any bind, then it would be optimal to do that number of tasks). Suppose that s is optimal given p,F and is not a cutoff strategy. Fixing s, finding the optimal punishments is a linear programming problem. By duality theory, we know that if the primal problem is $\max u^T y$ s.t. $A^T y\leq w$ and $y\geq 0$, then the dual problem is $\min w^T x$ s.t. $Ax\geq u$ and $x\geq 0$; the optimal solution to one problem corresponds to the Lagrange multipliers of the other, and if feasible solutions to the dual and primal achieve the same objective value then these are optimal for their respective problems.

The relaxed problem (dropping upward incentive constraints), written in the form of the primal problem, is given by

$$\max \sum_{f=0}^{F} (-v(f)) \sum_{a=0}^{p} -g(f,a)t_s(a) \text{ subject to}$$

$$\sum_{f=0}^{F} (-v(f))[g(f,a) - g(f,k)] \leq -(a-k)(c-b) \text{ for all } a \text{ s.t. } t_s(a) > 0 \text{ and all } k < a$$

$$\text{and } -v(f) \geq 0 \text{ for all } f = 0, 1, \ldots, F$$

The dual of this problem is then

$$\min \sum_{\{(k,a) \mid t_s(a)>0, k<a\}} -(a-k)(c-b)x_{ka} \text{ subject to}$$

$$\sum_{\{(k,a) \mid t_s(a)>0, k<a\}} x_{ka}[g(f,a) - g(f,k)] \geq -\sum_{a=0}^{p} g(f,a)t_s(a) \text{ for all } f = 0, 1, \ldots, F$$

$$\text{and } x_{ka} \geq 0 \text{ for all } f = 0, 1, \ldots, F$$

Let $v(f) = 0$ for all $f = 0, 1, \ldots, F-1$, and set $v(F) = \frac{c-b}{g(F,p^*)-g(F,p^*-1)}$, which makes the IC constraint bind in comparing p^* and p^*-1 tasks. We know the denominator is strictly negative by the assumption that $p-(p^*-1) \geq F$ and the fact that $g(F,a) \leq g(F,a-1)$ for all $a = 1, 2, \ldots, p$. This is feasible in the primal because all downward IC constraints will be slack after the first that binds, since $g(F,\cdot)$ has a MLRP (or by preservation of convexity in Lemma 7). Then the value of the primal is given by

$$\frac{c-b}{g(F,p^*) - g(F,p^*-1)} \sum_{a=0}^{p} g(F,a)t_s(a).$$

Let $x_{ka} = 0$ for all pairs (k,a) except for $a = p^*$ and $k = p^*-1$, since those IC constraints in the primal are slack. Let

$$x_{p^*,p^*-1} = -\frac{\sum_{a=0}^{p} g(F,a)t_s(a)}{g(F,p^*) - g(F,p^*-1)},$$

corresponding to the constraint for F binding, since $v(F) < 0$. This is feasible in the dual by the assumption in (17). Then the value of the dual is the same as that in the primal, which means that the optimal punishment involves $v(f) = 0$ for all $f = 0, 1, \ldots, F-1$ and $v(F) = \frac{c-b}{g(F,p^*)-g(F,p^*-1)}$.

However, because all downward IC constraints are satisfied, if s is not a cutoff strategy then at least one of the upward IC constraints that were dropped is violated, a contradiction to being an optimal strategy given p and F. $\square$

Proof of Theorem 5. By a similar argument as in Lemma 2, whenever $0 < p^* = p$ the contract should be linear, which can be optimal only if $p = M - 1$ (because punishments are not bounded below). Suppose that $p^* < p = M - 1$. To implement $p = M - 1$, it must be that $F = 1$. Then the hypotheses of Lemma 6 are satisfied, and the contract optimally implementing this has $v(0) = 0$ and $v(1)$ set to make doing p^* tasks indifferent to doing $p^* - 1$ tasks: that is, $v(F) = \frac{c-b}{g(F,p^*)-g(F,p^*-1)}$. Then the expected punishment when a tasks are done is given by

$$(c-b)\frac{g(F,a)}{g(F,p^*)-g(F,p^*-1)} = (c-b)\frac{\binom{M-1-a}{1}}{\binom{M-1-p^*}{1}-\binom{M-p^*}{1}} = -(c-b)(M-1-a).$$

