

Self-Deception and Choice*

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Abstract

We model agents who use self-deception to rationalize and justify actions that can eventually lead them into temptation. Formally, we extend Gul and Pesendorfer's (2001) framework to three time periods and obtain a special functional form for the temptation utility at the interim stage. Our representation portrays an agent who is tempted (i) to relax her normative attitude towards future indulgence and (ii) to turn a blind eye to any possibility of temptation altogether. Welfare implications of self-deception are discussed.

1 Introduction

“This self-deceit, this fatal weakness of mankind, is the source of half the disorders of human life.”

– *Adam Smith, The Theory of Moral Sentiments.*

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The seminal model of Gul and Pesendorfer [13] (henceforth GP) captures temptation in the form of desires that appear to the agent at the moment of choice and demand their satisfaction. This paper promotes an alternative view where tempting desires can employ more strategic and more sophisticated means to achieve their eventual satisfaction. We hold that agents can, in a state of *self-deception*, find ways of rationalizing actions that will eventually lead them into temptation. Desires can steer the agent through her reasoning.

The strategic nature of temptation is recognized in psychology. It studies not only the strategies that people use to resist temptation, but also the strategies they use *to allow their resolve to fail*. Baumeister, Heatherton, and Tice [3, p. 139] write:

Smokers face the choice of obtaining immediate gratification of their addiction or living for a longer period of time. To do this, smokers must ignore or rationalize the long-term consequences of smoking. Thus, for instance, they claim that the evidence is weak linking smoking to cancer, or they fall prey to thoughts that they are personally invulnerable..

Similarly, individuals who may need to lose weight for health reasons often find themselves in tempting situations...Such individuals are known to engage in irrational thought processes during such events (“Well, one cookie won’t harm me,”..). Thus, giving in to temptation also involves a number of cognitive strategies that are used to negate the perceived long-term consequences associated with indulgence.

Therefore, desires that directly impact the agent at the moment of choice may also affect the agent in the steps leading up to it by motivating her to construct *rationalizations* for behaviors she hitherto preferred to avoid. These rationalizations enable the agent to justify a course of action that eventually leads her into temptation by diminishing the negative content of the action, often emphasizing that the agent does not perceive or comprehend the ‘badness’ of her action.

Rationalizations are a necessary feature of self-deception, but also belongs to the related phenomenon of motivated reasoning (Kunda [18]).¹ The key

¹Kunda [18, pg 482] proposes that people who are “motivated to arrive at a particular

defining feature of self-deception is that rationalizations are accompanied with *internal tension*. Losonsky [20] writes:

[S]elf-deceivers who come to recognize that they were self-deceivers often report that although they had rejected the belief, say, that they were anorexic (or that their children were using drugs), “on another level I knew I was anorexic,” or, “I knew all along she was abusing drugs, but I refused to accept it.”... Self-deception involves some kind of recognition of the fact that the available evidence warrants the undesirable proposition more than the desirable one. This can be manifested in various ways. One way is in a recurring or nagging doubt that typically does not occur when subjects fix their beliefs. Similarly, self-deceivers can find themselves repeatedly and obsessively entertaining the undesirable proposition and going over the same line of reasoning that supports the desirable proposition.

Thus, self-deception involves a delicate balance between the agent’s attempt to deceive herself and a minimal degree of awareness of this. This manifests itself in the internal tension that accompanies self-deception and is absent in simple motivated reasoning. Self-deception cannot survive knowledge of self-deception, but it co-exists with suspicion of self-deception.

This paper builds a choice-theoretic model of what we refer to as temptation-driven self-deception.² We seek to behaviorally capture the phenomenon as described above, but also to distinguish it from overt temptation and motivated reasoning. Our model is built in a GP-style setup. Choices are made in three stages—ex ante, interim, and ex post. Consumption takes place only in the ex post stage, where the agent faces a menu to choose out of. This

conclusion attempt to be rational and to construct a justification of their desired conclusion that would persuade a dispassionate observer. They draw the desired conclusion only if they can muster up the evidence necessary to support it”.

²In the psychology literature, self-deception has also been discussed in the context of achieving long term goals or the need to adjust and function in a social environment. This accommodates the case of *positive illusions* (Taylor [29], Taylor and Brown [30]) that take the form of self-aggrandizing self-perceptions, exaggerated perception of control over surroundings and unrealistic optimism, which are viewed as serving an important adaptive function. We also mention that *wishful thinking* is related but distinct from self-deception because the latter survives in the ‘teeth of evidence’ (by explaining away or reinterpreting the evidence) while the former does not (Szabados [26]).

menu is itself chosen in the interim stage, and the menu of menus available to her in this stage is determined in the ex ante stage. Temptation acts directly and overtly on the agent in the ex post stage, and it acts on her indirectly via self-deception in the interim stage. Our primitive is the agent's preferences over menus of menus in the ex ante stage, which is assumed to be prior to the experience of any temptation or self-deception. At this stage, the agent is in a cold emotional state that allows her to have an adequate picture of her future choices and struggle with temptation.³ To illustrate the setting, imagine an agent who is tempted to drink heavily when it is feasible. Whenever she visits a neighborhood consisting of bars and restaurants, she rationalizes her way into bars. Recognising that she somehow ends up in bars whenever one is in the neighborhood, she selects what neighborhood to visit accordingly. Here, final consumption is alcohol consumption, bars and restaurants are menus, and a neighborhood is a menu or menus.

We adopt the natural counterparts of GP's conditions and highlight that, when appropriately reinterpreted, they provide a setting where self-deception can be studied. In particular, the notion of self-control in GP's model permits reinterpretation in terms of the existence of internal tension at the interim stage: the agent rationalizes a (desirable) ranking of menus in the interim stage but yet is never completely divorced from her ex ante perspective. Intuitively, nagging doubts about the soundness of one's reasoning may restrict the extent to which one follows it. Imagine, for instance, a cancer patient who honestly avows the belief that she will not die of cancer, but never finds herself committing to any serious long term plans either. This is where the model is not explicable in terms of motivated thinking.

Within this GP style framework, we introduce two novel axioms that express self-deception and separate our model from one of overt temptation by menus. The axioms express the idea that self-deception is *motivated* and *restricted*: (i) it arises only if it ultimately leads to a greater satisfaction of ex post desires, and (ii) only those rationalizations can be adopted that mask her inferior motives behind a suitable facade of interim rationality. The corresponding representation for preference is obtained in Theorem 2. This representation portrays an agent who is tempted to *relax her ex ante normative perspective* at the interim stage in the direction of anticipated ex

³Empirical foundations for such a 'special period 0' perspective are provided in Noor [23] on the basis of the hypothesis that distant temptations have smaller impact on current choices than do immediate temptations. Thus, our ex ante stage may be interpreted as a time sufficiently distant from the interim stage.

post desires, and possibly also *turn a blind eye* to future temptations. Thus, rationalizations arise subjectively in this paper, as part of the representation for our axioms.

Our more general result (Theorem 3) goes beyond temptation-driven self-deception and incorporates an unconditionally-exaggerated view of self-control ability. This extension has interesting welfare implications. For instance, it implies that welfare can decrease if menus containing temptation (‘vice’ menus) are mixed with menus containing normatively attractive alternatives (‘virtuous’ menus). This is reflected in the following preference over menus of menus:

$$\{a, b\} \succ \{a, a \cup b\},$$

where a is a virtuous menu and b a vice menu. Intuitively, larger menus lend themselves to more excuses. Thus, it can be easier for the agent to deceive herself into choosing $a \cup b$ from $\{a, a \cup b\}$ than into choosing b from $\{a, b\}$. This provides a rationale for separating vice and virtue, which is presumably the main purpose of alcohol beverage consumption (ABC) laws or zoning laws for casinos.

1.1 Related Literature

There is an active philosophical debate about self-deception. The traditional *intentionalists* understand self-deception in light of interpersonal deception, thus requiring the self-deceiver to believe p but intentionally bring about the belief $\neg p$ (Sartre [25], Fingarette [12]). The difficulties raised by implied paradoxes has led more recent intentionalists to weaken the requirement of holding contradictory beliefs to just intentionally bringing about a belief – motivated by some desire or emotion – despite an initial recognition that the evidence may not warrant this belief (Talbot [28], Bermudez [5]). Levy [19] suggests that the agent can “avoid the evidence, or situations in which he is likely to be confronted by the evidence, can rationalize the evidence he has by imagining unlikely but possible explanations for each piece, and so on” and Talbot [28] suggests that the agent can exercise selectivity in attention, memory, evidence-gathering and reasoning. In this view, intentionality is not abandoned, and it remains prereflective.⁴ *Non-intentionalists* allow that such

⁴It is argued that intentionality underlies the internal tension typically identified with self-deception, which may be revealed in opacity and indirection (Levy [19]), emotional resistance to evidence (Talbot [28]) or hypersensitivity to criticism, confrontation or op-

forms of self-deception may be possible, but emphasize that most cases of self-deception can be also be explained in terms of unintentional motivationally-biased information processing, without resorting to unconscious beliefs and intentions (Barnes [2], Mele [21]). These various theories are not clearly distinguishable behaviorally in general, but in our setting the presence or absence of internal tension can be distinguished behaviorally.

