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A Game Theoretic Approach to Behavioral Economics

by

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Chapter 1

Introduction

1.1 Motivation

This thesis explores games that are played between individuals who exhibit non-standard preferences. The first two essays focus on firm behavior, while the third one focuses on the behavior of individuals. In the first essay (Chapter 2), I explore the welfare effect of advertising as a memory aid to consumers with limited attention. In the second essay (Chapter 3), jointly authored with Aleksandr Yankelevich, we explore asymmetries in firm pricing that stem from differences in consumer characteristics such as proximity to a particular firm. In the third essay (Chapter 4), I explore whether repetition can generate strategic altruism when player exhibit altruism towards kin, but kin recognition is noisy.

1.2 Essay 1: Advertising to Consumers with Limited Attention

Overall, an important role of advertising is to convince consumers that they want the product and to buy it from the brand advertising it. However, experimental studies suggest that consumers might not want to devote much atention to certain purchasing decisions. In particular, this is true for goods that consumers perceive as homogeneous and that are relatively unimportant to them. When making a purchasing decision, instead of focusing on the set of all feasible alternatives, consumers restrict attention to a subset of all feasible alternatives, called consideration set. The consumer consideration set is formed by evoking a certain number of brands. Consumers compare only the brands in consideration set, and purchase the one with the preferred attribute. The fact that consumers restrict attention to a subset of all feasible alternaitives gives rise to a new role for advertising, which is reminding consumers of the firm's brand in purchasing situations. An ad is salient when it reminds consumers of the brand that is being advertised in buying situations. When consumers have limited attention, firms seek to affect the probability with wich they get evoked into the consideration set by engaging in salient advertising. However, because consumers' attention is limited, an advertisement that induces a consumer to enter the market might not always lead him to purchase from the brand being advertised. In that case we say that the ad fails to be salient, and it acts like a public good by promoting the product (as opposed to promoting the brand).

We analyze the welfare implications of salient advertising in a homogeneous goods market with consumers with limited attention. In equilibrium ,firms engage in at least some degree of salient advertising if the marginal cost of advertising is not high. There are cases where welfare is highest with an intermediate advertising technology that leads to an equilibrium with positive but imperfect salience. When salient advertising substitutes for lower prices, and the total cost is sufficiently low, expected firm profits are greater than without advertising. If consumers are sensitive to advertising and increases in salience are smaller for higher prices, then the average price paid by consumers is less than without advertising, so consumer utility increases as well.

1.3 Essay 2: Asymmetric Search

Price dispersion occurs in markets for homogeneous goods because consumers vary in the amount of information they possess about firm prices or in the cost of obtaining that information. This gives firms contrasting pricing motives: keep prices high to extract profit from consumers who remain relatively uninformed about prices in rival firms, lower them to attract consumers who are relatively informed. As a result, firms play mixed pricing strategies where they run sales of different magnitudes. Because firms are frequently assumed to be homogeneous and the consumer search order is assumed to be random, in equilibrium, all firms choose prices from the same price distribution. However, in many real world market scenarios, different firms carrying the same products expect to encounter different types and numbers of consumers. If we compare two stores, say a supermarket and convenience store, which might expect to be visited by different proportions of different types of consumers, although one might expect the latter to have higher prices most of the time, it probably does not have higher prices all of the time. In this essay, we are able to study such a situation by supposing that some consumers can sample price freely at their local firm, but must pay for an additional sample elsewhere and that the proportions of local consumers in different firms are different.

In equilibrium, we find that the firm with more local consumers has higher prices on average and runs fewer sales, but that its sales can be just as large as those of the firm with fewer local consumers. However, we find that the highest price firms are willing to charge may be significantly lower than the consumer valuation for the good when the cost of sampling an additional firm is sufficiently small. An additional finding of note is that although average prices fall when the proportion of consumers who sample prices freely rises, the firm with more locals may run fewer sales. This turns out to have important implications for advertising policy to lower-income consumers, particularly with regard to traditional advertising using informative circulars.

1.4 Essay 3: Kin-Targeted Altruism

This essay studies a prisoners' dilemma played between two people who exhibit altruistic preferences towards kin. The probability that a player's opponent is kin is common knowledge. Instead of observing the degree of relatedness with the other player, each player observes a noisy private signal. When the game is played once, players cooperate only with those identified as kin. However, when the prisoners' dilemma is played for two periods instead of one, uncertainty about relatedness brings strategic considerations into the game even if the odds of being related are small. Depending on the parameters, there are Perfect Bayesian Equilibria in which players cooperate in the first round even after getting a negative kin signal. Since a player can make inferences about her opponent's signal based on first period actions, a non-relative mimics kin to induce cooperation in the second period.

Chapter 2

The Bunny and the Batteries: Advertising to Consumers with Limited Attention

2.1 Introduction

According to Advertising Age's annual assessment of advertising, total spending on advertising in the United States in 2009 was approximately 250 billion, almost two percent of GDP. Much of this expenditure corresponds to firms' investment in advertising.[1] The economics literature focuses on three functions of firm advertising: altering consumer preferences (Braithwaite, 1928; Kaldor, 1950), increasing consumer information about firms and their goods (Stigler, 1961; Telser, 1964; Butters, 1977; Grossman and Shapiro, 1984; Robert and Stahl, 1993), and augmenting the utility of consuming a particular good (Stigler and Becker, 1977;

[1] *Ad Spending by Medium and Sector.* The 2011 Entertainment, Media and Advertising Market Research Handbook, 225-227.

Becker and Murphy, 1993).[2] Experimental studies suggest that, in many purchasing situations, there is a lack of motivation to devote substantial cognitive processing efforts to brand selection. In this context, an important role of advertising is to to get noticed and remembered by consumers. In particular, we say that an ad is salient when its main purpose is to make consumers remember the brand. The existing literature on advertising might suggest that salient advertising decreases overall welfare, since it does not contain any new information about the product. However, in a context where consumers devote limited attention to purchasing decisions, the welfare effects become less obvious.

To understand the effects of salient advertising, we first consider how consumers in the market decide which brand to buy. Building on research in psychology, experimental studies of consumers posit that once the consumer decides to buy the product, he does not necessarily consider all available alternatives. Instead, he considers his list of viable brand alternatives, referred to as his consideration set (Miller and Berry, 1998; and Romaniuk and Sharp, 2004). He evaluates other attributes such as price of the brands in his consideration set, and then makes his decision of which brand to buy. Bullmore (1999) succinctly summarizes this decision process: "Most of us have clusters of brands which we find perfectly satisfactory. We will allocate share of choice within this repertoire according to chance, promotions, advertising, availability, price, impulse or recommendation." For example, a consumer shopping for toothpaste at the supermarket has an idea of which brands he is willing to buy even before stepping into the toothpaste aisle. He then compares the prices of the first couple of brands that come into his mind and buys the cheapest. Alternatively, a consumer who is considering buying a car

[2] See Bagwell (2007) for an extensive overview of advertising.

chooses to visit a subset of local car dealerships. After looking at the different makes, he purchases the one with the best gas mileage. As such, firms find it crucial to be on top of consumers' minds. By engaging in salient advertising, firms attempt to increase the probability of entering consumer consideration sets. That is, firms must not only convince consumers that they want to buy the product, but, most importantly, that they want to buy it from the brand advertising it. When an ad is salient, it increases the probability a consumer will evoke that brand when forming his consideration set.

However, a firm's attempt to make an ad salient might fail. Since most modern advertising occurs in the midst of multi firm competition, if we are to think of advertisements as a means to reinforce a brand in consumer memory, it is important to consider the effects of rival ads on memory as it pertains to purchasing decisions. Experimental evidence suggests that in the face of myriad ads from different firms (Kent, 1993; Kent and Allen, 1993), consumers are likely to get confused about which brand a particular ad refers to. This occurs because there is typically a lag between consumers' exposure to advertising and their decision to purchase the advertised good (Keller, 1987) and is accentuated when consumers review similar ads without an intention of purchasing what is advertised (Burke and Srull, 1988). In a particularly well known instance, the Energizer Bunny ad ended up promoting just batteries instead of Energizer: viewers could not remember which brand the ad belonged to because Duracell –Energizer's main competitor– had also used bunnies in previous ads (Lipman, 1990). Therefore, advertising can function as a public good: the fact that a firm engages in salient advertising does not guarantee that all consumers will perceive the ad as salient. If an ad is not perceived as salient by consumers it will only serve to promote the product (as opposed to the brand),

and, will not affect any of the existing brands' probability of being remembered.

This paper sets up a theoretical model of salient advertising in order to analyze its effects on welfare. In our model, firms have the option of engaging in imperfect salience. That is, to choose the probability with which the ad will turn out to be salient in one realization. Firms can also engage in perfect salience (choosing ads that are always salient) or no salience at all (ads that do not attempt to promote the brand). Consumers form consideration sets by making a number of evocations. The probability that a particular firm is evoked is influenced by firms' investment in the degree of salience. We find that in equilibrium, if salient advertising is either sufficiently expensive or sufficiently cheap, all firms either forgo or engage in perfect salient advertising, respectively. In both cases, the equilibrium price distributions are identical and both firms have an equal probability of being evoked. This means that profit is strictly lower in the low cost case, making advertising wasteful. The effect of salient advertising is lost because both firms advertise with equal intensity. Thus, the high-cost advertising technology that leads to the no-salience equilibrium Pareto dominates the low-cost technology that leads to the perfect salience equilibrium. Intermediate levels of advertising cost lead to an equilibrium with imperfect salient advertising. Unlike in the previous two cases where advertising is ineffective, there are realizations in which firms can successfully use advertising to get consumers to pay attention to their brand. The effects on prices and firm profit are ambiguous whenever imperfect salience is involved, depending on the advertising technology, consumer valuation and production cost. There are cases where welfare is highest under an intermediate advertising technology that leads to an equilibrium with positive but imperfect salience. When effective salient advertising substitutes for lower prices and the total cost is low enough, expected firm profits are greater than without advertising. However, if consumers are sensitive to advertising and increases in salience are smaller for higher prices, then the average price paid by consumers is less than without advertising, so consumer utility increases as well.

The remainder of this paper is organized as follows. Section 2.2 sets up the model and analyzes its equilibrium. Section 2.3 analyzes the welfare impact of technology changes, Section 2.4 does a brief literature review, and Section 2.5 concludes.

2.2 The Model

There are two identical firms that compete to sell a homogeneous good. Each firm pays a constant marginal cost of c to produce the good. Firm i makes a twofold decision: it sets price, p_i, and determines the degree of salience in its advertising $a_i^s \in [0, 1]$, incurring a cost of $A(a_i^s)$, where A is strictly convex. The cost of choosing no salience, $A(0)$, is normalized to zero. If a firm chooses a certain degree of salience $a_i^s \in (0, 1)$, it means that firm i's ad will turn out to be salient with probability a_i^s (imperfect salience). Likewise, choosing $a_i^s = 1$ means that firm i's ad will always turn out to be salient (perfect salience), and choosing $a_i^s = 0$ means that firm i's ad will never turn out to be salient (no salience).

On the other side of the market, there is a unit mass of identical consumers with valuation $v > c$ for the good. Before making a purchase, consumers form their consideration set by making two evocations. The degree of salience in the ads (a_1^s, a_2^s) determines the probability that an individual consumer evokes a particular firm. With probability $(1-a_1^s)(1-a_2^s)$, none of the ads will be salient, so consumers will evoke each firm with equal probability. With probability $a_1^s(1-a_2^s)$, firm 1's ad will be salient and firm 2's ad will not be salient. In this case, a consumer will evoke firm 1 with $\mu > 1/2$ and firm 2 with probability $1-\mu$, where μ is a fixed parameter of the model. Finally, when both ads turn out to be salient, which happens with probability $a_1^s a_2^s$, consumers will evoke each firm with equal probability. That is, when both firms' ads are salient, the effect of salience is lost because both ads cancel each other out, and the result will be the same as in the case where none of the ads is salient.

To form their consideration sets, consumers make two evocations with replacement. If only one firm is evoked, the consumer purchases from that firm. If two different firms are evoked, the consumer chooses the firm offering the lower price. If both firms have the same price, each of them has an equal probability of being chosen. In this framework, salient advertising raises the probability that a firm i will be evoked, but does not guarantee that it will be the only firm to be evoked.

Firms and consumers play the following game. First, the two firms in the market simultaneously choose prices and choose the degree of salient advertising. Then, consumers in the market observe each firm's ads and form their consideration sets. Then, consumers observe the prices of the firms in their consideration sets and make a purchasing decision. A pricing strategy for firm i is a price distribution F_i. An advertising strategy is a degree of salient advertising $a_i^s \in [0,1]$.

2.2.1 Equilibrium

We restrict attention to the symmetric Nash equilibrium and dispense with subscripts on F, a^s and p. First, we consider the baseline case with no advertising, where firms only compete in prices. Consumers make two evocations to form their consideration sets, and each firm is remembered with equal probability. In the baseline case, when salient advertising is not allowed, there is price dispersion, as shown in the following Proposition.