Consequently, expected punishment is independent of p^*, and decrease in a. Because benefits are also increasing in a, the contract is dominated by complete promise-keeping. Promise-keeping, in turn, is dominated by not keeping any promises if $\lambda < \frac{c-b}{b}$. $\square$

For the following lemma, we say that a function $\psi : \{0, 1, \ldots, R\} \to \mathbb{R}$ is *concave* if $\psi(r+1) - \psi(r) \leq \psi(r) - \psi(r-1)$ for all $r = 1, \ldots, R-1$.[16] We say that a function $\phi : \mathcal{Z} \to \mathbb{R}$, where $\mathcal{Z} \subseteq \mathbb{R}$, is *double crossing* if there is a convex set $A \subset \mathbb{R}$ such that $A \cap \mathcal{Z} = \big\{z \in \mathcal{Z} : \phi(z) < 0\big\}$.

Lemma 7. *Let $\mathcal{R} = \{0, 1, \ldots, R\}$, and let $\{q_z\}_{z\in\mathcal{Z}}$ be a collection of probability distributions on $\mathcal{R}$ parameterized by z, taking either discrete values $z \in \mathcal{Z} = \{0, 1, \ldots, Z\}$ or continuous values $z \in \mathcal{Z} = [0, 1]$. If*

1. *There exists $k > 0$ such that $z = k\sum_{r=0}^{R} r q_z(r)$ for all $z \in \mathcal{Z}$;*
2. *Either $q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)$ (for all $z = 1, \ldots, Z-1$ if z is discrete) or $\frac{\partial^2}{\partial z^2} q_z(r)$ (for all $z \in (0, 1)$ if z is continuous), as a function of r, is double crossing;*
3. *$\psi : \{0, 1, \ldots, R\} \to \mathbb{R}$ is concave;*

then $\Psi(z) = \sum_{r=0}^{R} \psi(r) q_z(r)$ is concave.[17]

Proof. Since $z = k\sum_{r=0}^{R} r q_z(r)$, $\sum_{r=0}^{R}(mr+b)q_z(r) = \frac{m}{k}z + b$ for any real m and b. Hence, for any m and b, if z is discrete then

$$\sum_{r=0}^{R}(mr+b)\big(q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)\big) = 0,$$

[16] Convexity generalizes naturally to functions on discrete subsets of $\mathbb{R}$ (see, for example, Kiselman (2005)). The definition used here is one of several possible equivalent definitions.

[17] A more general mathematical result along these lines appears in Fishburn (1982).

for all $z = 1, \dots, Z-1$, while if z is continuous then

$$\sum_{r=0}^{R}(mr+b)\frac{\partial^2}{\partial z^2}q_z(r) = \frac{\partial^2}{\partial z^2}(mz+b) = 0$$

for all $z \in (0,1)$.

For any m and b, if z is discrete, the second difference of $\Psi(z)$ is

$$\begin{aligned}\Psi(z+1) - 2\Psi(z) + \Psi(z-1) &= \sum_{r=0}^{R}\psi(r)\big(q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)\big) \\ &= \sum_{r=0}^{R}\big(\psi(r) - mr - b\big)\big(q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)\big),\end{aligned} \tag{19}$$

while if z is continuous then the second derivative of $\Psi(z)$ is

$$\frac{\partial^2}{\partial z^2}\Psi(z) = \sum_{r=0}^{R}\psi(r)\frac{\partial^2}{\partial z^2}q_z(r)\big) = \sum_{r=0}^{R}\big(\psi(r) - mr - b\big)\frac{\partial^2}{\partial z^2}q_z(r). \tag{20}$$