To our knowledge, the idea of temptation-driven self-deception that we consider has not been studied in economics. The behavioral economics literature discusses positive illusions (Benabou and Tirole [4]) and wishful thinking (Brunnermeier and Parker [6]). The internal tension that defines self-deception is absent in these models – the agent simply responds to a motivation to distort beliefs in a particular manner. The decision-theoretic literature on temptation has modelled temptation only as arising in its naked, unstrategic form and being relevant mainly at the moment of choice. One paper that comes close to ours is Cherepanov, Feddersen and Sandroni [7], who provide a decision-theoretic model of an agent who follows her preferences only to the extent that she can justify it. The primitive of their model is a choice function and the representation suggests that the agent has a preference over alternatives and a set of rationales (binary relations) that she uses to ascertain choice: for any given menu, she first determines the maximizer of each rationale, and then her choice follows the rationale whose maximizer is most preferred. While this may serve as a model of motivated reasoning, it does not serve as a model of self-deception because the choice domain (choice from menus) is not rich enough to express the existence of any internal tension. Indeed, the authors show that their model either subsumes or is observationally equivalent to others that involve substantially different stories.

The idea that an agent may be tempted to change her view of world (specifically, beliefs over a state space) is considered in Epstein [10] and Epstein and Kopylov [11]. The temptation to change beliefs is unexplained, and in particular, its existence is not motivated in the sense of serving a strategic function. In our paper, a temptation to change the ex ante perspective (about her propensity for self-control, etc) serves the purpose of, and owes its existence to, the desire for tempting final consumption. There is no temptation by final consumption in Epstein [10] and Epstein and Kopylov [11].

The idea that temptation may influence the choice of menu is present in posing evidence (Gur and Sackeim [14]).

Noor [22, 23] and Noor and Ren [24], but the mode in which temptation acts there is presumed to be direct and overt. Our paper maintains that an agent may be tempted by menus, but it hypothesizes that the agent is only tempted by menus that she can justify choosing. Indeed, we find that the models of tempting menus in [22, 24] cannot be regarded as one of self-deception.

A version of our self-deception model also shows up in Noor [23]. There are two important differences, however. First, the model appears in [23] mainly as a means to axiomatically unify other temptation models in the literature. In contrast, the current paper starts by behaviorally defining a necessary condition for self-deception, and it turns out that the representation theorem delivers the same model. A novel interpretation of that model is obtained here as a result. Second, our choice domain differs substantially from [23] and so do our axioms. Intuitively, our axioms are imposed on the agent’s perspective in a cold state – in a special period 0 – where she anticipates all temptation (by self-deception or otherwise) but is not subject to it yet. In contrast, axioms in [23] are imposed directly on choice in a hot state, when the agent is subject to all kinds of temptation. A counterpart of our main axiom does not appear in [23].

On a technical level our results exploit Kopylov’s [15, 17] extensions of GP’s model to dynamic settings with more than two periods and to representations with finitely many additive components.

The remainder of this paper proceeds as follows. The introduction concludes with a mention of related literature. Section 2 introduces the primitives of the model and presents a benchmark case. Section 3 presents our model of self-deception. Section 4 extends this model to permit a virtuous self-perception and derives some of its implications for welfare policy. Section 5 concludes. Proofs are relegated to appendices.

2 Preliminaries

To aid exposition in subsequent sections, we first present a basic three-period extension of GP’s model.

Let $X = \{x, y, z, \dots\}$ be the set $\Delta(Z)$ of all Borel probability measures on a compact metric space Z of deterministic consumptions. Let d be the Prohorov metric of the weak convergence topology on X . More generally, let X be the class of all Anscombe–Aumann acts f that map a finite state space

Ω into $\Delta(Z)$, and let d be the corresponding product metric in X .⁵

Suppose that choices are made in three stages—ex ante, interim, and ex post—and these choices determine the decision maker’s consumption in X after the ex post stage. Let $\mathcal{M}_1 = \{a, b, c, \dots\}$ be the set of all interim *menus*—non-empty compact subsets $a \subset X$. Interpret any menu $a \in \mathcal{M}_1$ as a course of action that, if taken at the interim stage, restricts the ex post choice to the set $a \subset X$. Endow the space $\mathcal{M}_1$ with the Hausdorff metric μ_1 and define mixtures

$$\alpha a + (1 - \alpha)b = \{\alpha x + (1 - \alpha)y : x \in a, y \in b\}$$

for all $\alpha \in [0, 1]$ and menus $a, b \in \mathcal{M}_1$.

Similarly, let $\mathcal{M}_0 = \{A, B, C, \dots\}$ be the set of all ex ante menus—non-empty compact subsets $A \subset \mathcal{M}_1$. Interpret any menu $A \in \mathcal{M}_0$ as a course of action that, if taken ex ante, restricts the interim choice to the set $A \subset \mathcal{M}_1$. Endow the space $\mathcal{M}_0$ with the Hausdorff metric μ_0 and define mixtures

$$\alpha A + (1 - \alpha)B = \{\alpha a + (1 - \alpha)b : a \in A, b \in B\}$$

for all $\alpha \in [0, 1]$ and menus $A, B \in \mathcal{M}_0$. Then both $\mathcal{M}_0$ and $\mathcal{M}_1$ are compact (see Theorem 3.71 in Aliprantis and Border [1]) and the mixture operations in these spaces are continuous.

Let a binary relation $\succeq$ on $\mathcal{M}_0$ be the decision maker’s weak preference over ex ante menus. Write the symmetric and asymmetric parts of this relation as $\sim$ and $\succ$ respectively. Note that our model does not take the decision maker’s interim and ex post choices as primitive, but instead derives her anticipation of these choices from her ex ante preference.

Adapt GP’s list of axioms for the preference $\succeq$.

Axiom 1 (Order). $\succeq$ is complete and transitive.

Axiom 2 (Continuity). For all menus $A \in \mathcal{M}_0$, the sets $\{B \in \mathcal{M}_0 : B \succeq A\}$ and $\{B \in \mathcal{M}_0 : B \preceq A\}$ are closed.

Axiom 3 (Independence). For all $\alpha \in [0, 1]$ and menus $A, B, C \in \mathcal{M}_0$,

$$A \succeq B \quad \Rightarrow \quad \alpha A + (1 - \alpha)C \succeq \alpha B + (1 - \alpha)C.$$

⁵Menus of lotteries are first used by Gul and Pesendorfer [13] and—for finite Z —by Dekel, Lipman, and Rustichini [8]. Menus of acts are proposed by Epstein [10].

Axiom 4 (Set-Betweenness). *For all menus $a, b \in \mathcal{M}_1$ and $A, B \in \mathcal{M}_0$,*

$$\{a\} \succeq \{b\} \Rightarrow \{a\} \succeq \{a \cup b\} \succeq \{b\}, \quad (1)$$

$$A \succeq B \Rightarrow A \succeq A \cup B \succeq B. \quad (2)$$

Order and Continuity are standard conditions of rationality. To motivate Independence, interpret any mixture $\alpha A + (1 - \alpha)C$ as a lottery that yields the menus A and C with probabilities α and $1 - \alpha$ respectively and is resolved after the ex post stage. In this interpretation, the decision maker's interim choice $\alpha a + (1 - \alpha)c$ in $\alpha A + (1 - \alpha)C$ and her ex post choice $\alpha x + (1 - \alpha)y$ in $\alpha a + (1 - \alpha)c$ determine her consumptions $x \in a \in A$ and $y \in c \in C$ contingent on the resolution of the lottery between the menus A and C . If the timing of the resolution of this objective uncertainty is irrelevant for preference, then the decision maker should be indifferent between the menu $\alpha A + (1 - \alpha)C$ and a hypothetical lottery $\alpha \circ A + (1 - \alpha) \circ C$ that yields the menus A or C with probabilities α and $1 - \alpha$ respectively, but is resolved immediately after the ex ante stage. (Here the preference $\succeq$ is extended from the original domain $\mathcal{M}_0$ to lotteries over menus.) Then the standard separability argument suggests that

$$A \succeq B \Rightarrow \alpha \circ A + (1 - \alpha) \circ C \succeq \alpha \circ B + (1 - \alpha) \circ C$$

because the possibility of getting the menu C with probability $1 - \alpha$ should not affect the decision maker's comparison of A and B . Independence follows.

Set-Betweenness is imposed separately on the preference $\succeq$ over the entire $\mathcal{M}_0$ and on the restriction of $\succeq$ to singleton menus $\{a\}$ that provide a strict commitment to the menu $a \in \mathcal{M}_1$ at the interim stage, but may still require self-control ex post when choice in a has to be made. It is assumed that the decision maker's ex ante evaluation of any such menu $\{a\}$ is based on her anticipation of two factors:

- the consumption $x_a \in a$ that she will choose if a is feasible ex post,
- the self-control that she will use to resist the strongest temptation $y_a \in a$ in this menu.

This informal assumption suggests that for all menus $a, b \in \mathcal{M}_1$,

$$x_b \in a, y_a \in b \Rightarrow \{a\} \succeq \{b\}. \quad (3)$$

Indeed, if $x_b \in a$ and $y_a \in b$, then the decision maker should expect that if she chooses x_a from the menu a at the ex post stage, then she will (i) obtain the same consumption that she plans to choose from b and (ii) resist the temptation y_a , which belongs to b and hence, should not be harder to resist than the *strongest* temptation y_b in b . Therefore, the ranking $\{a\} \succeq \{b\}$ is intuitive because the menu a offers a weakly better combination of consumption benefits and self-control costs than b does. Condition (3) implies (1).⁶ Analogously, condition (2) assumes that the decision maker should evaluate any menu $A \in \mathcal{M}_0$ based on her anticipated interim choice $a_A \in A$ and the most tempting alternative $b_A \in A$ in this menu. Note that if temptations are cumulative or uncertain (as in Dekel, Lipman, Rustichini [9]), then both parts of Set-Betweenness can be violated.