Proposition 2.1. *In the baseline case, where consumers have limited attention but salient advertising is not allowed, firms play a mixed pricing strategy. The lower bound of the support is* $\underline{p} = c + \frac{1}{3}(v - c)$, *the upper bound is the consumer valuation* v, *and the firm price distribution is* $F(p) = 1 - \frac{1}{2}\left(\frac{v-p}{p-c}\right)$. *Expected profit is equal to* $\frac{v-c}{4}$.

The proof of Proposition 2.1 relies on the following Lemmas.

Lemma 2.1. *There are no atoms in the equilibrium price distribution.*

Lemma 2.2. *The upper bound of the equilibrium firm price distribution is* v.

The proof of Lemma 2.1 follows from the proof of Lemma A, shown in the appendix. The reasoning is that, if both firms have an atom at a certain price p, they split all consumers with two firms in their consideration sets. Firms can benefit by undercutting the other firm, thus getting all consumers with two firms in their consideration sets. Lemma 2.2 follows because firms make no profit at prices above v, but they always have an incentive to raise prices to v for those consumers who only evoked them into their consideration sets. Price dispersion occurs for the following reason. Some consumers only evoked one firm into their consideration sets, so firms want to exploit their willingness to pay. However, other consumers evoked both firms into their consideration sets and will buy from the lowest priced firm, so firms also want to charge lower prices to attract these consumers. Equal profit on the support of the firm price distribution allows us to solve for the lower bound and the firm price distribution. All proofs are in the Appendix.

When there is advertising, there is still price dispersion in equilibrium (see Lemma A in the Appendix). By strict convexity of the cost of advertising function A, there exists a unique optimal level of advertising for each price in the support of the firm price distribution. Therefore, an optimal advertising strategy is a mapping from the support of the firm price distribution F to $[0,1]$. The characterization of equilibrium depends on the marginal cost to marginal benefit of advertising ratio at the top of the firm price distribution, $A^{'}(a^s(v))/(v-c)$. When the marginal cost to marginal benefit ratio of advertising is high or low, both firms either forgo advertising or engage in perfect salient advertising, respectively. On the other hand, intermediate values of this ratio lead to an equilibrium with imperfect salient advertising.

Let $\beta = \int_{\underline{p}}^{v} a^s(x)dF(x) \in [0,1]$. Expected firm profit is given by the following expression:

$$\begin{aligned} \mathbb{E}\Pi(p, a^s(p)) &= a^s(p)\beta\Pi_{ss}(p) + (1-a^s(p))\beta\Pi_{ns}(p) \\ &\quad + a^s(p)(1-\beta)\Pi_{sn}(p) + (1-a^s(p))(1-\beta)\Pi_{nn}(p) - A(a^s(p)) \end{aligned} \tag{2.1}$$

Where

$$\Pi_{nn}(p) = \Pi_{ss}(p) = [\tfrac{1}{4} + \tfrac{1}{2}(1-F(p))](p-c)$$

$$\Pi_{ns}(p) = [(1-\mu)^2 + 2\mu(1-\mu)(1-F(p))](p-c)$$

$$\Pi_{sn}(p) = [a^2 + 2\mu(1-\mu)(1-F(p))](p-c)$$

The following Proposition summarizes the equilibrium result.[3]

Proposition 2.2. *There exists a unique symmetric Nash equilibrium where both firms have supports $[\underline{p}, v]$. The lower bound $\underline{p}$, the firm price distribution and the*

[3]There are other two cases corresponding to medium-high and medium-low cost of advertising technologies, which are a combination of cases 1 and 3 and 2 and 3 respectively. We decided not to include them because we can't get closed form solutions and the conditions are hard to interpret.

degree of salient advertising depend on the advertising technology as follows.

1. *High-cost Advertising technology,* $\frac{A'(0)}{v-c} > \mu^2 - \frac{1}{4}$*: no salience in equilibrium,* $a^s(p) = 0\ \forall p \in [\underline{p}, v]$*. The lower bound of the support is* $\underline{p} = c + \frac{1}{3}(v-c)$ *and the firm price distribution is* $F(p) = 1 - \frac{1}{2}\left(\frac{v-p}{p-c}\right)$*. Expected profit is equal to* $\frac{v-c}{4}$*.*

2. *Low-cost Advertising Technology,* $\frac{A'(1)}{v-c} > \frac{1}{3}(\mu^2 - \frac{1}{4})$*: perfect salience in equilibrium,* $a^s(p) = 1\ \forall p \in [\underline{p}, v]$*. The lower bound of the support is* $\underline{p} = c + \frac{1}{3}(v-c)$ *and the firm price distribution is* $F(p) = 1 - \frac{1}{2}\left(\frac{v-p}{p-c}\right)$*. Expected profit is equal to* $\frac{v-c}{4} - A(1)$*.*

3. *Medium-cost Advertising Technology,* $\frac{A'(0)}{v-c} \leq \frac{A'(a^s(v))}{v-c} < \frac{1}{4} - (1-\mu)^2$ *and* $\frac{A'(1)}{v-c} \geq \frac{A'(a^s(v))}{v-c} > \mu^2 - \frac{1}{4}$*: imperfect salience in equilibrium,* $a^s(p) \in (0,1)\ \forall p \in [\underline{p}, v]$*. The lower bound of the support is*

$$\underline{p} = c + \frac{A'(a^s(\underline{p}))}{2(\mu - \frac{1}{2}) - \frac{A'(a^s(v))}{v-c}}$$

   and the firm price distribution is

$$F(p) = 1 - \frac{1}{2}\left[\frac{\frac{A'(a^s(v))}{v-c} - \frac{A'(a^s(p))}{p-c}}{\frac{A'(a^s(v))}{v-c} - (\mu - \frac{1}{2})}\right]$$

   The expected probability that a firm's ad is salient is given by

$$\beta = \frac{(\mu^2 - \frac{1}{4}) - \frac{A'(a^s(v))}{v-c}}{2(\mu - \frac{1}{2})^2}$$

   Expected profit is equal to

$$(v-c)\left[s(v)(1-\beta)(\mu^2 - \tfrac{1}{4}) - (\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu)\beta(1 - a^s(v))\right] + \tfrac{1}{4}(v-c) - A(a^s(v))$$

As in the baseline case, the proof relies on Lemma A and Lemma 2.2 (see Appendix).

In the medium-cost advertising technology case, the equilibrium expected degree of salience depends on the ratio of the gain of having a salient ad when the opponent firm does not to the loss generated by the opponent's ad being salient at the upper bound of the price distribution. When firm 1 is pricing at v and its ad realization is salient but firm 2's ad realization is not salient, the proportion of the population who will only evoke firm 1 into their consideration set increases in $\mu^2 - \frac{1}{4}$. Thus, there will be more consumers who pay full price for the good because they failed to remember firm 2, generating an expected gain of $(\mu - \frac{1}{4})(v - c)$. Because of the medium cost of advertising, this expected gain will exceed the marginal cost of investing in the degree of salience $a^s(v)$. However, when firm 2's ad is salient, it decreases the efficiency of firm 1 having a salient ad realization. When firm 2's ad realization is salient, the expected gain of firm 1 is given by $\left[\frac{1}{4} - (1 - \mu^2)\right](v - c)$. That is, a salient realization of firm 2 generates a loss to firm 1, equal to $\left[\left(\mu^2 - \frac{1}{4}\right) - \left(\frac{1}{4} - (1 - \mu^2)\right)\right](v - c)$. The higher the expected loss provoked by the opponent firm's ad being salient, the lower the expected degree of salience, and vice versa.

The equilibrium firm price distribution is determined by a similar reasoning. When firm 2's ad is salient, which happens with probability β, firm 1 having a salient ad increases the proportion of the population that will evoke both firms into their consideration set. When firm 2's ad is not salient, firm 1 having a salient ad increases the proportion of the population that will only have firm 1 in their consideration set, therefore decreasing the proportion of people with two firms in their consideration set. Depending on the value of β, firm 1's ad being salient will generate an expected increase or decrease in the proportion of people who remember both firms. When $\beta < \frac{1}{2}$, firm 1's salience generates an expected decrease in the proportion of the population whose consideration set consists of both firms equal to $2[\frac{A^{'}(a^s(v))}{v-c} - (\mu - \frac{1}{2})]$. When this is the case, the increase in the proportion of people whose consideration consists of only firm 1, given by $\frac{A^{'}(a^s(v))}{v-c}$, who will definitely buy from firm 1 and generate a marginal benefit of $p-c$, should exceed the marginal cost of of engaging in the degree of advertising $a^s(p)$. That is, salience functions as a substitute for lower prices. On the other hand, when $\beta > \frac{1}{2}$, when firm 1's ad is salient, the expected number of people who remember both firms increases in $2[(\mu - \frac{1}{2}) - \frac{A^{'}(a^s(v))}{v-c}]$. This implies that salient advertising is complementary to charging lower prices. Also, the benefits earned by firm 1 when it is the only firm in consumers' consideration sets are not enough to cover the cost of salience.

The lower bound of the firm price distribution depends on the expected increase in the proportion of the population who will remember firm 1, given by $2(\mu - \frac{1}{2}) - \frac{A^{'}(a^s(v))}{v-c}$. Given that firm 1 has the lowest possible price, being remembered is sufficient for making a sale. If this number is high, then the lower bound of the price distribution will be closer to the marginal cost of production, since the firm will need to extract less from each individual consumer to compensate for the marginal cost of engaging in salient advertising. If, on the other hand, this number is low, $\underline{p}$ will be higher in the medium-cost case. The ratio $\frac{A^{'}(s(\underline{p}))}{2\left(\mu - \frac{1}{2}\right) - \frac{A^{'}(a^s(v))}{v-c}}$ determines how above the marginal cost of production a firm has to charge in order to cover the marginal cost of engaging in degree of salient advertising $a^s(\underline{p})$. If this is smaller than $\frac{1}{3}(v-c)$, then the lower bound of the price distribution is lower in the medium-cost advertising technology case than in the other two cases. This is more likely to happen in markets where at least one of the following is true: consumer valuation is high, production cost is low, or the probability of a salient firm being remembered, μ, is closer to 1.

Definition 2.1. *We say that consumers are* responsive to salience at the lower bound of the firm price distribution $\underline{p}$ *whenever* $2\left[(\mu - \frac{1}{2}) - \frac{A'(a^s(v))}{v-c}\right] > \frac{A'(s(\underline{p}))}{\frac{1}{3}(v-c)}$.

It follows from the previous discussion that whenever consumer are responsive to salience at $\underline{p}$, the lower bound of the firm price distribution in the medium cost case is smaller that the lower bound in the high and low cost cases.

2.3 Welfare Analysis

In this section we analyze the welfare impacts of switching the advertising technology, given by the cost of advertising function A. We fix the consumer valuation v and cost of production c, and focus solely on the advertising technology, represented by A_i for $i = h,\ m,\ l$ referring to high-cost, medium-cost and low-cost advertising technologies respectively.

Proposition 2.3. *Switching advertising technology from A_l to A_h always improves welfare.*

The proof is straightforward, and follows from observing that the firm profit stated in Proposition 3.3.3 case 1 is always strictly greater than the firm profit stated in Proposition 3.3.3 case 2. Consumer welfare remains unchanged because the supports and the firm price distributions are identical. That is, the low-cost technology is always Pareto dominated by the high-cost technology.

Whenever the medium-cost technology is involved, the welfare analysis is ambiguous. First of all, firm profit might increase or decrease when switching from high-cost to medium-cost technology. Imperfect salience introduces a potential gain, given by the realizations in which only one firm's ad turns out to be salient. In this case, the salient firm is remembered with a higher probability, and, thus, has a higher likelihood of being the only firm in the consumer consideration set. This means that the likelihood that a firm sells at higher prices increases. However, we must also take into account that there is an associated potential loss, generated by the opposite case. That is, when only one firm's ad realization is not salient, it has a lower probability of being in the consumer consideration set, so its likelihood of selling at higher prices falls. A sufficient condition for imperfect salience to generate an expected gain is that the advertising function $a^s(p)$ be increasing in p, which implies that $a^s(v) > \beta$. This means that advertising works as a substitute for lower prices: firms engage in a higher degree of salient advertising for higher prices to increase the likelihood of being the only brand in the consumer consideration set. However, for lower prices, it becomes less important to engage in salient advertising because the likelihood of having the lowest price increases.

The effect of switching from high-cost technology to medium-cost technology on prices is also ambiguous. As mentioned in the previous section, when consumers are responsive to salience at the lower bound of the price distribution, both firms charge lower prices in the medium-cost of advertising case. Also, it must be the case that salient advertising $a^s(p)$ is increasing in prices, to guarantee that the marginal cost of advertising increases in prices. In order to guarantee lower expected prices, it must be the case that the equilibrium firm price distribution in the high and low cost cases, denoted by F_c, first order stochastically dominates the equilibrium firm price distribution from the medium cost case, denoted by F_m. Whenever the marginal cost of salient advertising is strictly concave in prices, F_c always first order stochastically dominates F_m. A necessary condition for strict concavity of marginal cost with respect to price is that $a^s(p)$ is concave in p, which means that the rate at which advertising increases should decrease as prices get higher. If the marginal cost of advertising is not strictly concave in prices, a sufficient condition to guarantee first order stochastic dominance is that the density at the upper bound of the firm price distribution obeys $f(v) < \frac{1}{2(v-c)}$.

The following proposition summarizes the case where the medium-cost technology Pareto dominates the high-cost technology.