By assumption, either $q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)$ (if z is discrete) or $\frac{\partial^2}{\partial z^2}q_z(r)$ (if z is continuous), as a function of r, is double crossing. Furthermore, since ψ is concave, we can choose m and b such that, wherever $\big(q_{z+1}(r) - 2q_z(r) + q_{z-1}(r)\big)$ or $\frac{\partial^2}{\partial z^2}q_z(r)$ is nonzero, $\psi(r) - mr - b$ either has the opposite sign or is zero. From Eq. 19 and Eq. 20, above, we can then conclude that $\Psi(z)$ is concave. □

Lemma 8. *For any M and p, and cutoff strategy s,*

1. *The value of Eq. 10 is strictly increasing and concave in λ.*
2. *If $p < M-1$ and $p_1^* < p_2^* \leq p - F + 1$, the value of Eq. 10 for p_2^* strictly single crosses the value of Eq. 10 for p_1^* from below, as a function of λ.*
3. *If $z \in \mathbf{Z}_{++}$, $p + z \leq M - 1$, and $p^* \leq p - F + 1$, the value of Eq. 10 for $p+z, p^*+z$ strictly single crosses the value of Eq. 10 for p, p^* from below, as a function of λ.*

Proof. We prove each part separately below.

1. The value of Eq. 10 is the expectation of $\beta(a) \equiv 2(2b-c)s(a) + 2\frac{(c-b)g(F,s(a))}{g(F,p^*)-g(F,p^*-1)}$ with respect to the binomial distribution over a. For any cutoff strategy s, $(2b-c)s(a)$ is clearly

concave. The second term of $\beta(a)$ is a negative constant times $g(F, s(a))$. Itself, $g(F, s(a)) = \lambda^F \binom{p-s(a)}{F} \Big/ \binom{p}{F}$, which is convex:

$$\begin{aligned}
&\binom{p-s(a+1)}{F} - 2\binom{p-s(a)}{F} + \binom{p-s(a-1)}{F} \\
&= \begin{cases} \binom{p-a}{F}\left(\frac{F}{p-(a+1)-F} - \frac{F}{p-a}\right) & \text{if } a \leq p^* - 1, \\ \binom{p-(p^*-1)}{F} - \binom{p-p^*}{F} & \text{if } a = p^*, \\ 0 & \text{if } a \geq p^* + 1. \end{cases}
\end{aligned}$$

which is positive because $F \geq 1$, and $p - p^* + 1 \geq F$. Hence $\beta(a)$ is concave. Finally, the binomial distribution satisfies double-crossing, since

$$\frac{\partial^2}{\partial \lambda^2}\left(\binom{p}{a}\lambda^a(1-\lambda^{p-a}\right) = \binom{p}{a}(1-\lambda)^{p-2-a}\lambda^{a-2}\left(a^2 - \left(1 + 2(p-1)\lambda\right)a + p(p-1)\lambda^2\right)$$

is negative if and only if $a^2 - \left(1 + 2(p-1)\lambda\right)a + p(p-1)\lambda^2 < 0$. Hence by Lemma 7, Eq. 10 is concave in λ.

To see that Eq. 10 is increasing in λ, the benefit of each task is linear in a, increasing in p^* and independent of λ, which is a parameter of first-order stochastic dominance for the binomial distribution.

2. For a cutoff strategy s, we need only check that the expected punishment for completing $s(a)$ tasks,

$$\frac{(c-b)g(F, s(a))}{g(F, p^*) - g(F, p^*-1)},$$

has increasing differences in a and p^*, since λ cancels out of the above (e.g., see Topkis 1998). Let us denote a p^*-cutoff strategy by s_{p^*} to account for the indicator function in the strategy. Since $c - b > 0$, the sign of the second difference depends on

$$\begin{aligned}
&\frac{g(F, s_{p^*+1}(a+1)) - g(F, s_{p^*+1}(a))}{g(F, p^*+1) - g(F, p^*)} - \frac{g(F, s_{p^*}(a+1)) - g(F, s_{p^*}(a))}{g(F, p^*) - g(F, p^*-1)} \\
&= \begin{cases} 0 & \text{if } a \geq p^* + 1 \\ 1 & \text{if } a = p^* \\ \frac{g(F,a+1) - g(F,a)}{g(F,p^*+1) - g(F,p^*)} - \frac{g(F,a+1) - g(F,a)}{g(F,p^*) - g(F,p^*-1)} & \text{if } a \leq p^* - 1. \end{cases}
\end{aligned}$$

Concentrating on the third case, since $g(F, a)$ is decreasing in a, it suffices to show that

$$\binom{p-p^*}{F} - \binom{p-p^*+1}{F} > \binom{p-p^*+1}{F} - \binom{p-p^*+2}{F}. \tag{21}$$

But this is exactly analogous to the calculation in part (1).