The following condition is used to obtain uniqueness in representation results below.

Axiom 5 (Regularity). *There are $x, y, x', y' \in X$ such that*

$$\begin{aligned} \{\{x\}\} &\succ \{\{x, y\}\} \succ \{\{y\}\} \\ \{\{x'\}\} &\succ \{\{x'\}, \{y'\}\} \succ \{\{y'\}\}. \end{aligned}$$

The two rankings in this axiom are intuitive if the agent plans to resist the tempting consumption y in the menu $\{x, y\}$ at the ex post stage, and to resist the tempting menu $\{y'\}$ in $\{\{x'\}, \{y'\}\}$ at the interim stage.

Say that a function $u : X \rightarrow \mathbb{R}$ is *linear* if for all $\alpha \in [0, 1]$ and $x, y \in X$,

$$u(\alpha x + (1 - \alpha)y) = \alpha u(x) + (1 - \alpha)u(y).$$

Let $\mathcal{U}$ be the set of all continuous linear functions $u : X \rightarrow \mathbb{R}$. Similarly, define linearity for functions on $\mathcal{M}_1$ and let $\mathcal{U}_1$ be the set of all continuous linear functions $V : \mathcal{M}_1 \rightarrow \mathbb{R}$.

Theorem 1. *The preference $\succeq$ satisfies Axioms 1–4 if and only if $\succeq$ is represented by a utility function U_0 such that for all $A \in \mathcal{M}_0$ and $a \in \mathcal{M}_1$,*

$$U_0(A) = \max_{a \in A} \left[U(a) - \max_{b \in A} (V(b) - V(a)) \right] \quad (4)$$

$$U(a) = \max_{x \in a} [u(x) - \max_{y \in a} (v(y) - v(x))], \quad (5)$$

⁶To show this claim, take any menus $\{a\} \succeq \{b\}$. Let $c = a \cup b$. Then $x_a, x_b, y_a, y_b \in c$. By (3), if $x_c \in a$, then $\{a\} \succeq \{c\}$; if $x_c \in b$, then $\{b\} \succeq \{c\}$. In either case, $\{a\} \succeq \{c\}$. By (3), if $y_c \in a$, then $\{c\} \succeq \{a\}$; if $y_c \in b$, then $\{c\} \succeq \{b\}$. In either case, $\{c\} \succeq \{b\}$.

where $u, v \in \mathcal{U}$ and $V \in \mathcal{U}_1$.

Moreover, if $\succeq$ satisfies Regularity, then it has another representation (4) with components $u', v' \in \mathcal{U}$ and $V' \in \mathcal{U}_1$ if and only if $u' = \alpha u + \beta_u$, $v' = \alpha v + \beta_v$, and $V' = \alpha V + \beta_V$ for some $\alpha > 0$ and $\beta_u, \beta_v, \beta_V \in \mathbb{R}$.

This theorem provides a joint characterization for GP's utility representations (4) and (5) over $\mathcal{M}_0$ and $\mathcal{M}_1$ respectively. The restriction of U to $\mathcal{M}_1$ can be interpreted as the decision maker's ex ante normative perspective on what menu *should* be chosen at the interim stage. Temptations that impact her at this stage are captured by V , and the nonnegative component $\max_{b \in A} (V(b) - V(a))$ is interpreted as the self-control cost of choosing a from A . Similarly, the function u can be interpreted as the ex ante normative perspective on what should be chosen at the ex post stage, and the nonnegative term $\max_{y \in a} (v(y) - v(x))$ as the mental cost of ex post self-control. These interpretations suggest that in order to balance her normative perspective with the costs of self-control, the decision maker should plan to maximize $U + V$ and $u + v$ respectively at the interim and ex post stages.

Before turning to our models of distorted self-perception, note the benchmark case with $V = 0$ when the decision maker does not expect to have any temptations at the interim stage. In this case, she obeys *strategic rationality* so that for all $A, B \in \mathcal{M}_0$

$$A \succeq B \quad \Rightarrow \quad A \sim A \cup B.$$

Note that she may still anticipate costly temptations at the ex post stage, as she may exhibit a preference for commitment $\{a\} \succ \{a \cup b\}$ for some menus $a, b \in \mathcal{M}_1$.

3 Self-Deception

We model *self-deception* as an interim temptation in GP's setup. It should be noted, however, that self-deception and temptation are different phenomena in general (Szabados [27]). Knowledge of a craving is typical in cases of temptation, and indeed is *presupposed* in the notion of self-control. While nagging suspicion comes with self-deception, self-deception cannot survive knowledge of self-deception: the enterprise of self-deception is prompted by motives that cannot be spelled out without destroying the enterprise itself. Since GP's model is tailored for overt temptation and as such involves a degree of sophistication and self-awareness, using their model for self-deception

demands cautious reinterpretation. (An empirical distinction between overt temptation and self-deception is made in the context of the Rationalizable Self-Deception axiom below.)

We adopt the following interpretation. In the interim stage, the agent's desires subconsciously prompt her to reevaluate her *ex ante* perspective by constructing rationalizations. She finds her new interim perspective compelling and rational, but she also recollects her *ex ante* views, which include a recognition of her tendency to rationalize. Her self-deception survives as she *rationalizes her ex ante views*, such as by arguing that “at that time I was not appreciating the fact that ...”. At the same time, however, she cannot deny that she could be deceiving herself – she experiences nagging doubts about the motives underlying her reasoning. Consequently, her decisions may put some weight on her *ex ante* perspective, although this involves a psychological cost of underweighting her compelling interim reasoning.

Observe that in this story, *ex ante* sophistication does not destroy the possibility of interim self-deception because the latter can rationalize the former. Also, the internal tension underlying interim self-deception gives rise to features that are analogous to the notion of self-control in GP's model: self-control means placing weight on the *ex ante* preference despite current desires, and in the current context this describes the self-restraint that is prompted by nagging doubts that accompany self-deception. Finally, observe that this feature distinguishes the model from one of motivated reasoning. An agent who follows her motivated reasons need not experience any internal tension. In this case, there would be a divorce between interim and *ex ante* perspectives. Behaviorally, this would be captured in the special case of Set-Betweenness that GP refer to as No Self-Control, which is ruled out by Continuity here.⁷

We introduce two axioms that express the nature of self-deception. The first expresses the fact that self-deception is directed: it must be *motivated* by expected *ex post* consumption.

Axiom 6 (Motivated Self-Deception). *For all $a, b \in \mathcal{M}_1$,*

$$\{a \cup b\} \succ \{b\} \quad \Rightarrow \quad \{a\} \sim \{a, a \cup b\}.$$

⁷In the current context, this is given by: $A \succeq B \Rightarrow A \sim A \cup B$ or $A \cup B \sim B$. This suggests that in the interim stage, either the interim and *ex ante* perspectives coincide (in which case the agent follows her *ex ante* perspective without experiencing any cost), or she follows an interim perspective different from an *ex ante* one without any regard of the latter.

The ranking $\{a \cup b\} \succ \{b\}$ suggests that the anticipated ex post choice in the menu $a \cup b$ belongs to a rather than to b . Motivated Self-Deception (MSD for short) states that the agent does not deceive herself into choosing $a \cup b$ over a if the bigger menu does not affect her anticipated ex post consumption. When both menus a and $a \cup b$ lead to the same final choice, they also yield the same degree of satisfaction of desires. Consequently, there can simply be no *motivation* for the agent to deceive herself into choosing $a \cup b$ over a . Self-deception has no strategic value in such cases.

While MSD is a natural requirement for self-deception, it may also be satisfied by models of *overt temptation* where interim desires are sensitive to ex post choice. (For instance, see Noor and Ren [24] where the agent is tempted by guiltless indulgence.) Our second axiom delineates cases of self-deception that are behaviorally distinct from those of overt temptation. We illustrate with the following example. An agent can either not drink x_0 , drink moderately x_m or drink heavily x_h . Let $a = \{x_m\}$, $b = \{x_0, x_h\}$, and $a \cup b = \{x_0, x_m, x_h\}$. For instance, the menu a may be obtained at a formal social event, the menu b at a bar where any drinking is heavy, and $a \cup b$ at a home party. The agent has a normative preference to drink less, but she cannot resist the temptation to drink heavy at the bar. Then she would commit to a rather than to b ex ante because the former menu leads to less drinking ex post, and by Set-Betweenness, she exhibits the ranking $\{a\} \succeq \{a \cup b\} \succeq \{b\}$.