Proposition 2.4. *Welfare is higher under A_m rather than under A_h whenever all of the following conditions hold:*

1. *Imperfect salience generates an expected gain that exceeds its total cost at the upper bound of the price distribution, $A_m(a^s(v))$.*
2. *The degree of salient advertising is increasing in prices.*
3. *Consumers are responsive to salience at the lower bound of the price distribution.*
4. *The marginal cost of advertising is strictly concave in p or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.*

When the degree of salient advertising is increasing in prices, investing in imperfect salience generates an increase in expected revenue. Whenever this increase is sufficiently high to make up for the cost of investing in salience, firm profit is strictly higher under imperfect salience than under no-salience. Expected prices are lower due to conditions 3 and 4, as explained above. For a more detailed proof, see the Appendix.

In the following case, we establish conditions for the low-cost technology to be strictly worse than the medium-cost technology. Since the equilibrium price distribution and its lower bound are identical to the ones from the high-cost advertising technology, the intuition is very similar to that of the previous result. However, in this case, having to incur the cost of perfectly salient advertising makes firm profit even lower. The complete conditions for this case are stated in the following proposition.

Proposition 2.5. *Welfare is higher under A_m rather than under A_h in the following cases.*

1. *When $A_l(1) > A_m(a^s(v))$ and all of the following conditions hold.*
   (a) *The degree of salient advertising is increasing in prices.*
   (b) *Consumers are responsive to salience at the lower bound of the price distribution.*
   (c) *The marginal cost of advertising is strictly concave in p or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.*
2. *When $A_l(1) < A_m(a^s(v))$ and all of the following conditions hold.*
   (a) *The degree of salient advertising is increasing in prices.*
   (b) *Imperfect salience generates an expected gain that exceeds $A_m(a^s(v)) - A_l(1)$.*
   (c) *Consumers are responsive to salience at the lower bound of the price distribution.*

(d) *The marginal cost of advertising is strictly concave in p or the density at the upper bound of the firm price distribution $f(v) < \frac{1}{2(v-c)}$.*

2.4 Literature Review

In the Economics literature, other models have also considered consumer purchasing decisions as a two-step process. Manzini and Mariotti (2007) and Lleras et al. (2010) suggest that having too many options to choose from is overwhelming for consumers, and hence, they end up restricting attention to subsets (possibly strict) of all feasible alternatives. In both models, a consumer forms his consideration set using one particular rationale or consideration filter (such as salience or similarity), and then chooses one alternative from the consideration set using a different rationale (such as utility maximizing). Other models have studied how firm advertising can be used to shape or discriminate from consideration sets. Eliaz and Spiegler (2011) allow advertising to modify consideration sets in the following way. First, the consumer chooses a preliminary consideration set out of the set of all feasible alternatives. Then, salient advertising seeks to convince the consumer that other products are close substitutes for the product under consideration and should be included in his consideration set. In Tyson (2011), salient advertising cannot determine which elements enter the consideration set, but can be used as a filter to choose an element from the consideration set whenever it contains more than one brand.

As in Burdett and Judd's (1983) noisy sequential search framework, we get price dispersion because every consumer has a positive probability of evoking one or more firms. However, in this paper, the distribution is endogenously determined by advertising. This model also differs from models of informative advertising with price dispersion (e.g., Butters, 1977; Robert and Stahl, 1993) where consumers cannot purchase from a particular firm without either obtaining a price ad from that firm or having paid a cost to search that firm. A consumer who enters the market for a good is aware of all potential competitors in that market, but he only compares the prices of brands he has evoked. This contrasts both Chioveanu's (2008) model of persuasive advertising, where all consumers choose the lowest priced good unless they are "convinced" by advertising to become loyal to a certain brand and the framework of Haan and Moraga-Gonzlez (2009) where consumers are more likely to search firms with more salient advertising, but where search is required before a firm is considered.

2.5 Conclusion

In this paper, we have analyzed salient advertising to consumers with limited attention. Ads that are salient remind consumers of the brand that is being advertised in buying situations and may increase the probability of being the only brand in a consumer's consideration set. Ads that are not salient focus on promoting the product (as opposed to the brand), and do not affect the probability of any firm being evoked into the consumer consideration set. When the advertising technology is expensive, firms do not engage in salient advertising at all. On the other hand, when the advertising technology is cheap, firms engage in perfectly salient advertising, which means that their corresponding ads are always salient. Finally, when the advertising technology involves medium costs, firms engage in imperfect salient advertising. This means that a firm's ad has a certain expected probability of being salient. As a consequence, a firm will benefit when its ad realization is salient and the opponent firm's ad realization is not salient. Similarly, a firm will be harmed when its own ad realization is not salient and the opponent firm's ad realization is salient. That is, only in the imperfect salience case will salient ads have an effect on consumers and their consideration sets. When firms engage in perfect salience, the effect of salience is lost because both firms ads cancel each other out.

In terms of welfare, the high-cost advertising technology case with no salience always Pareto dominates the low-cost advertising technology case with perfect salience. While consumer welfare remains unchanged because the equilibrium price distributions and their supports are identical, firms are strictly worse off with perfect salience because they have to incur the additional cost of perfectly salient ads (that are ineffective in the end). This equilibrium result resembles a prisoners' dilemma or a tug of war competition when costs of advertising are low. In the low-cost advertising technology case, banning advertising would favor both firms, leading to an increase in profit.

The welfare implications of salient advertising are ambiguous whenever the medium-cost technology is involved. Under certain circumstances, imperfect salience leads to lower expected prices and higher firm profits than the no-salience case, so welfare is higher with a medium-cost advertising technology rather than with a high-cost technology. This happens in markets where consumers are very sensitive to salient advertising or have a very high willingness to pay for the product, or the product is relatively cheap to produce. In this case, a ban in advertising strictly reduces both consumer and firm welfare.

An example of the regulation implications of this model is the ban on cigarette advertising in the United States, that became effective in 1971 for radio and television ads[4], and extended to all forms of outdoor (i.e. billboards) advertising in 1999.[5] The main purposes of the ban are to reduce cigarette consumption among smokers. The ads that permeated the media before the ban suggest that cigarette advertising was perfectly salient (the Marlboro man, Joe Camel, Virginia Slims, etc.). In this context, our model has different predictions than the Becker and Murphy model of advertising. According to Becker and Murphy, Marlboro smokers get utility from seeing the Marlboro man. After the ban on advertising, consumers cannot feel like the Marlboro man anymore, so their utility of smoking decreases. Therefore, they should reduce their smoking, and hence, improve their health. In our model, smokers are going to smoke anyways. They cannot see the Marlboro man anymore, but they cannot see Joe Camel either. In cigarette buying situations, they will remember Marlboro with the same probability they would have when there was cigarette advertising and keep on smoking the same quantity. Welfare does improve due to the ban, but the ones who benefit from it are the tobacco companies, who do not have to invest in salient advertising anymore.

[4] Public Health Cigarette Smoking Act, 1970.

[5] Tobacco Master Settlement Agreement, 1998.

2.6 Appendix

2.6.1 Proof of Lemma 2.1

Before introducing the proof of Lemma 2.1 we need to introduce the following Lemma, which is a general version of Lemma 2.1.

Lemma A. *Suppose that a firm that engages in degree of salient advertising $a^s(p)$ at price p. The equilibrium firm price distribution has no atoms.*

Proof. Suppose the equilibrium price distributions have an atom at price $\hat{p}$. Let $\hat{a}^s(\hat{p})$ be the degree of salient advertising at $\hat{p}$. Firm i's expected profit at $\hat{p}$ is

$$\begin{aligned} \mathbb{E}\Pi(\hat{p}, \hat{a}^s(\hat{p})) = \hat{a}^s(\hat{p})\beta\Pi_{ss}(\hat{p}) + (1-\hat{a}^s(\hat{p}))\beta\Pi_{ns}(\hat{p}) + \hat{a}^s(\hat{p})(1-\beta)\Pi_{sn}(\hat{p}) \\ +(1-\hat{a}^s(\hat{p}))(1-\beta)\Pi_{nn}(\hat{p}) - A(\hat{a}^s(\hat{p})) \end{aligned} \tag{2.2}$$

where

$$\Pi_{nn}(\hat{p}) = \Pi_{ss}(\hat{p}) = \left\{\tfrac{1}{4} + \tfrac{1}{2}[1 - F(\hat{p}) + \tfrac{1}{2}\mathbb{P}(p = \hat{p})]\right\}(\hat{p} - c)$$

$$\Pi_{ns}(\hat{p}) = \left\{(1-\mu)^2 + 2\mu(1-\mu)[1 - F(\hat{p}) + \tfrac{1}{2}\mathbb{P}(p = \hat{p})]\right\}(\hat{p} - c)$$

$$\Pi_{sn}(\hat{p}) = \left\{a^2 + 2\mu(1-\mu)[1 - F(\hat{p}) + \tfrac{1}{2}\mathbb{P}(p = \hat{p})]\right\}(\hat{p} - c)$$

Suppose that firm i shifts mass from $\hat{p}$ to $\hat{p} - \varepsilon$, without changing the degree of salient advertising. Expected profit becomes

$$\begin{aligned} \mathbb{E}\Pi(\hat{p}-\varepsilon, \hat{a}^s(\hat{p})) = \hat{a}^s(\hat{p})\beta\Pi_{ss}(\hat{p}-\varepsilon) + (1-\hat{a}^s(\hat{p}))\beta\Pi_{ns}(\hat{p}-\varepsilon) \\ +\hat{a}^s(\hat{p})(1-\beta)\Pi_{sn}(\hat{p}-\varepsilon) + (1-\hat{a}^s(\hat{p}))(1-\beta)\Pi_{nn}(\hat{p}-\varepsilon) - A(\hat{a}^s(\hat{p})) \end{aligned} \tag{2.3}$$

where

$$\Pi_{nn}(\hat{p}-\varepsilon) = \Pi_{ss}(\hat{p}-\varepsilon) = \left\{\tfrac{1}{4} + \tfrac{1}{2}[1 - F(\hat{p}-\varepsilon)]\right\}(\hat{p} - \varepsilon - c)$$

$$\Pi_{ns}(\hat{p}-\varepsilon) = \{(1-\mu)^2 + 2\mu(1-\mu)[1 - F(\hat{p}-\varepsilon)]\}(\hat{p} - \varepsilon - c)$$

$$\Pi_{sn}(\hat{p}-\varepsilon) = \{a^2 + 2\mu(1-\mu)[1 - F(\hat{p}-\varepsilon)]\}(\hat{p} - \varepsilon - c)$$

For ε small enough, profit at $\hat{p}-\varepsilon$ is strictly higher than profit at $\hat{p}$, a contradiction. $\square$

Since Lemma 2.1 refers to the baseline case with no salient advertising, its proof follows by setting $a^s(p) = 0$ for every p in the support.

2.6.2 Proof of Lemma 2.2

Proof. A firm pricing at $\bar{p} > v$ makes no profit. Suppose that the upper bound is $\bar{p} < v$. At $\bar{p}$, firm i will only sell if it is evoked both times, since it will be underpriced for sure. Thus, it can increase its profit by raising its price to v, a contradiction. $\square$

2.6.3 Proof of Proposition 2.1

Proof. Since there is no advertising, expected profit is given by

$$\mathbb{E}\Pi(p, a^s(p)) = [\tfrac{1}{4} + \tfrac{1}{2}(1 - F(p))](p - c) \tag{2.4}$$

Setting profit at v equal to profit at p, solve for the price distribution

$$F(p) = 1 - \frac{1}{2}\left[\frac{v-p}{p-c}\right] \tag{2.5}$$

Setting $F(\underline{p})$ in Expression (2.5) equal to zero, solve for the lower bound of the price distribution

$$\underline{p} = c + \tfrac{1}{3}(v - c) \tag{2.6}$$

Evaluating Expression (2.4) at v, we get that expected profit is equal to $1/4(v-c)$.

$\square$

2.6.4 Proof of Proposition 3.3.3

We will prove each of the three cases separately, by solving for equilibrium. There is price dispersion, as follows from Lemma A and Lemma 2.2.

High Marginal Cost of Advertising

Proof. When the marginal cost of advertising is high, firms choose no advertising in equilibrium for all prices in the support $[\underline{p}, v]$. Thus, expected profit is given by Expression (2.4) in the proof of Proposition 2.1. For a corner solution with no advertising, it must be the case that the marginal benefit of no advertising is less than the marginal cost of no advertising for every price in the support, given that the opponent chooses not to advertise.

$$[\mu^2 - \tfrac{1}{4} - 2(\mu - \tfrac{1}{2})(1 - F(p))](p - c) < A^{'}(0) \tag{2.7}$$

Note that the left hand side of Expression (2.7) is increasing in p. Therefore, a necessary and sufficient condition for Expression (2.7) to hold at every $p \in [\underline{p}, v]$ is

$$A^{'}(0) \geq (a^2 - \tfrac{1}{4})(v - c) \tag{2.8}$$

Setting profit at v equal to profit at p, solve for the price distribution, given by Expression (2.5) in the proof of Proposition 2.1. Setting $F(\underline{p})$ in Expression (2.5) equal to zero, solve for the lower bound of the price distribution, shown in Expression (2.6). As in the baseline case, evaluating Expression (2.4) at v, we get that expected profit is equal to $1/4(v - c)$.