3. Without loss, the limit of summation in Eq. 10 may be replaced with M, since $\binom{p}{a} = 0$ for $a > p$. Then, similarly to above, we need only check that

$$\frac{\binom{p+z-s_{p^*+z}(a)}{F}}{\binom{p+z-(p^*+z)}{F} - \binom{p+z-(p^*+z-1)}{F}}$$

has increasing differences in a and z. The sign of the second difference is determined by

$$\frac{\binom{p+z+1-s_{p^*+z+1}(a+1)}{F} - \binom{p+z+1-s_{p^*+z+1}(a)}{F}}{\binom{p-p^*}{F} - \binom{p-p^*+1}{F}} - \frac{\binom{p+z-s_{p^*+z}(a+1)}{F} - \binom{p+z-s_{p^*+z}(a)}{F}}{\binom{p-p^*}{F} - \binom{p-p^*+1}{F}}$$
$$= \begin{cases} 0 & \text{if } a \geq p^* + z + 1 \\ 1 & \text{if } a = p^* + z \\ \frac{\binom{p+z-a}{F} - \binom{p+z-a+1}{F} - \binom{p+z-1-a}{F} + \binom{p+z-a}{F}}{\binom{p-p^*}{F} - \binom{p-p^*+1}{F}} & \text{if } a \leq p^* + z - 1. \end{cases}$$

The case $a \leq p^*+z-1$ reduces to checking the numerator is negative, since the denominator is negative. Again, by substituting in the definition of the binomial coefficient, this is equivalent to $\frac{p+z-a+1-F}{p+z-a} \leq 1$, which holds because $F \geq 1$ and $p + z - a + 1 > 0$.

□

Proof of Theorem 6. The first nontrivial case is $M = 3$, in which, by Lemma 1, the only possible promise levels are $p = 1$ (with $F = 1$) and $p = 2$ (with $F = 1$). In both cases Theorem 5 implies the contract must be promise-keeping.

For the case $M = 4$, by Lemma 1 the only possible promise levels are $p = 2$ (with $F = 2$) and $p = 3$ (with $F = 1$). Theorem 5 implies that the last case again reduces to promise-keeping with linear contracts, and that $p = 2$ (with $F = 2$) is suboptimal unless it is a cutoff strategy with $p^* = 1$. In this case the assumptions of Lemma 6 are satisfied.[18]

Finally, for the case $M = 5$, by Lemma 1 the only possible promise levels are $p = 2$ (with $F = 2$), $p = 3$ (with $F = 2$), and $p = 4$ (with $F = 1$). The last case again reduces to promise-keeping with linear contracts by Theorem 5. In light of Lemma 1, strategies must be increasing for the contract to be optimal, and by Theorem 5, they cannot have empty promises if $p^* = p$. Then there is only a cutoff strategy remaining for $p = 2$, with $p^* = 1$ (same as for $M = 4$). Moreover, there is only one non-cutoff strategy for the case that $p = 3$ that could potentially be optimal: $s(a) = 0$ for $a < 2$, and $s(a) = 2$ for $a \geq 2$. To rule this out, observe that the assumptions in Lemma 6 are satisfied

[18] It is easy to see numerically that for the case $F = 2$ the part of the summand in Eq. 17 that is in parentheses is always nonnegative, for all $\lambda \in (0, 1)$ and choices of p, p^*, a that are feasible given that $M \leq 5$.