We claim that a temptation to go to the bar rather than stay at home,

$$\{a \cup b\} \succ \{a \cup b, b\},$$

should be common in cases of overt temptation, especially when the sophistication implicit in MSD is present: if the agent's temptation is sensitive to ex post choice, if she anticipates drinking heavily in the bar and only moderately at home, and *if she is not constrained to maintain the facade of rationality that is necessary for self-deception*, then nothing keeps her from desiring the bar. The following axiom for self-deception imposes an inability to rationalize going to the bar, on the grounds that such rationalizations would expose the agent's underlying motives:

Axiom 7 (Rationalizable Self-Deception). *For all menus $a, b \in \mathcal{M}_1$,*

$$\{a \cup b, b\} \succeq \{a \cup b\}.$$

Rationalizable Self-Deception (RSD for short) asserts that the agent never deceives herself into choosing a menu b rather than the larger menu $a \cup b$ when she deems $\{a \cup b\} \succeq \{b\}$ ex ante. (Note that if $\{b\} \succ \{a \cup b\}$, then $\{a \cup b, b\} \succeq \{a \cup b\}$ follows from Set-Betweenness.) When $a \cup b$ offers greater temptation than b , such deception is not motivated to begin with. When, as in the case of the bar vs home, it offers as much temptation as b , then any rationalization for choosing b would have to overturn the fact that $a \cup b$ offers strictly more options without more temptation. The axiom rules out such rationalizations. Intuitively, such rationalizations may be too transparent to generate the *semblance of rationality* needed for self-deception.

To see that RSD accommodates intuitive cases of self-deception, consider the following three possible rationalizations the agent can use to justify going to the bar.

- Overestimation of propensity for self-control: “If I go to the bar I know I will be tempted to drink, but I have the will-power to abstain.”
- Underestimation of susceptibility to temptation: “I am not even going to be tempted to drink in the bar”.
- Relaxation in normative standards: “I am only going to live once, so I really *should* allow myself to enjoy life a little.”.

Observe how the first two rationalizations exploit the fact that the bar contains the normatively superior alternative x_0 . Indeed, under the exaggerated self-control or underestimated temptation, the bar leads to a normatively better outcome than attending the social event, where some drinking is unavoidable. The third rationalization leads the agent to view the bar in a positive light, without necessarily invoking a distorted view of her self-control ability or susceptibility to temptation.

Each of these rationalizations is consistent with RSD because none of them can help the agent justify going to the bar over staying home. This is immediately evident (given that the bar offers only a subset of the options available at home) except perhaps in the following case: Suppose the agent tells herself that she has substantial will-power, and that she will choose x_0 in the bar but x_m at home. This would seem to make the bar look normatively better in terms of final consumption. However, if x_m is optimally chosen when heavy drinking is possible then it must be better (perhaps after accounting

for self-control costs) than choosing x_0 when heavy drinking is possible. But the latter describes precisely what she expects in the bar.

We refrain from claiming that there cannot exist cases of self-deception that violate RSD. A rationalization for choosing the bar must make the presence of x_m look bad – note that the bar provides commitment relative to the home by excluding x_m . If the agent chooses x_m at home, then the agent may view her ex post choice as her temptation, or she may completely reverse her normative and temptation perspectives. Whether or not this is characteristic of agents who delicately try to get past their normative defenses by appealing to reason, we note that it is readily attributable to an agent who is overtly consumed by her temptation.

3.1 Representation Result

Say that functions $u, v \in \mathcal{U}$ are *independent* if for all $\alpha, \beta, \gamma \in \mathbb{R}$,

$$\alpha u + \beta v + \gamma = 0 \quad \Rightarrow \quad \alpha = \beta = \gamma = 0.$$

Note that u and v are independent if and only if the functions $u, v, u + v$ represent three different rankings on X .

Theorem 2. $\succeq$ satisfies Axioms 1–7 if and only if $\succeq$ has a utility representation (4)–(5) such that for all $a \in \mathcal{M}_1$,

$$V(a) = \kappa U(a) + \lambda \max_{y \in a} v(y), \quad (6)$$

where $\kappa \geq \lambda > 0$, and $u, v \in \mathcal{U}$ are independent.

Moreover, $\succeq$ has another representation (4), (5), (6) with parameters $\kappa', \lambda' \in \mathbb{R}$ and functions $u', v' \in \mathcal{U}$ if and only if $\kappa' = \kappa$, $\lambda' = \lambda$, $u' = \alpha u + \beta_u$, and $v' = \alpha v + \beta_v$ for some $\alpha > 0$ and $\beta_u, \beta_v \in \mathbb{R}$.

Here the temptation utility V can be interpreted as a distortion of the interim normative utility U in the direction of the ex post desires v . This distortion takes the form

$$V(a) = (\kappa - \lambda) \max_{x \in a} \left[\frac{\kappa}{\kappa - \lambda} u(x) + \frac{\lambda}{\kappa - \lambda} v(x) - \max_{y \in a} (v(y) - v(x)) \right] \quad (7)$$

if $\kappa > \lambda$, or

$$V(a) = \kappa \max_{x \in a} (u(x) + v(x)) \quad (8)$$

if $\kappa = \lambda$. To interpret, compare (7) with (5) to see that interim desires V in (7) modify the ex ante perspective U by replacing the ex ante normative perspective u with

$$u^* = \frac{\kappa}{\kappa - \lambda}u + \frac{\lambda}{\kappa - \lambda}v.$$

The perspective underlying u^* distorts u in the direction of the temptation utility v . It is as if the agent is tempted to believe that her ex ante normative perspective u was too stoic, and thus to *relax her normative standards* and adopt u^* . The case (8) is a limiting case of (7) where the agent views $u + v$ as her normative perspective and turns a *blind eye* to any possibility of temptation. It is as if by adjusting her normative perspective she believes that she is resolving all internal conflict. Observe that in either case, since $\kappa \geq \lambda > 0$, it can never be that the distorted normative preference puts too much weight on the temptation preference. This is strongly reminiscent of the fact that self-deception requires the agent to maintain a facade of rationality. This constrains her from adopting rationalizations that reveal their underlying motives too clearly.

Observe that interim choice maximizes

$$\begin{aligned} U(a) + V(a) &= (1 + \kappa)U(a) + \lambda \max_{y \in a} v(y) \\ &= (1 + \kappa - \lambda) \max_{x \in a} \left[\frac{1 + \kappa}{1 + \kappa - \lambda} u(x) + \frac{\lambda}{1 + \kappa - \lambda} v(x) - \max_{y \in a} (v(y) - v(x)) \right], \end{aligned}$$

which has a similar interpretation to (7). Thus, after engaging in rationalizations ((7) or (8)) and placing some weight on her ex ante perspective, the agent's interim behavior correctly recognizes her ex post desires, but distorts her normative perspective. Note that the interim commitment ranking is represented by the function $\frac{1 + \kappa}{1 + \kappa - \lambda}u + \frac{\lambda}{1 + \kappa - \lambda}v$ that assigns the weight of at least $\frac{2}{3}$ to the ex ante commitment utility u . This is an expression not only of the fact that the agent's rationalizations place weight on u , but also that interim choice places some weight on the ex ante perspective.

A notable observation is that the agent's anticipated ex post choice is preserved at the interim stage, as it maximizes the function $\frac{1 + \kappa}{1 + \kappa - \lambda}u + \frac{\lambda}{1 + \kappa - \lambda}v + v$ that is ordinally equivalent to $u + v$. Indeed, the interim temptation perspective under both (7) and (8) leaves anticipated choice undistorted. We learn, therefore, that while (7) and (8) accommodate a distortion in normative perspective and possible blindness to temptation, *they cannot accommodate a distortion in anticipated self-control ability*. Evidently, of the three rationalizations our axiom is consistent with, the third must always hold and the

second may hold simultaneously, but the first can never hold. Moreover, anticipated choice is never distorted by any rationalization she adopts.

These conclusions are not tied to the functional form, but rather express themselves in behavior. The statement that our self-deceived agent is necessarily tempted to change her normative perspective is captured in the fact that there must exist *singleton* menus $a, b \in \mathcal{M}_1$ such that $\{a\} \succ \{a, b\}$. Since the evaluation of singletons does not involve any (non-trivial) evaluation of ex post choice, this expresses an interim conflict surrounding only what *should* be consumed ex post. The statement that our self-deceived agent is not tempted to misperceive ex post choice (and thus self-control ability) is reflected in the following behavior:⁸

$$\{a\} \succ \{a \cup b\} \implies \{a, b\} \sim \{a, a \cup b\}. \quad (9)$$

That is, if b contains tempting alternatives, then there is never a situation where the temptation by $a \cup b$ differs from the temptation by b . A difference would arise if, for instance, the agent's actual anticipated choice from $a \cup b$ was in b (ex ante anticipated lack of self-control) but she was tempted to misperceive her choice from $a \cup b$ to lie in a (temptation-anticipated exertion of self-control). In such a case the agent would be tempted to view $a \cup b$ more favorably than b , and consequently, $\{a, b\} \succ \{a, a \cup b\}$.

On an intuitive level, MSD and RSD are consistent with rationalizations that distort anticipated choice. However, such rationalizations are ruled out when these axioms are adopted in conjunction with other axioms, particularly Independence.

4 Multiple Self-Deceptions

Our self-deception model rules out the possibility of a virtuous distortion in her perceived ability to exert self-control. We present here an extension that accommodates such virtuous distortion of self-perception without departing

⁸To show this claim, take any $a, b \in \mathcal{M}_1$ such that $\{a\} \succ \{a \cup b\}$. Then $V(a \cup b) \geq V(b)$ because $U(a \cup b) \geq U(b)$ and $\max_{y \in a \cup b} v(y) = \max_{y \in b} v(y)$. Consider two cases.