$\square$

Low Marginal Cost of Advertising

Proof. When the marginal cost of engaging in salient advertising is low, in equilibrium, firms choose to engage in perfectly salient advertising. That is, firms choose $a^s(p) = 1$ for all $p \in [\underline{p}, v]$. In this case, expected profit is given by

$$\begin{aligned}\mathbb{E}\Pi(p, a^s(p)) = \{a^s(p) &\left[(\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu) + 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\ &- \left[(\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu) + 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\ &+ \tfrac{1}{4} + \tfrac{1}{2}(1 - F(p))\}(p - c) - A(a^s(p))\end{aligned} \tag{2.9}$$

For this to be an equilibrium, it must be the case that

$$\left[(\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu) + 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right] > A^{'}(1) \tag{2.10}$$

The equilibrium firm price distribution is also given by Expression (2.5), and the lower bound by Expression (2.6). Taking the derivative of Expression (2.9) with respect to a^s, we find that the marginal benefit is increasing in p. Therefore, a sufficient condition for Expression (2.10) to hold at every $p \in [\underline{p}, v]$ is

$$A^{'}(1) \leq \tfrac{1}{3}(\mu^2 - \tfrac{1}{4}) \tag{2.11}$$

In this case, expected profit is strictly lower than in the equilibrium with no salience. This happens because firms incur in an additional expenditure, $A(1)$, which yields an expected profit of $1/4(v-c) - A(1)$ whenever $1/4(v-c) \geq A(1)$.

$\square$

Medium Marginal Cost of Advertising

Proof. When the marginal costs of advertising are in a medium range, the equilibrium level of advertisement $s(p) \in (0,1)$ for every $p \in [\underline{p}, v]$. Expected profit is given by

$$\begin{aligned}
\mathbb{E}\Pi(p, a^s(p)) = \{a^s(p)\beta \left[-2(\mu - \tfrac{1}{2})^2 + 4(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\
+a^s(p) \left[\mu^2 - \tfrac{1}{4} - 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\
-\beta \left[(\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu) + 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\
+\tfrac{1}{4} + \tfrac{1}{2}(1 - F(p))\}(p - c) - A(a^s(p))
\end{aligned} \tag{2.12}$$

Where $\beta = \int_{\underline{p}}^{v} a^s(x) f(x) dx \in (0,1)$ is the other firm's expected investment in salience.

An interior solution requires that marginal benefit be equal to marginal cost for every p

$$\begin{aligned}
\{\beta \left[-2(\mu - \tfrac{1}{2})^2 + 4(\mu - \tfrac{1}{2})^2(1 - F(p))\right] \\
+ \left[\mu^2 - \tfrac{1}{4} - 2(\mu - \tfrac{1}{2})^2(1 - F(p))\right]\}(p - c) = A^{'}(a^s(p))
\end{aligned} \tag{2.13}$$

Evaluating Equation (2.13) at v, we can solve for β

$$\beta = \frac{(\mu^2 - \frac{1}{4}) - \frac{A^{'}(a^s(v))}{v-c}}{2(\mu - \frac{1}{2})^2} \tag{2.14}$$

Necessary and sufficient conditions for $\beta \in (0, 1)$

$$A^{'}(0) \leq A^{'}(a^s(v)) \leq (\mu^2 - \tfrac{1}{4})(v - c) \tag{2.15}$$

$$A^{'}(1) \geq A^{'}(a^s(v)) \geq (\mu - \tfrac{1}{2})(\tfrac{3}{2} - \mu)(v - c) \tag{2.16}$$

Also from Equation (2.13), solve for $F(p)$

$$F(p) = 1 - \frac{1}{2}\left[\frac{\frac{A^{'}(a^s(v))}{v-c} - \frac{A^{'}(a^s(p))}{p-c}}{\frac{A^{'}(a^s(v))}{v-c} - (\mu - \frac{1}{2})}\right] \tag{2.17}$$

Set Expression (2.17) equal to zero to solve for the lower bound $\underline{p}$

$$\underline{p} = c + \frac{A^{'}(a^s(\underline{p}))}{2(\mu - \frac{1}{2}) - \frac{A^{'}(a^s(v))}{v-c}} \tag{2.18}$$

Condition (2.15) guarantees that the denominator in Expression (2.18) is positive, so $\underline{p} > c$.

In order to obtain the density function, take derivative with respect to p in Expression (2.17)

$$f(p) = \frac{1}{2}\left[\frac{\frac{A^{''}(a^s(p))a^{s'}(p)}{p-c} - \frac{A^{'}(a^s(p))}{(p-c)^2}}{\frac{A^{'}(a^s(v))}{v-c} - (\mu - \frac{1}{2})}\right] \tag{2.19}$$

Note that $(\frac{3}{2} - \mu)(\mu - \frac{1}{2}) \leq (\mu - \frac{1}{2}) \leq (\mu + \frac{1}{2})(\mu - \frac{1}{2})$. The equilibrium firm price distribution function in Expression (2.17) must be a number between zero and one. Also, the density function in Expression (2.19) must be nonnegative. Thus, we can identify two cases.

1. $\frac{A'(a^s(v))}{v-c} < (\mu - \frac{1}{2})$ and $A''(a^s(p))a^{s\prime}(p) \leq \frac{A'(a^s(p))}{(p-c)}$: The denominator of Expression (2.17) is negative, so the numerator must also be negative. This does not add any information about the sign of $s'(p)$.

2. $\frac{A'(a^s(v))}{v-c} > (\mu - \frac{1}{2})$ and $A''(a^s(p))a^{s\prime}(p) \geq \frac{A'(a^s(p))}{(p-c)}$: The denominator of Expression (2.17) is positive, so the numerator must also be positive. This implies that $a^s(p)$ must be increasing in p.

Using Expression (2.14) and Expression (2.19)

$$\frac{1}{2\left[\frac{A'(a^s(v))}{v-c} - (\mu - \frac{1}{2})\right]} \int_{\underline{p}}^{v} a^s(x) \left(\frac{A''(a^s(x))a^{s\prime}(x)}{x-c} - \frac{A'(a^s(x))}{(x-c)^2}\right) dx = \frac{\mu^2 - \frac{1}{4} - \frac{A'(a^s(v))}{v-c}}{2(\mu - \frac{1}{2})^2} \tag{2.20}$$

Equation (2.20) solves for $a^s(v)$. Then, set Equation (2.12) evaluated at p equal to Equation (2.12) evaluated at v to solve for $a^s(p)$.

Also, we need to make sure that profit is positive

$$\{-a^s(v)\left[\mu^2 - \frac{1}{4} - \frac{A'(a^s(v))}{v-c}\right] \frac{\mu-\frac{1}{2}}{2(\mu+\frac{1}{2})} + (\mu^2 - \frac{1}{4})a^s(v)\}(v-c) + \frac{1}{4}(v-c) - A(a^s(v)) \geq 0 \tag{2.21}$$

□

2.6.5 Proof of Proposition 2.4

Proof. Whether profit is higher in the high-cost technology case or in the medium-cost technology case depends on whether the following inequality holds.

$$(v-c)\left[-\beta(1-a^s(v))(\tfrac{1}{4}-(1-\mu)^2)+a^s(v)(1-\beta)(\mu^2-\tfrac{1}{4})\right] > A_m(a^s(v)) \quad (2.22)$$

$\square$

The left hand side of the inequality represents the change in expected revenue generated by investing in imperfect salience. Total revenue is equal to the term on the left hand side of the inequality plus $\frac{1}{4}(v-c)$, which is also the equilibrium expected revenue from the high-cost advertising technology. If the left hand side of the inequality is positive, it means that investing in imperfect salience generates an increase in expected revenue. A necessary and sufficient condition for the term inside the brackets to be positive is that $a^s(p)$ be increasing in p. This guarantees that $a^s(v) > \beta$. If the inequality in Expression (2.22) holds, investing in imperfect salience generates an expected benefit that exceeds its total cost. Hence, profit is strictly higher in the medium-cost advertising technology case than in the high-cost technology case.

As explained at the end of the equilibrium analysis, whenever consumers are responsive to salience at the lower bound of the equilibrium price distribution corresponding to the medium-cost advertising technology case, from now on denoted $\underline{p}_m$ ans $a^s(p)$ is increasing in p, p_m is strictly lower than the lower bound of the high-cost firm price distribution, denoted p_c. Additionally, $a^s(p)$ increasing in p guarantees that $F_m(p_c) > 0$. In order to ensure first order stochastic dominance of F_m, it must be the case that for every $p \in [\underline{p}_c, v]$, $F_c(p) < F_m(p)$. Rearranging terms, we get that it must be the case that

$$\frac{A'(a^s(v))}{v-c} - (\mu - \tfrac{1}{2}) < \frac{\frac{A'(a^s(v))}{v-c} - \frac{A'(a^s(p))}{p-c}}{\frac{v-p}{p-c}} \tag{2.23}$$

Note that the left hand side of the inequality is constant. If the right hand side is increasing in p, given that the inequality holds at $\underline{p}_c$, the Expression (2.23) holds for each $p \in [\underline{p}_c, v]$. This is always the case whenever the composite function $M(p) = A'(a^s(p))$ is strictly concave. Otherwise, if the right hand side is non increasing in p, it must be the case that when p approaches v, the inequality still holds. Taking limits on both sides as $p \to v$, we get that Expression (2.23) holds whenever $f(v) < \frac{1}{2(v-c)}$, where $f(v)$ is the density function from Expression (2.19).

2.6.6 Proof of Proposition 2.5

Proof. Comparing expected profit from the low-cost technology case, equal to $1/4(v-c) - A_l(1)$ and expected profit from the medium-cost case, given by Expression (2.21), we get that expected profit is higher in the medium-cost case whenever

$$\left[\frac{\mu-\frac{1}{2}}{2(\mu+\frac{1}{2})} A'_m(a^s(v)) + \tfrac{1}{2}(\mu+\tfrac{3}{2})(\mu-\tfrac{1}{2})(v-c)\right] a^s(v) \geq A_m(a^s(v)) - A_l(1) \tag{2.24}$$

If $A_m(a^s(v)) < A_l(1)$, then it is not necessary for imperfect salience to generate an expected increase in revenue. As long as the expected decrease in revenue is smaller than $A_l(1) - A_m(a^s(v))$, profit is higher in the medium-cost case. On the other hand, if $A_m(a^s(v)) > A_l(1)$, then engaging in imperfect salience should generate an increase in expected revenue sufficient to cover $A_m(a^s(v)) - A_l(1)$. Remember that a sufficient condition for imperfect salience to generate an increase in expected revenue is that $a^s(p)$ be increasing in p.

Given that the equilibrium firm price distribution and its lower bound are identical in the lo-cost technology case and in the high-cost technology case, the analysis for prices is identical to that in the proof of Proposition 2.4.

Chapter 3

Asymmetric Sequential Search

3.1 Introduction

There is a rich literature examining how costly price search by consumers leads to price dispersion in homogeneous goods markets (Burdett and Judd 1983; Stahl 1989; Janssen et al. 2005). Because the consumer search order is usually assumed to be random, all firms expect to be visited by the same types of consumers in equal proportion and all firms have the same price distribution in equilibrium. However, there are many markets where different firms expect to encounter different types of consumers. For instance, while supermarkets serve various types of buyers, higher pricing convenience stores sell many of the same goods primarily to one type: individuals with a high opportunity cost of shopping elsewhere. Similarly, low price outlets may provide consumers with the same products sold by general retailers, but since they are located far from residential areas they mainly cater to consumers with a low opportunity cost of searching.[1] In this paper, we are

[1]For empirical evidence, see Chung and Myers 1999; Hausman and Leibtag 2007; Griffith et al. 2009.

interested in analyzing how such demand asymmetries affect prices and welfare when consumers engage in sequential price search.

In our model, two firms compete by setting prices for a homogeneous good. There are two types of consumers in the market. Fully informed consumers (referred to as shoppers) have no opportunity cost of time and observe both prices at no cost,[2] whereas non-shoppers engage in optimal sequential search. When searching sequentially, a consumer with a price at hand continues searching only as long as the marginal benefit of doing so is higher than the constant marginal cost. We adjust the commonly used assumption that the first price sample is free for all consumers by supposing that non-shoppers can only obtain the first price for free at their local firm. When one of the two firms that comprise the market has a larger local population than the other, a symmetric equilibrium no longer exists. That is, the equilibrium price distributions for the two firms differ.

We characterize the unique equilibrium of this game and provide several comparative statics. In equilibrium, firms randomize over lower prices to attract shoppers, and over higher prices to realize greater profits from local non-shoppers, who end up not searching. The equilibrium price distribution of the firm with the larger local population first order stochastically dominates that of the other firm, and as a result, the firm with the larger local population will have higher prices on average. This is because the firm with the larger local population loses more profit from non-shoppers when it lowers its price. Since both firms have the same equilibrium support, the firm with the larger local population also has an atom

[2]Interpretations of shoppers in the literature are as individuals who read sales ads (Varian 1980), as consumers who derive enjoyment from shopping (Stahl 1989), as a coalition of consumers who freely share their search information (Stahl 1996), and as users of search engines (Janssen and Non 2008).

at the upper bound of the distribution. This is interpreted to mean that it is less likely to run a sale. Moreover, the firm's propensity to run a sale decreases as the share of the total population that is local to it increases. Interestingly, both firms will continue to run sales with positive probability even when all non-shoppers are local to one firm because this firm still has an incentive to try to capture shoppers.