for $p = 3$ and $M = 5$, so a non-cutoff strategy cannot be optimal. The cutoff strategies (p, p^*) remaining are given by $(x, 0)$, $(2, 1)$, $(3, 1)$, $(3, 2)$, and $(4, 4)$ are potentially optimal. We know by the single crossing result for fixed $p = 3$ that $(3, 2)$ single crosses $(3, 1)$ from below, and also single crosses $(2, 1)$ from below. By Lemma 8 the value functions for each p^* are concave in λ, so that once the linear value function for $p = 4$ is optimal it remains so. □

Proof of Theorem 7. Define $\sigma_1 \equiv \frac{b-c}{\lambda_1 \underline{v}}$, $\sigma_2 \equiv \frac{b-c}{\lambda_2 \underline{v}}$, and $\gamma \equiv -\frac{b}{\underline{v}}$. Using this notation,

$$\begin{aligned} \overline{\text{IC}}_1 &\Leftrightarrow p_2 \leq M_2 - p_1\sigma_2 \text{ whenever } p_1 \geq M_2 - p_2, \\ \overline{\text{IC}}_2 &\Leftrightarrow p_2 \leq \frac{1}{\sigma_1}(M_1 - p_1) \text{ whenever } p_2 \geq M_1 - p_1. \end{aligned}$$

Under the assumption that $\lambda_i \geq \frac{b-c}{\underline{v}}$ we know $\sigma_i \in (0, 1)$ and $\overline{\text{IC}}_i$ is satisfied in the region $p_i \leq M_{-i} - p_{-i}$ for $i = 1, 2$. Next, observe that

$$\begin{aligned} \overline{\text{IR}}_1 &\Leftrightarrow p_2 \geq \frac{\sigma_2}{\gamma} p_1, \\ \overline{\text{IR}}_2 &\Leftrightarrow p_2 \leq \frac{\gamma}{\sigma_1} p_1. \end{aligned}$$

For the individually rational region to be nonempty, one needs $\sqrt{\lambda_1\lambda_2} \geq \frac{c-b}{b}$, which is satisfied by the assumption $\lambda_i \geq \frac{c-b}{b}$ for $i = 1, 2$.

The intersection of $\overline{\text{IC}}_1$ and $\overline{\text{IC}}_1$, using the form those take in the region $\{(p_1, p_2) \mid p_2 \geq M_1 - p_1,\ p_1 \geq M_2 - p_2\}$, is given by

$$p_1 = \frac{M_1 - \sigma_1 M_2}{1 - \sigma_1\sigma_2}, \quad p_2 = \frac{M_2 - \sigma_2 M_1}{1 - \sigma_1\sigma_2}.$$

This intersection occurs above IR_1 if, plugging p_1 above into $\overline{\text{IR}}_1$, we have

$$\frac{M_2 - \sigma_2 M_1}{1 - \sigma_1\sigma_2} \geq \frac{\sigma_2}{\gamma} \frac{M_1 - \sigma_1 M_2}{1 - \sigma_1\sigma_2},$$

or when $\frac{M_1}{M_2} \leq \frac{\gamma + \sigma_1\sigma_2}{\sigma_2(\gamma+1)}$; and is below $\overline{\text{IR}}_1$ otherwise.

Similarly, the intersection occurs below $\overline{\text{IR}}_2$ if

$$\frac{M_2 - \sigma_2 M_1}{1 - \sigma_1\sigma_2} \leq \frac{\gamma}{\sigma_1} \frac{M_1 - \sigma_1 M_2}{1 - \sigma_1\sigma_2},$$

or when $\frac{M_1}{M_2} \geq \frac{(1+\gamma)\sigma_1}{\sigma_1\sigma_2 + \gamma}$; and is above $\overline{\text{IR}}_2$ otherwise.

The slope of $\overline{\text{IC}}_1$ when it binds is $-\sigma_2$ and the slope of $\overline{\text{IC}}_2$ when it binds is $-\frac{1}{\sigma_1}$. The social objective takes the form

$$(b-c)\left(\frac{\sigma_1-\gamma}{\sigma_1}p_1+\frac{\sigma_2-\gamma}{\sigma_2}p_2\right)$$

and has slope $-\frac{\sigma_2}{\sigma_1}\frac{\sigma_1-\gamma}{\sigma_2-\gamma}$. The solution is then obtained by comparing slopes in each case.

□

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