- (i) $\{a\} \sim \{a, a \cup b\} \succ \{a \cup b\}$. Then $V(a) \geq V(a \cup b) \geq V(b)$ and hence, $\{a\} \sim \{a, b\}$.
- (ii) $\{a\} \succ \{a, a \cup b\}$. By MSD, $\{a \cup b\} \sim \{b\}$, that is, $U(a \cup b) = U(b)$. By (6), $V(a \cup b) = V(b)$. Thus, $\{a, b\} \sim \{a, a \cup b\}$.

from the GP framework. Although the general model will not be compatible with an interpretation involving sophisticated desires with a single motive, it helps identify some intuitive behaviors ruled out by the model in the previous section and lends itself to discussion of welfare.

In the self-deception model, temptation by b is motivated by the choice that the agent expects to make in b . The following axiom accommodates temptations that are motivated also by the normative content in b .

Axiom 8 (Weak Motivated Self-Deception). *For any $a, b \in \mathcal{M}_1$ such that $\{a \cup b\} \succ \{b\}$ and $\{a\} \succ \{a, a \cup b\}$, there is $z \in b$ such that $\{\{z\}\} \succ \{\{x\}\}$ for all $x \in a$.*

This condition (WMSD for short) relaxes MSD and allows the menu $a \cup b$ to tempt a when the anticipated ex post choices in both menus are the same, but the menu $a \cup b$ provides a more virtuous interim self-perception. Formally, the rankings $\{a \cup b\} \succ \{b\}$ and $\{a\} \succ \{a, a \cup b\}$ are allowed only if $a \cup b$ has an element z that is normatively better than any alternative in a . It is as if the agent has an exaggerated view of her propensity for self-control, that is, an excessively virtuous self-image.

A departure from a temptation-driven self-deception is evident here: interim temptation is no longer intimately connected with a desire to achieve ex post satisfaction of desires. An excessively virtuous self-image may lead the agent to temptation only by accident, and in some situations may even *defeat* efforts at ex post desire satisfaction. It follows that the tendency toward a virtuous self-perception satisfies a different desire – presumably a direct desire for a positive self-image – that is distinct from ex post temptation. We thus interpret the axiom as adding a second kind of self-deception, one involved in *positive illusions*. However, due to the abstract nature of our framework, we cannot strictly speaking justify this interpretation relative to, say, wishful thinking.

On the other hand, RSD remains intuitive even if the agent's interim self-deception has virtuous rationalization and motivation.

Theorem 3. *$\succeq$ satisfies Axioms 1–6 and WMSD if and only if $\succeq$ has a utility representation (4)–(5) such that for all $a \in \mathcal{M}_1$,*

$$V(a) = \kappa U(a) + \lambda \max_{y \in a} v(y) + \mu \max_{z \in a} u(z) \quad (10)$$

where $\kappa \geq \lambda > 0$, $\mu \geq 0$, and $u, v \in \mathcal{U}$ are independent.

Moreover, $\succeq$ has another representation (4), (5), (6) with parameters $\kappa', \lambda', \mu' \in \mathbb{R}$ and functions $u', v' \in \mathcal{U}$ if and only if $\kappa' = \kappa$, $\lambda' = \lambda$, $\mu' = \mu$, $u' = \alpha u + \beta_u$, and $v' = \alpha v + \beta_v$ for some $\alpha > 0$ and $\beta_u, \beta_v \in \mathbb{R}$.

The difference between (10) and (6) is the additional term $\mu \max_{z \in a} u(z)$ in the interim temptation utility V . Although the model offers up to three rationalizations for choosing a given menu (namely, distorted normative preference, distorted temptation preference and perceived virtuosity), the rationalizations may be *inconsistent*. For instance, if the menu contains irresistible temptation, then one rationalization will recognize this and justify it according to a more relaxed normative perspective, while the other will refuse to recognize it.⁹ The rationalizations may also *neutralize* each other, such as when a is more virtuous than b and b contains greater temptation. These are reflections of the fact noted above that the model is not one of temptation-driven self-deception. Such self-deception would presumably involve a search for the strongest possible case for making a ‘bad’ decision, and such a case would be as devoid of inconsistencies as possible. Nevertheless, as a model of an agent who engages in rationalizations more broadly, it permits some substantive discussion of welfare, to which we now turn.

The implication (9) of the pure self-deception model does not hold in general. Formally, write $a \succeq_0 b$ if there exists $z \in a$ such that $\{\{z\}\} \succeq \{\{x\}\}$ for all $x \in b$. Then the preference $\succeq$ represented by (10) satisfies¹⁰

$$\{a\} \succ \{a \cup b\} \text{ and } a \succeq_0 b \quad \Rightarrow \quad \{a, b\} \succeq \{a, a \cup b\}.$$

This condition implies that if b is a ‘vice’ menu and a is a ‘virtuous’ menu, then the agent is generally better off if virtue and vice are kept separate (as in $\{a, b\}$) rather than combined (as in $\{a, a \cup b\}$). The intuition is that

⁹Behaviorally, suppose $\{\{x\}\} \succ \{\{x, y\}\} \sim \{\{y\}\}$. Then $\{x, y\}$ may be more tempting than both $\{x\}$ and $\{y\}$. That is, $\{\{x\}, \{y\}\} \succ \{\{x\}, \{y\}, \{x, y\}\}$. Since one rationalization favors $\{x\}$ and the other favors $\{y\}$, the fact that $\{x, y\}$ is more tempting than either implies the simultaneous use of both rationalizations.

¹⁰To show this claim, take any $a, b \in \mathcal{M}_1$ such that $\{a\} \succ \{a \cup b\}$ and $\max_{z \in a} u(z) \geq \max_{z \in b} u(z)$. Then $V(a \cup b) \geq V(b)$ because $U(a \cup b) \geq U(b)$ and $\max_{y \in a \cup b} v(y) = \max_{y \in b} v(y)$. Consider two cases.

- (i) $\{a\} \sim \{a, a \cup b\}$. Then $V(a) \geq V(a \cup b) \geq V(b)$ and hence, $\{a\} \sim \{a, b\}$.
- (ii) $\{a\} \succ \{a, a \cup b\}$. By WMSD, $\{a \cup b\} \sim \{b\}$, that is, $U(a \cup b) = U(b)$. Therefore, $V(a \cup b) \geq V(b)$ implies $\{a, b\} \succeq \{a, a \cup b\}$.

the union $a \cup b$ lends itself to *more* excuses for the agent to lead herself into temptation. Indeed, in this case, given whatever rationalizations the agent may adopt to justify choosing b from $\{a, b\}$, virtuous self-perception is an additional rationalization that can be invoked to justify choosing $a \cup b$ from $\{a, a \cup b\}$. The behavioral implication suggests, for instance, that a procrastinator is better-off if a completely flexible option is not a feasible choice: if a is the option of completing the task sooner and b is the option of completing it later, then having the opportunity to make this decision later (as in $\{a, a \cup b\}$) makes her worse-off relative to a situation where she has to decide today whether to complete the task sooner or later (as in $\{a, b\}$). For another example, view a menu as a physical location selling particular alternatives and a menu of menus as a town. Then agents in a town are better-off if virtue and vice are sold at distinct locations (as in $\{a, b\}$) relative to when vice is always bundled with virtue (as in $\{a, a \cup b\}$). Zoning laws for casinos may be welfare improving in this sense.

A common view is that optimal welfare policy for agents with self-control problems constitutes the provision of commitment opportunities. In the above setting, this would correspond to providing the agent with $\{a, a \cup b\}$, in which she may keep all her options by selecting $a \cup b$ or avoid temptation by choosing the commitment option a . The above discussion suggests that when agents are subject to self-deception, then the simple provision of commitment opportunities may not always be optimal.

5 Comparative Self-Deception

To interpret the parameters κ , λ , and μ in terms of choice behavior, consider a pair of preferences $\succeq$ and $\succeq^*$ over $\mathcal{M}_0$. Call this pair *regular* if both $\succeq$ and $\succeq^*$ satisfy Axioms 1–5 and BSD, and the two rankings agree on the domain of singleton menus so that

$$\{a\} \succeq \{b\} \quad \Leftrightarrow \quad \{a\} \succeq^* \{b\}$$

for all $a, b \in \mathcal{M}_1$.

By Theorem 3, any regular pair of preferences $\succeq$ and $\succeq^*$ can be represented by (4)–(6) with components $(u, v, \kappa, \lambda, \mu)$ and $(u^*, v^*, \kappa^*, \lambda^*, \mu^*)$ respectively. Moreover, the functions U and U^* represent the same preference on $\mathcal{M}_1$ and hence, by GP’s Theorem, one can take $u = u^*$ and $v = v^*$.

Say that $\succeq^*$ is *more self-deceptive* than $\succeq$ if for all menus $a, b \in \mathcal{M}_1$,

$$\{a\} \succ \{a, b\} \quad \Rightarrow \quad \{a\} \succ^* \{a, b\}, \quad (11)$$

This definition requires that any self-deception that is tempting for $\succeq$ should be tempting for $\succeq^*$ as well.

Theorem 4. *Let $\succeq$ and $\succeq^*$ be a regular pair of preferences. Then $\succeq^*$ is more self-deceptive than $\succeq$ if and only if the two preferences have representations (4)-(6) such that $\frac{\lambda^*}{\kappa^*} \geq \frac{\lambda}{\kappa}$ and $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$.*

This result suggests that the ratios $\frac{\lambda}{\kappa}$ and $\frac{\mu}{\kappa}$ are both positively related to the intensity of self-deception in our model. (Note that $\frac{\lambda^*}{\kappa^*} \geq \frac{\lambda}{\kappa}$ and $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$ imply $\frac{\mu^*}{\kappa^*} \geq \frac{\mu}{\kappa}$.) If $\kappa > \lambda$, then the weight $\frac{\lambda}{\kappa - \lambda}$ that is put on the function v in (7) is a positive monotonic transformation of $\frac{\lambda}{\kappa}$ and hence, can serve as an index of self-deception as well.