It turns out that the likelihood of a sale does not have to increase in the proportion of shoppers. As this proportion increases, firms tend to run better sales, but the frequency of sales can increase or decrease depending on how local non-shoppers in the two firms are affected. Both firms are clearly motivated to run better sales to attract the new shoppers in the population. However, where these new shoppers are coming from affects the frequency of sales. For instance, suppose that firm 1 has more local non-shoppers than firm 2 and that the growing number of shoppers comes primarily out of the population of non-shoppers local to firm 2. To compensate for the better sales that it runs to attract the new shoppers, firm 1 exploits its local non-shoppers by running sales less often. Nevertheless, in terms of consumer welfare, the effect of better sales always dominates.

In most models of sequential search (Stahl 1989, 1996; Janssen et al. 2005), heterogeneous consumers sample firm prices at random, so changes in the composition of consumers affect all firms in the same way. It is by biasing the sampling behavior of some of these consumers to favor a particular firm that we are able to examine the impact of demand differences on firm pricing. Arbatskaya (2007) examines an ordered search model where consumers with heterogeneous search costs *all* face the same search order. In her pure strategy equilibrium, firms at the bottom of the search order always have lower prices than those at the top. On the other hand, Armstrong et al. (2009) analyze a model where consumers with different tastes all search a prominent firm first, but find that the prominent firm has a lower price than the rest. Reinganum's (1979) model addresses the opposite question: it asks how firm cost heterogeneity affects price dispersion when all consumers are homogeneous. The papers that come closest to addressing our research question are Narasimhan (1988) and Jing and Wen (2008), which consider a market with shoppers and non-shoppers who decide where to buy according to an exogenous rule. In Narasimhan, a proportion of non-shoppers is only allowed to buy from one firm while the rest must purchase from the other firm. In Jing and Wen all non-shoppers purchase from the same firm unless the other firm underprices it by an exogenously determined amount. By endogenizing the purchasing decision, we are able to show how changes in the proportion of shoppers or the level of asymmetry affect non-shoppers' search strategies and firm pricing behavior.

The remainder of this paper is organized as follows. Section 3.2 presents the model. Section 3.3 develops the equilibrium and Section 3.4 explores the comparative statics. Section 3.5 concludes and details directions for future research. Section 3.6 consists of appendices containing formal proofs.

3.2 The model

Two firms, labeled 1 and 2, sell a homogeneous good. Firms have no capacity constraints and an identical constant cost of zero of producing one unit of the good.[3] There is a unit mass of almost identical consumers with inelastic (unit) demand and valuation $v > 0$ for the good. A proportion $\sigma \in (0, 1)$ of consumers have 0 cost of search, and will be referred to as shoppers. The remaining $1 - \sigma$

[3]The price of the good can be viewed as a price cost margin.

consumers, called non-shoppers, pay a positive search cost $c \in (0, v)$ for each firm they visit except for their local firm, which they search first. A fraction $\lambda \in [0, 1-\sigma]$ of all non-shoppers are local to firm 1, while the remaining $1-\sigma-\lambda$ are local to firm 2. Non-shoppers search sequentially. Upon observing the price at their local firm, they must decide whether to search the second one. We assume costless recall—that is, consumers who have observed both prices can freely choose to purchase the good at the lower price.

Firms and consumers play the following game. First, firms 1 and 2 simultaneously choose prices taking into consideration their beliefs about the rival firm's pricing strategy and about consumer search behavior. A pricing strategy consists of a price distribution F_i over $[\underline{p}_i, \bar{p}_i]$, where $F_i(p)$ represents the probability that firm $i = 1, 2$ offers a price no higher than p. After firms choose price distributions, prices are realized. Shoppers observe the price realizations of both firms and choose where to buy the product. Non-shoppers local to firm i only observe firm i's price realization. Given their information and beliefs about firm j's price distribution, non-shoppers choose a search strategy that specifies whether or not to search the non-local firm, whether or not to buy the product, and where to buy it (in case both firm price realizations are observed). Parameters v, c, σ, λ, as well as the rationality of all agents in the model are commonly known.

3.3 Equilibrium analysis

The equilibrium concept that we use is Sequential Equilibrium.[4] In this context, Sequential Equilibrium requires that non-shoppers who observe an off-equilibrium

[4]For an extension of the definition of sequential equilibrium to infinite action games, see Manelli (1996).

price at their local firm treat such deviations as mistakes when forming beliefs about the non-local firm's strategy. Thus, non-shoppers believe that the non-local firm plays its equilibrium strategy at all information sets.

3.3.1 Consumer behavior

The marginal benefit of searching firm j for a non-shopper local to firm i after having observed a price of p_λ at firm i is given by

$$\int_{\underline{p}_j}^{p_\lambda} (p_\lambda - p)\mathrm{d}F_j(p) \tag{3.1}$$

Expression (3.1) denotes the non-shopper's expected surplus from searching firm j. After searching firm j, the non-shopper will purchase at the lower of p_λ and the price observed at firm j. Thus, the marginal benefit of search arises from the opportunity to find a price lower than p_λ at firm j.

After integration by parts, Expression (3.1) becomes

$$\int_{\underline{p}_j}^{p_\lambda} F_j(p)\mathrm{d}p \tag{3.2}$$

A non-shopper will benefit from searching his non-local firm if and only if Expression (3.2) is no less than his cost of search c. That is,

$$\int_{\underline{p}_j}^{p_\lambda} F_j(p)\mathrm{d}p \geq c \tag{3.3}$$

The optimal strategy for a non-shopper local to firm i is to search firm j if and only if the price realization at his local firm is greater than some reservation price, denoted r_j. In subsection 3.3.3 we will show that the equilibrium reservation price is the mapping that gives the value of p_λ that makes Expression (3.3) hold with equality when such a p_λ exists and is less than or equal to v.

3.3.2 Firm pricing

Before characterizing the equilibrium of the game, we will narrow down the possible sets of prices that firms can charge.

Proposition 3.1. *In equilibrium, the firm supports can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_1 = r_2\}$.

2. *Single atom, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* i *has an atom at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$, $r_j \leq r_i$.

3. *Two atoms, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* j *has an atom at* $r_i < \min\{v, r_j\}$; *mutual break over* (r_i, p^u) *for* $p^u \in (r_i, \bar{p})$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$, *firm* i *has an atom at* $\bar{p}$.

4. *Two atoms, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; *firm* j *has an atom at* $\bar{p}_j = r_i < \min\{v, r_j\}$; *firm* i *has a break over* $(r_i, \bar{p}_i)$ *for* $\bar{p}_i = \min\{v, r_j\}$ *and an atom at* $\bar{p}_i$.

The proof of this proposition is contained in the appendix. Notice that in every case firms never price higher than v, so there is always complete consumer participation in the market. The completely symmetric case in Proposition 3.1 occurs if and only if $\lambda = (1 - \sigma)/2$, that is, if both firms have the same fraction of local non-shoppers. As such, it is a special case of support type 2. In support type 2, because firms never price above the smaller of the two reservation prices, non-shoppers never search in equilibrium. Support type 3 is the only one where some non-shoppers may search beyond their local firm, while support type 4 is the only case where the firm supports are not the same. However, in Proposition 3.2 we show that the last two support types cannot occur in equilibrium.

The corollary below is a technical result needed for existence of equilibrium. It follows immediately from the proof of Proposition 3.1. It states that when non-shoppers are indifferent between staying at their local firm and searching, whenever the upper bound of the firm supports is lower than v, they must stay in order for equilibrium to exist.

Corollary 3.1. *For equilibria with support types 2, 3 or 4 in Proposition 3.1 to exist, all non-shoppers must stop searching after observing a price of r_j at local firm i,*[5] *unless $v < r_j$.*

3.3.3 Equilibrium

Let $\rho_j(\sigma, \lambda, c)$ be the mapping that gives the value of p_λ that makes Expression (3.3) hold with equality. From Proposition 3.1, we know that there are no atoms at the lower bound of the support of firm price distributions. Since Expression (3.2) is increasing in p, we know that such a value of p_λ exists and that it is unique.

Going forward, we will consider only the case $\lambda \in ((1-\sigma)/2, 1-\sigma]$, so that firm 1 will always be treated as the one with more local non-shoppers. The following proposition completely describes the unique Sequential Equilibrium of this game for this case. The case $\lambda \in [0, (1-\sigma)/2)$ follows analogously and we leave it to the reader.

Proposition 3.2. *There exists a unique Sequential Equilibrium where both firms*

[5] This means $\gamma = 1$ in the proof of Proposition 3.1.

have supports $\left[\underline{p}, \bar{p}\right]$*, where* $\bar{p} = \min\{v, r_2^*\}$ *and* $\underline{p} = \frac{\lambda}{\lambda+\sigma}\bar{p}$*, and* r_1^* *and* r_2^* *are the equilibrium reservation prices for non-shoppers local to firm 2 and firm 1 respectively.* $r_1^* = \infty$,

$$r_2^* = \begin{cases} \rho_2(\sigma,\lambda,c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma+\lambda}{\lambda}\right)\right]^{-1} & \textit{if } \rho_2(\sigma,\lambda,c) \leq v \\ \infty & \textit{otherwise} \end{cases}$$

Firm 1 distributes prices according to

$$F_1(p) = \begin{cases} \frac{1-\lambda}{\sigma}\left[1 - \underline{p}/p\right] & \underline{p} \leq p < \bar{p} \\ 1 & p = \bar{p} \end{cases}$$

with $\Pr\left[p_1 = \bar{p}\right] = \frac{2\lambda+\sigma-1}{\sigma+\lambda}$*, and firm 2 distributes prices according to* $F_2(p) = \frac{\sigma+\lambda}{\sigma}\left[1 - \underline{p}/p\right]$.

Proof. We prove existence by directly computing and characterizing the equilibrium. A sketch of the proof of uniqueness is included in the appendix, where we rule out support types (3) and (4) in Proposition 3.1.

In equilibrium, a firm must be indifferent between any price in its support. Therefore, for any p_1 in the support of F_2, $\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(p_1, F_2(p_1))$. Given that the unique equilibrium has support type (2),[6]

$$\underline{p}(\sigma+\lambda) = p_1\{\sigma[1 - F_2(p_1)] + \lambda\} \tag{3.4}$$

We can solve for $F_2(p)$ to get

$$F_2(p) = \frac{\sigma+\lambda}{\sigma}\left(1 - \underline{p}/p\right) \tag{3.5}$$

Similarly, setting $\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(p_2, F_1(p_2))$, it is easy to show that

$$F_1(p) = \frac{1-\lambda}{\sigma}\left(1 - \underline{p}/p\right) \tag{3.6}$$

[6]Support type (2) subsumes support type (1) as the special case ($\lambda = (1-\sigma)/2$, $r_1^* = r_2^*$, $\Pr(p_i = \bar{p}) = 0$).

Since $\lambda > \frac{1-\sigma}{2}$, F_1 first order stochastically dominates F_2. Because $\bar{p}_1 = \bar{p}_2 = \bar{p}$, this implies an atom at firm 1's upper bound.

Setting $F_2(\bar{p}) = 1$, we can solve for $\underline{p}$ in terms of $\bar{p}$ and substitute it into $F_2(p)$, which becomes

$$F_2(p) = \frac{\sigma + \lambda}{\sigma}\left(1 - \frac{\lambda}{\sigma + \lambda}\frac{\bar{p}}{p}\right) \tag{3.7}$$

When $\rho_2(\sigma, \lambda, c) \leq v$, $\bar{p} = \rho_2(\sigma, \lambda, c)$. Optimal search requires that Expression (3.3) holds with equality. Substituting in Equation (3.7) yields

$$\int_{\underline{p}}^{\rho_2} \frac{\sigma + \lambda}{\sigma}\left(1 - \frac{\lambda}{\sigma + \lambda}\frac{\rho_2}{p}\right) \mathrm{d}p = c \tag{3.8}$$

Finally, integrating and solving for ρ_2, we get

$$\rho_2(\sigma, \lambda, c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma + \lambda}{\lambda}\right)\right]^{-1} \tag{3.9}$$

If $\rho_2(\sigma, \lambda, c) \leq v$, r_2^* is defined by Equation (3.9). Because firms are not concerned with prices above v, if $\rho_2(\sigma, \lambda, c) > v$, we define r_2^* as positive infinity. Since $F_1(p) < F_2(p)$, $\rho_1(\sigma, \lambda, c) > \min\{v, \rho_2(\sigma, \lambda, c)\}$, and we can define r_1^* as positive infinity. □

In the equilibrium described above, all non-shoppers search their local firm and make a purchase there, whereas all shoppers purchase from the firm with the lower price. Since non-shoppers always buy from their local firm, it is more costly for firm 1 to lower its price than it is for firm 2. Firm 1 takes advantage of its location by running fewer sales and pricing higher on average.

Proposition 3.2 immediately gives rise to the following result.

Corollary 3.2. *For $\lambda \geq \frac{1-\sigma}{2}$, $\rho_2(\sigma, \lambda, c)$ is decreasing in σ and increasing in λ.*

Corollary 3.2 tells us that as long as $r_2^* \leq v$, the bounds of the firm price distributions fall in the proportion of shoppers and rise as the firm with more local non-shoppers gains market power relative to the other firm.

3.4 Comparative statics

In this section we explore how changes in λ and σ affect firm price distributions in more depth. All proofs will be contained in the appendix.

3.4.1 Changes in the proportion of locals at firm 1

Proposition 3.3. *For $\sigma \in (0, 1)$,*

(i) As λ increases over $[\frac{1-\sigma}{2}, 1 - \sigma]$, the probability that firm 1 runs a sale decreases. Even when $\lambda = 1 - \sigma$, price dispersion persists.

(ii) Let $\frac{1-\sigma}{2} < \underline{\lambda} < \bar{\lambda} < 1 - \sigma$. Then for both firms, the price distribution contitional on $\bar{\lambda}$ first order stochastically dominates the price distribution conditional on $\underline{\lambda}$.

As λ increases, the atom at $p_1 = \bar{p}$ increases in size, but equals $1 - \sigma$ when $\lambda = 1 - \sigma$. Hence, there is always price dispersion in equilibrium (there is no pure strategy equilibrium). This results because $\sigma \in (0, 1)$ and both firms run sales to attract shoppers. The second part of Proposition 3.3 says that as firm 1 gains market power relative to firm 2, not only do the bounds on firm price distributions increase (as implied by Corollary 3.2), but also both distributions shift in a first order stochastic dominant sense. This is not completely straightforward for firm 2 because as λ increases there are two countervailing forces acting on its prices. On the one hand, when firm 2 has fewer locals, it has more incentive to lower prices to lure shoppers. On the other, because of the shift in firm 1's distribution, firm 2 no longer needs to lower prices as much to have the same probability of capturing all the shoppers as it did with a lower λ. Proposition 3.3 tells us that the latter effect dominates. The proof is included in the appendix.

3.4.2 Changes in the proportion of shoppers

When λ increases, by definition, firm one obtains a higher proportion of local non-shoppers. In contrast, when σ changes, it is not clear which firm(s) loses or gains local non-shoppers. For example, if σ increases there may be a fall in the number of local non-shoppers in firm 2 without any change in the number of non-shoppers local to firm 1. However, the fact that $\sigma + \lambda \leq 1$ imposes a constraint on how much σ can increase without affecting λ. As a result, when evaluating changes in σ, λ must be treated as a function of σ, $\varphi(\sigma)$ with derivative $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma \in [-1, 0]$. $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = 0$ means that as σ increases, only firm 2 loses local non-shoppers, $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = -1$ means that as σ increases, only firm 1 loses local non shoppers, and otherwise, both firms lose local non-shoppers as σ increases.

Proposition 3.4. *The probability that firm 1 runs a sale and both firm price distributions depend on σ and the size of $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda'$ as follows.*

(i) For λ' sufficiently close to zero, the atom increases in σ, and firm 1's price distribution increases in σ for lower prices, and decreases in σ for higher prices.

(ii) For λ' sufficiently close to -1, the atom decreases in σ, and firm 1's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ.

(iii) Firm 2's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ always.

According to Proposition 3.4, as the proportion of shoppers increases, both firms have incentives to run better sales to attract the new shoppers. That is, both firm price distributions will put more mass on lower prices. However, firm 1's atom at $\bar{p}$ may increase or decrease depending on which firm is losing more local non-shoppers. This can be interpreted respectively as a decrease or an increase in the frequency of sales run by firm 1. Suppose that that the new shoppers come primarily out of the population of non-shoppers local to firm 2. Firm 1 can compensate itself for the better sales that it runs to attract the new shoppers by pricing at $\bar{p}$ with higher probability because it still maintains its fraction of locals relative to the entire population of consumers (λ is constant). If, instead, firm 1's local non-shoppers are the ones becoming shoppers, then firm 1 will run more sales and have higher discounts because λ is decreasing.

3.4.3 Welfare implications

The welfare implications of Proposition 3.3 are straightforward. That is, expected welfare is decreasing in λ because when λ is higher there are fewer sales and those that are run tend to be worse. On the contrary, expected welfare is increasing in σ. When λ' is sufficiently close to -1, this is clear because there are more sales and those that are run tend to be better. However, when λ' is sufficiently close to zero, the welfare implications are not obvious because, while there are fewer sales, those that are run tend to be better. To show that expected welfare is still increasing in σ in this case, we define expected welfare, W, as follows:

$$W = (v - \mathbb{E}_1[p])\lambda + (v - \mathbb{E}_2[p])(1 - \lambda) \tag{3.10}$$

where $\mathbb{E}_i$ denotes the expected price under distribution F_i. Since firm 1 has a higher expected price in equilibrium, only its λ local non-shoppers are expected to purchase from it, while all other consumers are expected to buy from firm 2. It is easy to show that W increases in σ, so that the positive effect from better sales dominates the negative effect from the decline in the number of sales.

3.4.4 Limiting cases

Before concluding, we would like to explore the limiting cases when either all consumers are shoppers or all are non-shoppers. We start by looking at the case when σ approaches 1. From Corollary 3.2, we know that for sufficiently high σ, $\rho_2(\sigma, \lambda, c)$ is strictly lower than v. In addition, for any value of λ, when σ becomes large enough, $\lambda = 1 - \sigma$. Thus, the equilibrium reservation price r_2^* becomes the following expression:

$$c\left[1 + \frac{1-\sigma}{\sigma}\ln(1-\sigma)\right]^{-1} \tag{3.11}$$

As $\sigma \to 1$, Expression (3.11) approaches c and $\underline{p} \to 0$. As a consequence, Proposition 3.4 implies that as $\sigma \to 1$, the equilibrium firm price distributions collapse to a degenerate distribution at zero. Therefore, in the limit, the effects of location disappear and we are back to Stahl's (1989) result that when all consumers are shoppers, the unique Nash equilibrium is the competitive price.

The case when σ approaches 0 is not as clear cut as the previous one. The results differ depending on the value of λ. For a given value of $\lambda \in [\frac{1-\sigma}{2}, 1-\sigma)$, as $\sigma \to 0$, $\rho_2(\sigma, \lambda, c) \to \infty$, so that for sufficiently low σ, $\bar{p} = v$. Moreover, as $\sigma \to 0$, $\underline{p} \to \bar{p}$. Therefore, in the limit, when all consumers are non-shoppers, the entire distribution collapses to v. However, when $\sigma = 0$ and $\lambda = 1$, then to the contrary, the monopolistic outcome may not occur. The results of this subsection are summarized in Proposition 3.5.

Proposition 3.5.

(i) *When $\sigma = 1$, the unique equilibrium is $p_1 = p_2 = 0$.*

(ii) *When $\sigma = 0$ and $\lambda \in [0.5, 1)$, the unique equilibrium is $p_1 = p_2 = v$, $r_1^* = r_2^* = \infty$, and consumers purchase from their local firm.*

(iii) *When $\sigma = 0$ and $\lambda = 1$, the set of pure strategy equilibria[7] can be characterized as follows: $p_1 \in [0, v]$. If $p_1 \in [0, v)$, $p_2 = p_1 - c$, $r_2^* = p_1 = p_2 + c < v$. If $p_1 = v$, $p_2 \in [v - c, \infty)$, and $r_2^* = \min\{v, \infty\}$.*

Part (ii) of Proposition 3.5 states that Stahl's (1989) result persists even after accounting for location asymmetries. That is, when all consumers are non-shoppers and both firms have locals, the monopolistic outcome is the unique equilibrium of the game.

However, when $\lambda = 1$, there are multiple pure strategy equilibria where firm 2 underprices firm 1 by c. In such equilibria, since non-shoppers do not search, firm 1 would like to charge v. However, since $\lambda = 1$, firm 2 makes zero profit, so it can charge any price. By charging prices lower than $v - c$, firm 2 increases the marginal benefit of searching, causing r_2^* to drop below v. In order to keep its local non-shoppers from searching firm 2, firm 1 has to lower its price below v. This is contrary to what happens in Proposition 3.3 where $\sigma \in (0, 1)$ and a higher λ is associated with increasing prices.

[7]In this case there is no completely mixed strategy equilibrium. Firm 1 will always play a pure strategy in equilibrium. Even though firm 2 can play a mixed strategy in equilibrium, this does not matter to consumers because they never observe its price.

3.5 Conclusion

In this paper, we have studied the consequences of introducing a location asymmetry into a duopoly version of Stahl's (1989) seminal model of sequential consumer search. Contrary to Stahl, where all consumers can sample the first price for free at any firm, non-shoppers in our model can only obtain the first price quote for free at their local firm. When firms serve different proportions of local non-shoppers, Stahl's symmetric equilibrium can no longer exist. In this case, the price distribution of the firm with more locals (and hence, greater market power) first order stochastically dominates that of the other firm and the firm with more locals no longer runs sales all the time (as in symmetric models).

We have analyzed the following comparative statics results in this equilibrium. First, as the market power of the firm with more locals grows, it runs fewer sales and tends to offer smaller discounts in the sales it does run. Second, as the proportion of shoppers in the economy rises, this firm offers greater discounts on the sales it runs. However, it may run more or fewer sales depending on which firm is losing local non-shoppers.

A natural direction for future research is to assume that non-shoppers have a cost of recalling the first price after having searched the second firm. This has not been modeled in an asymmetric framework and we are particularly interested to learn how this feature will influence non-shopper search behavior. Another possible extension is the N firm analogue of this paper. With more than two firms reservation prices are no longer stationary and all consumers must determine an optimal sampling order for firms beyond their local one.

3.6 Appendices

3.6.1 Appendix 1: Proof of Proposition 3.1

Proposition 3.1. *In equilibrium, the firm supports can only take one of the four following forms:*

1. *Completely symmetric, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$; $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_1 = r_2\}$.

2. *Single atom, no breaks:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* i *has an atom at* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\},\ r_j \leq r_i$.

3. *Two atoms, mutual break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has an atom at* $r_i < \min\{v, r_j\}$*; mutual break over* (r_i, p^u) *for* $p^u \in (r_i, \bar{p})$*;* $\bar{p}_1 = \bar{p}_2 = \bar{p} = \min\{v, r_j\}$*, firm* i *has an atom at* $\bar{p}$.

4. *Two atoms, single break:* $\underline{p}_1 = \underline{p}_2 = \underline{p}$*; firm* j *has an atom at* $\bar{p}_j = r_i < \min\{v, r_j\}$*; firm* i *has a break over* $(r_i, \bar{p}_i)$ *for* $\bar{p}_i = \min\{v, r_j\}$ *and an atom at* $\bar{p}_i$.

The following claims complete the proof of Proposition 3.1.

Claim 3.1. $v \geq \max\{\bar{p}_1, \bar{p}_2\} \geq \underline{p}_1 = \underline{p}_2 = \underline{p} \geq 0.$

Proof. Let γ be the proportion of non-shoppers who do not search after observing a price of r_j at their local firm i. Suppose $\underline{p}_1 < \underline{p}_2 \leq v$. Firm 1's profit on $[\underline{p}_1, \underline{p}_2)$ equals

$$p_1\{\sigma + \lambda + (1 - \sigma - \lambda)[1 - F_2(r_1) + (1 - \gamma)\Pr(p_2 = r_1)]\} \tag{3.12}$$

which is increasing in p_1, a contradiction. Now suppose that $\underline{p}_1 \leq v < \underline{p}_2$. Then for $p_1 \in [\underline{p}_1, v)$, firm 1's profit is given by Equation (3.12), which is increasing in p_1, so it must be the case that $\underline{p}_1 = v$. If $\underline{p}_1 = v < \underline{p}_2$, then firm 2 makes zero profit on its support, and would increase profits by shifting mass to v. If $v < \underline{p}_1 \leq \underline{p}_2$, then both firms make zero profits and either can increase profit by shifting mass to v, so $\underline{p}_2 \leq \underline{p}_1$. By a similar argument, $\underline{p}_2 \geq \underline{p}_1$ and $v \geq \underline{p}_1 = \underline{p}_2 = \underline{p}$.

Firms will not charge prices below zero because these yield negative profit.[8] Similarly, at prices above v firms make zero profit. As a result, all consumers in the market make a purchase. □

Definition 3.1. *We say that firms have a mutual atom when each firm has an atom at the same price. We say that firms have a mutual break when each firm's equilibrium support has a break over the same price interval.*

Claim 3.2. *There are no mutual atoms.*

Proof. Let α^S be the proportion of shoppers who buy from firm 1 after having observed the same price in both firms. Let α^N be the proportion of non-shoppers who buy from their local firm after having observed the same price in both firms. Suppose that both firms have a mutual atom at p. When $p_1 = p_2 = p$, firm 1's profit is given by

$$\begin{aligned} p\{\alpha^S\sigma + \lambda[\mathbb{I}_{p<r_2} + [\gamma + \alpha^N(1-\gamma)]\mathbb{I}_{p=r_2} + \alpha^N\mathbb{I}_{p>r_2}] \\ + (1-\sigma-\lambda)(1-\alpha^N)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \end{aligned} \tag{3.13}$$

where $\mathbb{I}$ is an indicator function. Suppose that firm 1 sets $p_1 = p - \varepsilon$ instead of p. Then profits become

$$(p-\varepsilon)\{\sigma + \lambda + (1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1} + \mathbb{I}_{p>r_1}]\} \tag{3.14}$$

[8] If $\underline{p} = 0$, then there must be zero density at $\underline{p} = 0$.