Moreover, the equality $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$ is necessary for an unambiguous comparison of self-deception revealed by the two rankings $\succeq$ and $\succeq^*$. This equality requires roughly that the proportion of the virtuous and motivated components of self-deception should be the same for $\succeq$ and $\succeq^*$. In particular, this must be true when both $\succeq$ and $\succeq^*$ satisfy MSD and hence, $\mu = \mu^* = 0$.

6 Concluding Remarks

This paper explores the behavioral foundations for self-deception. It seeks to behaviorally capture the phenomenon as described in the Introduction, but also to distinguish it from overt temptation and motivated reasoning. Achieving the distinction requires us to adopt a rich choice domain. For instance, while self-deception may well take place in the ex post stage (choice from menus), it may not be distinguishable from overt temptation: an agent may find reasons to drink heavily while in a bar, but her behavior would be identical to that of some overtly tempted agent. Thus we are led to focus on self-deception prior to the time of final consumption, that is, in the agent's interim choice of bars while in a neighborhood. However, in order to identify self-deception at this stage and to distinguish it from motivated reasoning, the choice data we require is the agent's choice of neighborhoods. Thus, avoiding neighborhoods with particular bars – a preference for commitment – reveals the agent's concern with her capacity for self-deception.

The distinction between self-deception and motivated reasoning (namely, the existence of tension) comes down to that between the Set-Betweenness and No Self-Control axioms.

Rationalizations are germane to self-deception, but instead of taking these unobservables as part of our primitives, we derive them from choice behavior. On an intuitive level, we expect that the overly virtuous self-image inherent in the rationalization “I have the will-power to stop myself..” is an important strategic tool for desires to induce the agent to make decisions that will lead her into temptation. Yet, while such rationalizations are consistent with key behavioral properties of self-deception (the MSD and RSD axioms) on an intuitive level, they are ruled out in a GP-style model of self-deception (in particular, one that imposes Independence). A direction for future research is to provide a model of temptation-driven self-deception that can accommodate the strategic use of virtuous self-perceptions.

A APPENDIX: PROOFS

In the proofs, we use the following notation and terminology. For any function $u \in \mathcal{U}$ and any menu $a \in \mathcal{M}_1$, write

$$u(a) = \max_{x \in a} u(x),$$

and let

$$\mathcal{T}(u) = \{\alpha u + \beta : \alpha \geq 0, \beta \in \mathbb{R}\}$$

be the set of all non-negative transformations of the function u .

For any $S \in \mathbb{N}$, let S denote also the set $\{1, \dots, S\}$. Kopylov [16, Lemma A.1] shows that for any $u_1, \dots, u_S \in \mathcal{U}$, there are elements $x_1, \dots, x_S \in X$ such that for all $i, j \in S$, and

$$u_i \notin \mathcal{T}(u_j) \quad \Leftrightarrow \quad u_i(x_i) > u_i(x_j). \quad (12)$$

This equivalence implies that $u_i(x_i) \geq u_i(x_j)$ for all $i, j \in S$.

Turn to Theorem 1. The necessity of Axioms 1–4 for representation (1) is straightforward. Conversely, suppose that $\succeq$ satisfies Axioms 1–4. Kopylov [17, Theorem 1] shows that $\succeq$ has a utility representation

$$U_0(A) = \max_{a \in A, x \in a} \left[u(x) - \max_{y \in a} (v(y) - v(x)) - \max_{b \in A} (V(b) - V(a)) \right]$$

for some $u, v \in \mathcal{U}$ and $V \in \mathcal{U}_1$. Moreover, if $\succeq$ satisfies Regularity, then the triple (u, v, V) in this representation is unique up to a positive linear transformation. The utility function U_0 has the required form (4)-(5), where

$$U(a) = \max_{x \in a} [u(x) - \max_{y \in a} (v(y) - v(x))]$$

for all menus $a \in \mathcal{M}_1$. Let $w = u + v$ and $W = U + V$. Then

$$U_0(A) = \max_{a \in A} W(a) - \max_{b \in A} V(b) \quad (13)$$

$$U(a) = w(a) - v(a) \quad (14)$$

for all $A \in \mathcal{M}_0$ and $a \in \mathcal{M}_1$.

Turn to Theorems 2 and 3. Suppose that $\succeq$ satisfies Axioms 1–6 and WMSD. By Theorem 1, $\succeq$ is represented by (13). By Regularity, there are $x^*, y^* \in X$ such that

$$\{\{x^*\}\} \succ \{\{x^*, y^*\}\} \succ \{\{y^*\}\}.$$

Then $w(x^*) > w(y^*)$ and $v(y^*) > v(x^*)$, and hence, w and v are not redundant. Without loss in generality, assume that

$$u(x^*) = v(x^*) = w(x^*) = V(\{x^*\}) = 0. \quad (15)$$

The following two lemmas obtain the required form for V .

Lemma 5. *There are $\kappa, \rho, \mu \in \mathbb{R}$ such that for all $a \in \mathcal{M}_1$,*

$$V(a) = \kappa w(a) + \rho v(a) + \mu u(a). \quad (16)$$

Proof. We claim first that for all $a, b \in \mathcal{M}_1$,

$$w(a) = w(b), v(a) = v(b), u(a) = u(b) \quad \Rightarrow \quad V(a) = V(b). \quad (17)$$

Show this claim by contradiction. Consider any $a, b \in \mathcal{M}_1$ such that $w(a) = w(b)$, $v(a) = v(b)$, $u(a) = u(b)$, but $V(b) > V(a)$. By (14), $U(a) = U(b)$ and hence, $W(b) > W(a)$. As W is continuous, then there is $\varepsilon > 0$ such that

$$W(\varepsilon\{y^*\} + (1 - \varepsilon)b) > W(\varepsilon\{x^*\} + (1 - \varepsilon)a).$$

Let $a^* = \varepsilon\{x^*\} + (1 - \varepsilon)a$ and $b^* = \varepsilon\{y^*\} + (1 - \varepsilon)b$. As $w(x^*) > w(y^*)$, $v(y^*) > v(x^*)$, and $u(x^*) > u(y^*)$, then by linearity, $w(a^*) = w(a^* \cup b^*) > w(b^*)$, $v(b^*) = v(a^* \cup b^*) > v(a^*)$, and $u(a^*) = u(a^* \cup b^*) > u(b^*)$. By (14),

$$U(a^*) > U(a^* \cup b^*) > U(b^*).$$

As $W(b^*) > W(a^*)$, then there are two possible cases.

- $W(b^*) > W(a^* \cup b^*)$. Then $V(b^*) > V(a^* \cup b^*)$. By (13),

$$U(\{b^*, a^* \cup b^*\}) = W(b^*) - V(b^*) = U(b^*) < U(a^* \cup b^*),$$

which contradicts RSD.

- $W(a^* \cup b^*) > W(a^*)$. Then $V(a^* \cup b^*) > V(a^*)$. By (13), $\{a^*\} \succ \{a^*, a^* \cup b^*\}$, which contradicts WMSD because $u(a^*) > u(b^*)$.

This contradiction shows (17).

Take any four menus $a_1, a_2, a_3, a_4 \in \mathcal{M}_1$. Let $a = \cup_{i=1}^4 a_i$. There is i such that $w(a) \geq w(a_i)$, $v(a) \geq v(a_i)$, and $u(a) \geq u(a_i)$. Let $b = \cup_{j \neq i} a_j$. Then $w(a) = w(b)$, $v(a) = v(b)$, and $u(a) = u(b)$. By (17), $V(a) = V(b)$. Kopylov [15, Theorem 2.1] implies that the ranking that V represents on $\mathcal{M}_1$ is represented also by

$$V'(a) = \sum_{i=1}^S \gamma_i u_i(a) \quad (18)$$

such that $S \leq 3$, $\gamma_1, \dots, \gamma_S \in \{-1, 1\}$, and $u_i \notin \mathcal{T}(u_j)$ for all $i, j \in S$ such that $i \neq j$. As both V' and V are linear, then without loss in generality, $V' = V$.

We claim that for all $i \in \{1, \dots, S\}$,

$$u_i \in \mathcal{T}(w) \cup \mathcal{T}(v) \cup \mathcal{T}(u). \quad (19)$$

Wlog let $i = 1$, and suppose that $u_1 \notin \mathcal{T}(w) \cup \mathcal{T}(v) \cup \mathcal{T}(u)$. Take $x_1, \dots, x_{S+3}$ that satisfy (12), that is,

$$\begin{aligned} u_1(x_1) &> u_1(x_j) && \text{for all } j \neq 1 \\ u_i(x_i) &\geq u_i(x_j) && \text{for all } i > 1 \text{ and } j \neq i \\ w(x_{S+1}) &\geq w(x_j) && \text{for all } j \neq S+1 \\ v(x_{S+2}) &\geq v(x_j) && \text{for all } j \neq S+2 \\ u(x_{S+3}) &\geq u(x_j) && \text{for all } j \neq S+3. \end{aligned}$$

Let $a = \{x_1, \dots, x_{S+3}\}$ and $b = \{x_2, \dots, x_{S+3}\}$. Then $u_1(a) = u_1(x_1) > u_1(b)$, but $w(a) = w(b)$, $v(a) = v(b)$, $u(a) = u(b)$, and $u_j(a) = u_j(b)$ for all $j \neq 1$. Thus,

$$V(b) - V(a) = V'(b) - V'(a) = \gamma_1(u_1(x_1) - u_1(a)) \neq 0,$$

which contradicts (17).