Expression (3.14) is larger than Expression (3.13) when

$$\varepsilon < \frac{p\{\sigma(1-\alpha^S)+\lambda(1-\alpha^N)[(1-\gamma)\mathbb{I}_{p=r_2}+\mathbb{I}_{p>r_2}]+(1-\sigma-\lambda)\alpha^N[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]\}}{\sigma+\lambda+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]}$$

Suppose firm 2 chooses a price other than p. Lowering the price charged never reduces the number of sales, so the loss to firm 1 from lowering the price by ε is at most ε. However, when p is charged with positive probability, lowering the price by ε will with positive probability lead to a gain and with complementary probability, at worst lead to a loss of ε. Therefore, by shifting its atom at p to $p-\varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. By a similar argument, firm 2 will want to undercut a mutual atom for $\lambda \neq 1-\sigma$.

For the case $1-\alpha^S = \alpha^N = \lambda = 0$, firm 1 does not have a profitable deviation, but firm 2 does. □

Claim 3.3. *The only possible breaks in the equilibrium supports are:*

(i) if $\bar{p}_i < \bar{p}_j$ there is a break at $(\bar{p}_i, \bar{p}_j) \in S_j$

(ii) if $r = r_i = r_j < \bar{p}_i = \bar{p}_j$ there may be a mutual break with lower bound r, and

(iii) if $r_i \neq r_j$ and firm i has an atom at r_j, there may be a mutual break with lower bound r_j.

Proof. Let S_1 and S_2 denote the equilibrium supports for firms 1 and 2 respectively. Let $\hat{p} = \inf(S_1 \cap S_2)$ and $\hat{\hat{p}} = \sup(S_1 \cap S_2)$. Define $H = (p^d, p^u) \in \mathrm{int}(S_1 \cap S_2)$.

Suppose first, without loss of generality, that in equilibrium, firm 2 has no support over H, but that firm 1 does. Firm 1's expected profit at $p_1 \in H$ is

$$
\begin{aligned}
&p_1\{\ \sigma[1-F_2(p_1)] \\
&\quad + \lambda\{\mathbb{I}_{p_1<r_2} + [\gamma + (1-\gamma)[1-F_2(p_1)]]\mathbb{I}_{p_1=r_2} + [1-F_2(p_1)]\mathbb{I}_{p_1>r_2}\} \\
&\quad + (1-\sigma-\lambda)\{[1-F_2(r_1) + (1-\gamma)\Pr(p_2=r_1)]\mathbb{I}_{p_1<r_1} \\
&\quad + [1-F_2(r_1)]\mathbb{I}_{p_1=r_1} + [1-F_2(p_1)]\mathbb{I}_{p_1>r_1}\}\ \}
\end{aligned}
\tag{3.15}
$$

As firm 1 raises p_1 along H, its expected profit is increasing since $F_2(p_1)$ is constant along H (and equal to $F_2(r_1)$ if $r_1 \in H$). If $r_2 \notin H$, then firm 1 could increase profits by shifting all mass in H slightly below p^u (or to p^u if firm 2 doesn't have an atom there), a contradiction. If $r_2 \in H$, then firm 1 can increase profits by shifting all mass in (p^d, r_2) to slightly below r_2, and all mass in (r_2, p^u) either to slightly below r_2 or to p^u, again contradicting the equilibrium. A similar argument applies when firm 1 has no support over H, but firm 2 does. This tells us that any breaks in $S_1 \cap S_2$ are mutual.

Now suppose that neither firm randomizes over H in equilibrium. Suppose first that $p^d \neq r_1$, $p^d \neq r_2$ and that neither firm has an atom at p^d. Then either firm 1 has a strictly higher expected profit at p^u (or slightly below r_2 if $r_2 \in H$) than at p^d, or firm 2 has a strictly higher expected profit at p^u (or slightly below r_1 if $r_1 \in H$) than at p^d, or both, if neither firm has an atom at p^u, contradicting the equilibrium.

Suppose that firm i has an atom at $p^d \neq r_j$. Since there are no mutual atoms, firm i could increase profits by shifting its atom to p^u (or slightly below p^u if firm j has an atom there, or slightly below r_j if $r_j \in H$).

If $p^d = r_j \neq r_i$ and firm i has no atom at p^d, firm j's expected profit will be strictly higher at p^u (or slightly below p^u if firm i has an atom there, or to slightly below r_i if $r_i \in H$) than at p^d. But if firm i does have an atom at p^d, then it is possible that profits are the same at p^d and p^u for each firm. If $\gamma \neq 1$, firm i can profitably deviate by shifting its atom slightly below p^d. In doing so, it retains $1-\gamma$ non-shoppers who search after observing a price r_j and have a positive probability of purchasing from firm j. However, if $\gamma = 1$, neither firm has a profitable deviation. This may also be the case if, $p^d = r_2 = r_1$.

By Claim 3.1, we know that both S_1 and S_2 have the same lower bound $\underline{p}$, so $S_1 \Delta S_2 \in (\min\{\bar{p}_1, \bar{p}_2\}, \max\{\bar{p}_1, \bar{p}_2\}]$. Suppose, without loss of generality, that $\bar{p}_1 > \bar{p}_2$. At $p_1 \in (\bar{p}_2, \bar{p}_1]$, firm 1's expected profit is $p_1\lambda(\mathbb{I}_{p_1<r_2} + \gamma\mathbb{I}_{p_1=r_2})$. If $\bar{p}_1 > r_2$, then firm 1 can increase profits by shifting mass in $(r_2, \bar{p}_1]$ to r_2 or slightly below it if $\gamma = 0$. If $r_2 \geq \bar{p}_1$, then profits are strictly increasing in $p_1 \in (\bar{p}_2, \bar{p}_1)$, so firm 1 could increase profits by shifting mass in $(\bar{p}_2, \bar{p}_1]$ to $\min\{r_2, v\} - \varepsilon$ for $\varepsilon > 0$ sufficiently small. As a result, $S_1 \Delta S_2 = \{\min\{v, r_2\}\}$. If firm 2 has no atom at $\bar{p}_2$, firm 1's expected profit at $\bar{p}_1$ is $\bar{p}_1\lambda$, strictly higher than its expected profit of $\bar{p}_2\lambda$ at $\bar{p}_2$, a contradiction. If firm 2 has an atom at $\bar{p}_2 \neq r_1$, since there are no mutual atoms, firm 2 can profitably shift the atom to slightly below $\bar{p}_1$ (or slightly below r_1 if $r_1 \in (\bar{p}_2, \bar{p}_1]$). However, if $\gamma = 1$ and firm 2 has an atom at $\bar{p}_2 = r_1$, $\mathbb{E}\Pi_1(\bar{p}_1, F_2(\bar{p}_1)) = \mathbb{E}\Pi_1(\bar{p}_2, F_2(\bar{p}_2))$, and $F_1(r_1)$ is large enough, then neither firm has a profitable deviation. A similar argument applies when $\bar{p}_2 > \bar{p}_1$. $\square$

Corollary 3.3. *The equilibrium supports are the same except if* $\bar{p}_i = r_j < \bar{p}_j = \min\{r_i, v\}$.

Claim 3.4. *Firm i does not have an atom in the lower bound or the interior of firm j's equilibrium support, except possibly at r_j.*

Proof. Suppose without loss of generality that firm 2 has an atom at $p \in S_1 \backslash \{\bar{p}_1\}$, and suppose that $p \neq r_1$. Firm 1's expected profit at $p - \varepsilon$ when firm 2 charges p is given by Expression (3.14), whereas its expected profit at $p + \varepsilon$ is

$$(p + \varepsilon)\lambda(\mathbb{I}_{p+\varepsilon<r_2} + \gamma\mathbb{I}_{p+\varepsilon=r_2}) \tag{3.16}$$

Expression (3.16) is smaller than Expression (3.14) for

$$\varepsilon \ < \ \frac{p \ \{\sigma+\lambda[(1-\gamma)\mathbb{I}_{p+\varepsilon=r_2}+\mathbb{I}_{p+\varepsilon>r_2}]+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]\}}{\sigma+\lambda[1+\mathbb{I}_{p+\varepsilon<r_2}+\gamma\mathbb{I}_{p+\varepsilon=r_2}]+(1-\sigma-\lambda)[(1-\gamma)\mathbb{I}_{p=r_1}+\mathbb{I}_{p>r_1}]}.$$

Firm 1 can increase expected profits by shifting mass from $(p_2 - \varepsilon, p_2 + \varepsilon]$ to $p_2 - \varepsilon$ for the following reason. For any price that firm 2 charges, shifting mass to $p - \varepsilon$ never reduces the number of sales for firm 1, so it loses at most 2ε. However, when p is charged with positive probability, lowering the price by 2ε or less will, with positive probability, lead to a gain, and with complementary probability, at worst, lead to a loss of 2ε. Therefore, by shifting its mass between p and $p + \varepsilon$ to $p - \varepsilon$ for sufficiently small ε, firm 1 increases its expected profit, a contradiction. □

Claim 3.5. *If* $\bar{p}_1 = \bar{p}_2 = \bar{p}$ *then either*

(i) $\bar{p} = \min\{v, r_1, r_2\}$*, the supports have no breaks, and at most one firm can have an atom at* $\bar{p}$*, or*

(ii) $\bar{p} = \min\{v, \max\{r_1, r_2\}\}$*, there is a mutual break above* $\min\{r_1, r_2\} < \bar{p}$*, firm* i *has an atom at* r_j*, and firm* j *has an atom at* $\bar{p}$*.*

Proof. Suppose that $\bar{p}_1 = \bar{p}_2 = \bar{p}$ and neither firm has an atom at $\bar{p}$. From Claims 3.1 and 3.4 we know that $\underline{p} < \bar{p} \leq v$. Suppose $\bar{p} < \min\{v, r_2\}$. At $\bar{p}$, firm 1 expects profit of $\bar{p}\lambda$. For $\lambda \neq 0$, by raising its price to $\min\{v, r_2\} - \varepsilon$, firm 1 expects to gain $[\min\{v, r_2\} - \varepsilon - \bar{p}]\,\lambda > 0$ for sufficiently small $\varepsilon > 0$, a contradiction. Suppose instead $\bar{p} > \min\{v, r_2\}$. But then at $\bar{p}$ firm 1 expects no profit, a contradiction, so $\bar{p} = \min\{v, r_2\}$. If $\lambda = 0$, firm 1 expects no profit at $\bar{p}$ unless either $\bar{p}_1 < \bar{p}_2$ or firm 2 has an atom at $\bar{p}$, a contradiction. By a similar argument, $\bar{p} = \min\{v, r_1\}$, so $\bar{p} = \min\{v, r_1, r_2\}$.

From Claim 3.2, we know that at most one firm can have an atom at $\bar{p}$, say firm j. If $\gamma = 1$ or $v < r_i$, then following the argument in the paragraph above, $\bar{p} = \min\{v, r_i\}$. Otherwise, firm j cannot have an atom at $\bar{p}$ (using similar reasoning to that in the proof of Claim 3.3). Moreover, if $r_j \geq r_i$, then $\bar{p} = \min\{v, r_1, r_2\}$ and from Claim 3.3, we know that the firm supports have no breaks. Conversely, suppose $r_j < r_i$ (and therefore $r_j < v$). Without loss of generality, let $i = 1$. From Claim 3.4 we know that firm 2 cannot have an atom at r_2. At r_2, firm 1 expects profit of

$$r_2\{[\sigma + \lambda(1-\gamma)][1 - F_2(r_2)] + \lambda\gamma\} \tag{3.17}$$

At $p_1 \in (r_2, \bar{p})$, firm 1 expects profit of

$$p_1(\sigma + \lambda)[1 - F_2(p_1)] \tag{3.18}$$

But since $p_1 \in (r_2, \bar{p})$, by definition, $0 < F_2(r_2) \leq F_2(p_1)$, so for a small enough p_1, Expression (3.17) will be strictly greater than Expression (3.18) as long as $\gamma > 0$. Therefore, r_2 must be the lower bound for a break in S_1, so we must be in case (iii) of Claim 3.3. The second to last paragraph in the proof of Claim 3.3 implies that this equilibrium only exists for $\gamma = 1$. □

Notice that Claim 3.5 rules out case (ii) in Claim 3.3.