The claims (18) and (19) and the normalization (15) imply (16). $\square$

The previous lemma implies that

$$\begin{aligned} W(a) &= U(a) + V(a) = (\kappa + 1)w(a) + (\rho - 1)v(a) + \mu u(a) \\ &= (\kappa + 1)U(a) + (\kappa + \rho)v(a) + \mu u(a) \end{aligned} \quad (20)$$

for all menus $a \in \mathcal{M}_1$.

Lemma 6. *The functions u, v are independent. The parameters κ, ρ, μ are unique and satisfy $\rho \leq 0$, $\kappa + \rho > 0$, $\mu \geq 0$. If $\succeq$ satisfies MSD, then $\mu = 0$.*

Proof. Let $a = \{x^*\}$ and $b = \{y^*\}$. By (14), $\{a\} \succ \{a \cup b\} \succ \{b\}$. By WMSD, $V(a) \geq V(a \cup b)$. By (16),

$$V(a \cup b) - V(a) = \kappa(w(x^*) - w(x^*)) + \rho(v(y^*) - v(x^*)) + \mu(u(x^*) - u(x^*)) \leq 0.$$

Thus, $\rho \leq 0$. To prove the other claims of the lemma, consider two cases.

Case 1. w, v, u are redundant. Then u must be a positive linear transformation of w or v . If $u = \alpha v$ for some $\alpha > 0$, then $w = u + v$ and v are redundant. Thus, $u = \alpha w$ for some $\alpha > 0$. Then $v = (\alpha - 1)w$. As w and v are not redundant, then $\alpha \in (0, 1)$. For all $a \in \mathcal{M}_1$,

$$\begin{aligned} V(a) &= \kappa w(a) + \rho v(a) + \mu u(a) = (\kappa' + \rho)U(a) + \rho v(a), \\ W(a) &= (\kappa' + 1)U(a) + (\kappa' + \rho)v(a), \end{aligned}$$

where $\kappa' = \kappa + \mu\alpha$. Suppose that $\kappa' + \rho < 0$. Take $\alpha, \beta \in (0, \frac{1}{2})$ such that

$$1 < \frac{\alpha w(x^*) - w(y^*)}{\beta v(y^*) - v(x^*)} < 1 + \left| \frac{\kappa' + \rho}{\kappa' + 1} \right|. \quad (21)$$

Let $a = \{x^*, y^*\}$ and $b = \{\alpha y^* + (1 - \alpha)x^*, \beta x^* + (1 - \beta)y^*\}$. Then

$$\begin{aligned} w(a \cup b) - w(b) &= w(x^*) - w(\alpha y^* + (1 - \alpha)x^*) = \alpha(w(x^*) - w(y^*)) > 0 \\ v(a \cup b) - v(b) &= v(y^*) - v(\beta x^* + (1 - \beta)y^*) = \beta(v(y^*) - v(x^*)) > 0. \end{aligned}$$

By (21) and (20),

$$\begin{aligned} U(a \cup b) - U(b) &= \alpha(w(x^*) - w(y^*)) - \beta(v(y^*) - v(x^*)) > 0 \\ |(\kappa' + 1)(U(a \cup b) - U(b))| &< |\kappa' + \rho| \beta(v(y^*) - v(x^*)) \\ W(a \cup b) - W(b) &= (\kappa' + 1)(U(a \cup b) - U(b)) + (\kappa' + \rho)\beta(v(y^*) - v(x^*)) < 0. \end{aligned}$$

Therefore, $\{a \cup b\} \succ \{b\}$, but $\{b, a \cup b\} \sim \{b\}$, which contradicts RSD. Thus, $\kappa' + \rho \geq 0$. Thus, for all $x \in X$,

$$V(\{x\}) = (\kappa' + \rho)U(\{x\}) + \rho \frac{\alpha-1}{\alpha} u(x) = \gamma U(\{x\}),$$

where $\gamma = (\kappa' + \rho) + \rho \frac{\alpha-1}{\alpha}$ is positive. By (13) and (14), for all $x, y \in X$,

$$\{\{x\}\} \succeq \{\{y\}\} \Rightarrow \{\{x\}\} \sim \{\{x\}, \{y\}\} \succeq \{\{y\}\},$$

which violates Regularity.

Case 2. w, v, u are not redundant. Then u and v are independent, and there are $x, y, z \in X$ such that

$$\begin{aligned} w(x) &> w(y) \vee w(z) \\ v(y) &> v(x) \vee v(z) \\ u(z) &> u(x) \vee u(y). \end{aligned}$$

Suppose that $\mu < 0$. Take $\alpha \in (0, 1)$ such that

$$\alpha(\kappa + 1)(w(x) - w(y)) + \mu(u(z) - u(x)) < 0.$$

Let $a = \{x, y, z\}$ and $b = \{y, \alpha y + (1 - \alpha)x\}$. Then

$$\begin{aligned} w(a \cup b) - w(b) &= w(x) - w(\alpha y + (1 - \alpha)x) = \alpha(w(x) - w(y)) > 0 \\ v(a \cup b) - v(b) &= v(y) - v(y) = 0 \\ u(a \cup b) - u(b) &= u(z) - u(\alpha y + (1 - \alpha)x) \geq u(z) - u(x) > 0. \end{aligned}$$

By (14), $U(a \cup b) - U(b) = w(a \cup b) - w(b) > 0$. By (20),

$$\begin{aligned} W(a \cup b) - W(b) &= (\kappa + 1)(w(a \cup b) - w(b)) + \mu(u(a \cup b) - u(b)) \leq \\ &\quad \alpha(\kappa + 1)(w(x) - w(y)) + \mu(u(z) - u(x)) < 0. \end{aligned}$$

Therefore, $\{a \cup b\} \succ \{b\}$ and $\{b, a \cup b\} \sim \{b\}$. These rankings violate RSD. Thus, $\mu \geq 0$.

Suppose that $\kappa + \rho < 0$. Take $\alpha, \beta \in (0, \frac{1}{2})$ such that

$$1 < \frac{\alpha}{\beta} \frac{w(x) - w(y)}{v(y) - v(x)} < 1 + \left| \frac{\kappa + \rho}{\kappa + 1} \right|. \quad (22)$$

Let $a = \{x, y, z\}$ and $b = \{\alpha y + (1 - \alpha)x, \beta x + (1 - \beta)y, z\}$. Then

$$\begin{aligned} w(a \cup b) - w(b) &= w(x) - w(\alpha y + (1 - \alpha)x) = \alpha(w(x) - w(y)) > 0 \\ v(a \cup b) - v(b) &= v(y) - v(\beta x + (1 - \beta)y) = \beta(v(y) - v(x)) > 0 \\ u(a \cup b) - u(b) &= u(z) - u(z) = 0. \end{aligned}$$

By (22) and (20),

$$\begin{aligned} U(a \cup b) - U(b) &= \alpha(w(x) - w(y)) - \beta(v(y) - v(x)) > 0 \\ |(\kappa + 1)(U(a \cup b) - U(b))| &< |\kappa + \rho| \beta(v(y) - v(x)) \\ W(a \cup b) - W(b) &= (\kappa + 1)(U(a \cup b) - U(b)) + (\kappa + \rho)\beta(v(y) - v(x)) < 0. \end{aligned}$$

Therefore, $\{a \cup b\} \succ \{b\}$, but $\{b, a \cup b\} \sim \{b\}$, which contradicts RSD. Thus, $\kappa + \rho \geq 0$.

Suppose that $\kappa + \rho = 0$. Then for all $x' \in X$,

$$V(\{x'\}) = \kappa U(\{x'\}) + \mu u(x') = (\kappa + \mu)U(\{x'\}),$$

where $\kappa + \mu = -\rho + \mu \geq 0$. This equality contradicts Regularity (see the proof of Case 1.) Thus, $\kappa + \rho > 0$.

Suppose that $\succeq$ satisfies MSD. Let $a = \{x, y\}$ and $b = \{y, z\}$. Then $\{a \cup b\} \succ \{b\}$. By MSD,

$$V(a) - V(a \cup b) = \mu(u(x) - u(z)) \geq 0.$$

Thus, $\mu = 0$.

Finally, note that

$$\begin{aligned} \kappa &= \frac{V(\{x, y, z\}) - V(\{\alpha z + (1 - \alpha)x, y, z\})}{\alpha(w(x) - w(z))} \\ \rho &= \frac{V(\{x, y, z\}) - V(\{x, \alpha x + (1 - \alpha)y, z\})}{\alpha(v(y) - v(x))} \\ \mu &= \frac{V(\{x, y, z\}) - V(\{x, y, \alpha x + (1 - \alpha)z\})}{\alpha(u(z) - u(x))} \end{aligned}$$

for all sufficiently small α . These equations show that all of these parameters are unique. $\square$

Lemmas 5 and 6 imply that V has the required form

$$V(a) = \kappa U(a) + \lambda v(a) + \mu u(a) = \kappa w(a) + (\lambda - \kappa)v(a) + \mu u(a), \quad (23)$$

where $\kappa \geq \lambda = \kappa + \rho > 0$, $\mu \geq 0$, and $u, v \in \mathcal{U}$ are independent.

Conversely, suppose that $\succeq$ has representation (13), (14), and (23). Take any $a, b \in \mathcal{M}_1$ such that $\{a \cup b\} \succ \{b\}$. By (23),

$$V(a \cup b) - V(b) = \kappa(U(a \cup b) - U(b)) + \lambda(v(a \cup b) - v(b)) + \mu(u(a \cup b) - u(b)) \geq 0$$

because $U(a \cup b) > U(b)$, $v(a \cup b) \geq v(b)$, and $u(a \cup b) \geq u(b)$. By (13),

$$\{a \cup b\} \sim \{b, a \cup b\} \succ \{b\},$$

and $\succeq$ satisfies RSD. (If $\{b\} \succeq \{a \cup b\}$, then $\{b, a \cup b\} \succeq \{a \cup b\}$ follows from Set-Betweenness.)

Moreover,

$$V(a) - V(a \cup b) = \kappa(w(a) - w(a \cup b)) + (\lambda - \kappa)(v(a) - v(a \cup b)) + \mu(u(a) - u(a \cup b)) \geq \mu(u(a) - u(a \cup b))$$

because $w(a) = w(a \cup b)$, $v(a \cup b) \geq v(a)$, and $\lambda - \kappa \leq 0$. Therefore, the ranking $\{a\} \succ \{a, a \cup b\}$ implies that $\mu > 0$ and $u(b) > u(a)$. Thus $\succeq$ satisfies WMSD, and if $\mu = 0$, then $\succeq$ satisfies MSD.

As u and v are independent, then w and v are not redundant. Take $x, y \in X$ such that $w(x) > w(y)$ and $v(y) > v(x)$. By (14),

$$\{\{x\}\} \succ \{\{x, y\}\} \succ \{\{y\}\}.$$

Let $v' = (\kappa + \mu)u + \lambda v$ and $w' = u + v'$. As u and v are independent and $\lambda > 0$, then the functions w' and v' are not redundant. Take $x', y' \in X$ such that $w'(x') > w'(y')$ and $v'(y') > v'(x')$. By (13),

$$\{\{x'\}\} \succ \{\{x'\}, \{y'\}\} \succ \{\{y'\}\}.$$

Thus, $\succeq$ satisfies Regularity.

Turn to Theorem 4. Suppose that $\succeq$ and $\succeq^*$ have representations (4)-(6) with components $(u, v, \kappa, \lambda, \mu)$ and $(u, v, \kappa^*, \lambda^*, \mu^*)$ such that $\kappa \geq \lambda > 0$, $\kappa^* \geq \lambda^* > 0$, and $\mu, \mu^* \geq 0$.

Suppose that $\frac{\lambda^*}{\kappa^*} \geq \frac{\lambda}{\kappa}$ and $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$. Then $U = U^*$ and for all $a, b \in \mathcal{M}_1$,

$$\begin{aligned} \{a\} \succ \{a, b\} &\Rightarrow U(a) > U(b) \text{ and } V(b) > V(a) \Rightarrow \\ [U(b) - U(a)] + \frac{\lambda}{\kappa}[v(b) - v(a) + \frac{\mu}{\lambda}u(b) - \frac{\mu}{\lambda}u(a)] &> 0 > U(b) - U(a) \Rightarrow \\ [U^*(b) - U^*(a)] + \frac{\lambda^*}{\kappa^*}[v(b) - v(a) + \frac{\mu^*}{\lambda^*}u(b) - \frac{\mu^*}{\lambda^*}u(a)] &> 0 > U^*(b) - U^*(a) \Rightarrow \\ U^*(a) > U^*(b) \text{ and } V^*(b) > V^*(a) &\Rightarrow \{a\} \succ^* \{a, b\}. \end{aligned}$$

Thus, $\succeq^*$ is more self-deceptive than $\succeq$.

Conversely, suppose that $\succeq^*$ is more self-deceptive than $\succeq$. As u and v are independent, then the functions u , v , and $w = u + v$ are not redundant. By (12), there are $x, y, z \in X$ such that

$$\begin{aligned} w(x) &> w(y) \vee w(z) \\ v(y) &> v(x) \vee v(z) \\ u(z) &> u(x) \vee u(y). \end{aligned}$$

As $w, v, u \in \mathcal{U}$, then for any $\alpha, \gamma > 0$, there exist $x', y', z' \in X$ such that

$$\begin{aligned} w(x) &> w(x') > w(y) \vee w(y') \vee w(z) \vee w(z') \\ v(y) &> v(y') > v(x) \vee v(x') \vee v(z) \vee v(z') \\ u(z) &> u(z') > u(x) \vee u(x') \vee u(y) \vee u(y') \\ \frac{w(x) - w(x')}{v(y) - v(y')} &= \alpha \\ \frac{v(y) - v(y')}{u(z) - u(z')} &= \gamma. \end{aligned} \tag{24}$$

Show the inequalities $\frac{\lambda^*}{\kappa^*} \geq \frac{\lambda}{\kappa}$ and $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$ by contradiction. Consider three cases.

Case 1. $\frac{\mu^*}{\lambda^*} > \frac{\mu}{\lambda}$. Take γ such that $\frac{\mu^*}{\lambda^*} > \gamma > \frac{\mu}{\lambda}$ and α such that $1 > \alpha > 1 - \frac{\gamma\lambda - \mu}{\gamma\kappa}$. Take $x', y', z' \in X$ that satisfy (24). Let $a = \{x', y', z\}$ and $b = \{x, y, z'\}$. Then

$$\begin{aligned} U(a) - U(b) &= (w(x') - v(y')) - (w(x) - v(y)) = (1 - \alpha)(v(y) - v(y')) > 0 \\ V(b) - V(a) &= \left(-\kappa(1 - \alpha) + \lambda - \frac{\mu}{\gamma}\right)(v(y) - v(y')) > 0 \\ V^*(b) - V^*(a) &= \left(-\kappa^*(1 - \alpha) + \lambda^* - \frac{\mu^*}{\gamma^*}\right)(v(y) - v(y')) < 0 \end{aligned}$$

because

$$-\kappa(1 - \alpha) + \lambda - \frac{\mu}{\gamma} > 0 > -\kappa^*(1 - \alpha) + \lambda^* - \frac{\mu^*}{\gamma}.$$

Thus, $\{a\} \succ \{a, b\}$, but $\{a\} \sim^* \{a, b\}$, which contradicts the assumption that $\succeq^*$ is more self-deceptive than $\succeq$.

Case 2. $\frac{\mu^*}{\lambda^*} < \frac{\mu}{\lambda}$. Take γ such that $\frac{\mu^*}{\lambda^*} < \gamma < \frac{\mu}{\lambda}$ and α such that $1 < \alpha < 1 + \frac{\mu - \gamma\lambda}{\gamma\kappa}$. Take $x', y', z' \in X$ that satisfy (24). Let $a = \{x, y, z'\}$ and $b = \{x', y', z\}$. Then

$$U(a) - U(b) = (w(x) - v(y)) - (w(x') - v(y')) = (\alpha - 1)(v(y) - v(y')) > 0$$

$$V(b) - V(a) = \left(-\kappa(\alpha - 1) - \lambda + \frac{\mu}{\gamma}\right)(v(y) - v(y')) > 0$$

$$V^*(b) - V^*(a) = \left(-\kappa^*(\alpha - 1) - \lambda^* + \frac{\mu^*}{\gamma}\right)(v(y) - v(y')) < 0$$

because

$$-\kappa(\alpha - 1) - \lambda + \frac{\mu}{\gamma} > 0 > -\kappa^*(\alpha - 1) - \lambda^* + \frac{\mu^*}{\gamma}.$$

Thus, $\{a\} \succ \{a, b\}$, but $\{a\} \sim^* \{a, b\}$, which contradicts the assumption that $\succeq^*$ is more self-deceptive than $\succeq$.

Case 3. $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$ and $\frac{\lambda^*}{\kappa^*} < \frac{\lambda}{\kappa}$. Take α such that $1 - \frac{\lambda^*}{\kappa^*} > \alpha > 1 - \frac{\lambda}{\kappa}$. Take $x', y' \in X$ that satisfy (24). (z' is not required here.) Let $a = \{x', y', z\}$ and $b = \{x, y, z\}$. Then

$$U(a) - U(b) = (w(x') - v(y')) - (w(x) - v(y)) = (1 - \alpha)(v(y) - v(y')) > 0$$

$$V(b) - V(a) = (-\kappa(1 - \alpha) + \lambda)(v(y) - v(y')) > 0$$

$$V^*(b) - V^*(a) = (-\kappa^*(1 - \alpha) + \lambda^*)(v(y) - v(y')) < 0$$

because

$$-\kappa(1 - \alpha) + \lambda > 0 > -\kappa^*(1 - \alpha) + \lambda^*.$$

Thus, $\{a\} \succ \{a, b\}$, but $\{a\} \sim^* \{a, b\}$, which contradicts the assumption that $\succeq^*$ is more self-deceptive than $\succeq$.

Thus, $\frac{\mu^*}{\lambda^*} = \frac{\mu}{\lambda}$ and $\frac{\lambda^*}{\kappa^*} \geq \frac{\lambda}{\kappa}$.

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