3.6.2 Appendix B: Proof of Proposition 3.2 (uniqueness)

Proposition 3.2. *There exists a unique Sequential Equilibrium where both firms have supports* $\left[\underline{p}, \bar{p}\right]$, *where* $\bar{p} = \min\{v, r_2^*\}$ *and* $\underline{p} = \frac{\lambda}{\lambda+\sigma}\bar{p}$, *and* r_1^* *and* r_2^* *are the equilibrium reservation prices for non-shoppers local to firm 2 and firm 1 respectively.* $r_1^* = \infty$,

$$r_2^* = \begin{cases} \rho_2(\sigma,\lambda,c) = c\left[1 - \frac{\lambda}{\sigma}\ln\left(\frac{\sigma+\lambda}{\lambda}\right)\right]^{-1} & if\ \rho_2(\sigma,\lambda,c) \leq v \\ \infty & otherwise \end{cases}$$

Firm 1 distributes prices according to

$$F_1(p) = \begin{cases} \frac{1-\lambda}{\sigma}\left[1 - \underline{p}/p\right] & \underline{p} \leq p < \bar{p} \\ 1 & p = \bar{p} \end{cases}$$

with $\Pr[p_1 = \bar{p}] = \frac{2\lambda+\sigma-1}{\sigma+\lambda}$*, and firm 2 distributes prices according to* $F_2(p) = \frac{\sigma+\lambda}{\sigma}\left[1 - \underline{p}/p\right]$*.*

Proof. From the proof of Proposition 3.2, we know that F_1 and F_2 are the only possible equilibrium distributions when the equilibrium support set is represented by type (1) or (2) outlined in the body of the paper. It remains to show that there do not exist any probability distribution functions F_1 and F_2 that satisfy the conditions necessary for equilibrium with supports of type (3) or (4). We will sketch the proof for support type (4) when $\lambda > (1-\sigma)/2$. For a complete proof, contact the authors.

$F_1(p)$ and $F_2(p)$ are represented by Equation (3.6) and Equation (3.5) respectively over $\left[\underline{p}, \bar{p}_1 = r_2^*\right)$. Note that, $\lambda > (1-\sigma)/2 \Leftrightarrow F_1(p) < F_2(p)$, so $\bar{p}_1 = r_2^* < \bar{p}_2 = \min\{v, r_1^*\}$. A complete solution to this equilibrium requires the following set of equations to hold.

$$\begin{aligned} &\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(p_1, F_2(p_1)) \\ &\Leftrightarrow (\sigma+\lambda)\underline{p} = \{\sigma[1 - F_2(p_1)] + \lambda\}p_1 \end{aligned} \tag{3.19}$$

$$\begin{aligned} &\mathbb{E}\Pi_1(\underline{p}) = \mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) \\ &\Leftrightarrow (\sigma+\lambda)\underline{p} = [\sigma\Pr(p_2 = \bar{p}_2) + \lambda]r_2^* \end{aligned} \tag{3.20}$$

$$\begin{aligned}
&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(p_2, F_1(p_2)) \\
&\Leftrightarrow (1-\lambda)\underline{p} = \{\sigma[1-F_1(p_2)] + (1-\sigma-\lambda)\}p_2
\end{aligned} \tag{3.21}$$

$$\begin{aligned}
&\mathbb{E}\Pi_2(\underline{p}) = \mathbb{E}\Pi_2(\bar{p}_2) \\
&\Leftrightarrow (1-\lambda)\underline{p} = (1-\sigma-\lambda)\bar{p}_2
\end{aligned} \tag{3.22}$$

$$\int_{\underline{p}}^{\rho_2} F_1(p)\mathrm{d}p + (\rho_1 - \rho_2) = c \tag{3.23}$$

$$\int_{\underline{p}}^{\rho_2} F_2(p)\mathrm{d}p = c \tag{3.24}$$

$$\begin{aligned}
&\mathbb{E}\Pi_1(r_2^*, F_2(r_2^*)) > \mathbb{E}\Pi_1(\bar{p}_2 - \varepsilon, F_2(\bar{p}_2 - \varepsilon))\ \forall \varepsilon \in (0, \bar{p}_2 - r_2^*) \\
&\Leftrightarrow [\sigma \Pr(p_2 = \bar{p}_2) + \lambda]r_2^* \geq \bar{p}_2 \Pr(p_2 = \bar{p}_2)(\sigma + \lambda)
\end{aligned} \tag{3.25}$$

$$\Pr(p_1 = \bar{p}_1^*) = 1 - \lim_{\varepsilon \to 0^-} F_1(\bar{p}_1 - \varepsilon) \in (0,1) \tag{3.26}$$

$$\Pr(p_2 = \bar{p}_2) = 1 - \lim_{\varepsilon \to 0^-} F_2(\bar{p}_2 - \varepsilon) \in (0,1) \tag{3.27}$$

We use the following procedure to attempt to find an equilibrium. First, we use Equation (3.19) and Equation (3.21) to solve for $F_2(p)$ and $F_1(p)$ respectively, in terms of $\underline{p}$. Plugging $F_2(p)$ into Equation (3.24) and using Equation (3.20) to solve for $\underline{p}$ we obtain ρ_2 in terms of $\Pr(p_2 = \bar{p}_2)$. Plugging $F_1(p)$ into Equation (3.26) yields $\Pr(p_1 = \bar{p}_1)$ in terms of $\Pr(p_2 = \bar{p}_2)$. Rewriting $F_1(p)$ in terms of $\Pr(p_2 = \bar{p}_2)$ and plugging into Equation (3.23) yields ρ_1 in terms of $\Pr(p_2 = \bar{p}_2)$. Finally, using Equation (3.22) to solve for $\underline{p}$ and setting this equal to the solution obtained from Equation (3.20) we can rewrite r_1 as an alternate function of $\Pr(p_2 = \bar{p}_2)$. Setting the two expressions for ρ_1 equal to each other, we can now solve for $\Pr(p_2 = \bar{p}_2)$ in terms of the exogenous parameters. We can then use this to see if Inequality (3.25) holds. The solution to $\Pr(p_2 = \bar{p}_2)$ is an implicit function. A numerical analysis shows us that there is no solution in the interval $[0,1]$ (except when $\lambda = (1-\sigma)/2$ and $\Pr(p_2 = \bar{p}_2)$=$\Pr(p_1 = \bar{p}_1) = 0$). $\square$

Corollary 3.2. *For $\lambda \geq \frac{1-\sigma}{2}$, $\rho_2(\sigma, \lambda, c)$ is decreasing in σ and increasing in λ.*

Proof.

$$\frac{\partial \rho_2}{\partial \lambda} = -\frac{c(\ln \frac{\lambda}{\sigma+\lambda} + \frac{\sigma}{\sigma+\lambda})}{\sigma(1 + \frac{\lambda}{\sigma} \ln \frac{\lambda}{\sigma+\lambda})} > 0$$

Similarly,

$$\frac{\partial \rho_2}{\partial \sigma} = -\frac{c[\lambda^{'} - \frac{\lambda(1+\lambda^{'})}{\sigma+\lambda} - (\lambda^{'} - \frac{\lambda}{\sigma}) \ln \frac{\sigma+\lambda}{\lambda}]}{\sigma(1 - \frac{\lambda}{\sigma} \ln \frac{\sigma+\lambda}{\lambda})^2} < 0$$

Where $\lambda^{'}$ is defined as in Proposition 3.4. $\square$

3.6.3 Appendix C: Proofs of Propositions 3.3, 3.4 and 3.5

Proposition 3.3. *For $\sigma \in (0,1)$,*

(i) As λ increases over $[\frac{1-\sigma}{2}, 1-\sigma]$, the probability that firm 1 runs a sale decreases. Even when $\lambda = 1-\sigma$, price dispersion persists.

(ii) Let $\frac{1-\sigma}{2} < \underline{\lambda} < \bar{\lambda} < 1-\sigma$. Then for both firms, the price distribution contitional on $\bar{\lambda}$ first order stochastically dominates the price distribution conditional on $\underline{\lambda}$.

Proof.

Part (i):

$$\frac{\partial \Pr(p_1 = \bar{p})}{\partial \lambda} = \frac{\partial \left(\frac{2\lambda+\sigma-1}{\sigma+\lambda}\right)}{\partial \lambda} = \frac{1+\sigma}{(\sigma+\lambda)^2} > 0$$

Moreover, if $\lambda = 1-\sigma$, then $Pr(p_1 = \bar{p}) = 1-\sigma > 0$.

Part (ii):

For F_1: if $v < \rho_2$,

$$\frac{\partial F_1(p)}{\partial \lambda} = -\frac{1}{\sigma}\left\{\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{v}{p}\right] + \frac{(1-\lambda)\sigma}{(\sigma+\lambda)^2}\frac{v}{p}\right\}$$

If $v \geq \rho_2$,

$$\frac{\partial F_1(p)}{\partial \lambda} = -\frac{1}{\sigma}\left\{\left[1 - \frac{\lambda}{(\sigma+\lambda)}\frac{\rho_2}{p}\right] + \frac{1-\lambda}{\sigma+\lambda}\left[\frac{\sigma}{(\sigma+\lambda)}\frac{\rho_2}{p} + \frac{\lambda}{p}\frac{\partial \rho_2}{\partial \lambda}\right]\right\}$$

The second term inside the curly brackets in each of the two equations above is clearly positive. Given that $\bar{p}/\underline{p} = (\sigma+\lambda)/\lambda$, the first term is minimized at $\underline{p}$ and equal to zero there.

For F_2: if $v < \rho_2$,

$$\frac{\partial F_2(p)}{\partial \lambda} = -\frac{1}{\sigma}\left(1 - \frac{v}{p}\right) \leq 0$$

If $v \geq \rho_2$,

$$\frac{\partial F_2(p)}{\partial \lambda} = -\frac{1}{\sigma}\left(1 - \frac{\rho_2}{p} - \frac{\lambda}{p}\frac{\partial \rho_2}{\partial \lambda}\right) \leq 0 \qquad \square$$

Proposition 3.4. *The probability that firm 1 runs a sale and both firm price distributions depend on σ and the size of $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda'$ as follows.*

(i) For λ' sufficiently close to zero, the atom increases in σ, and firm 1's price distribution increases in σ for lower prices, and decreases in σ for higher prices.

(ii) For λ' sufficiently close to -1, the atom decreases in σ, and firm 1's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ.

(iii) Firm 2's price distribution for a lower value of σ first order stochastically dominates that for a higher value of σ always.

Proof. Let $\mathrm{d}\varphi(\sigma)/\mathrm{d}\sigma = \lambda^{'}$. rCl $\frac{\mathrm{d}\,\Pr(p_1 = \bar{p})}{\mathrm{d}\sigma = \frac{\mathrm{d}\left(\frac{2\lambda+\sigma-1}{\sigma+\lambda}\right)}{\mathrm{d}\sigma} = \frac{\lambda^{'}(1-\sigma)+(1-\lambda)}{(\sigma+\lambda)^2}}$ Since this is a strictly increasing function of $\lambda^{'}$ and is positive when $\lambda^{'} = 0$ but negative when $\lambda^{'} = -1$, $\mathrm{d}\,\Pr(p_1 = \bar{p})/\mathrm{d}\sigma$ is non-monotonic in σ.

For F_1: if $v < \rho_2$, rCl $\frac{\partial F_1(p)}{\partial\sigma = -\frac{1}{\sigma}\left(\lambda^{'}+\frac{1-\lambda}{\sigma}\right)\left[1-\frac{\lambda}{(\sigma+\lambda)}\frac{v}{p}\right] - \frac{1-\lambda}{\sigma(\sigma+\lambda)}\frac{v}{p}\left[\lambda^{'}-\frac{\lambda(1+\lambda^{'})}{\sigma+\lambda}\right]}$, If $v \geq \rho_2$, rCl $\frac{\partial F_1(p)}{\partial\sigma = -\frac{1}{\sigma}\left(\lambda^{'}+\frac{1-\lambda}{\sigma}\right)\left[1-\frac{\lambda}{(\sigma+\lambda)}\frac{\rho_2}{p}\right] - \frac{(1-\lambda)}{\sigma(\sigma+\lambda)}\left\{\left[\lambda^{'}-\frac{\lambda(1+\lambda^{'})}{\sigma+\lambda}\right]\frac{\rho_2}{p}+\frac{\lambda}{p}\frac{\partial\rho_2}{\partial\sigma}\right\}}$ When $\lambda^{'} = -1$, both of these expressions are positive. Likewise, when $\lambda^{'} = 0$, for p sufficiently low. However, when p is sufficiently high, both of these expressions become negative.

For F_2: if $v < \rho_2$,

$$\frac{\partial F_2(p)}{\partial\sigma} = \frac{\lambda^{'}\sigma - \lambda}{\sigma}\left[\frac{1}{\sigma}\left(1 - \frac{v}{p}\frac{\lambda}{\sigma+\lambda}\right) - \frac{1}{p}\frac{1}{\sigma+\lambda}\right] \; > \; 0$$

If $v \geq \rho_2$,

$$\frac{\partial F_2(p)}{\partial\sigma} = \frac{1}{\sigma}\left[\left(\lambda^{'} - \frac{\lambda}{\sigma}\right)\left(1 - \frac{\rho_2}{p}\frac{b}{\sigma+\lambda}\right) - \frac{1}{p}\left(\frac{\lambda^{'}\sigma - \lambda}{\sigma+\lambda} + \lambda\frac{\partial\rho_2}{\partial\sigma}\right)\right] \; > \; 0$$

□

Proposition 3.5.

(i) When $\sigma = 1$, the unique equilibrium is $p_1 = p_2 = 0$.

(ii) When $\sigma = 0$ and $\lambda \in [0.5, 1)$, the unique equilibrium is $p_1 = p_2 = v$, $r_1^* = r_2^* = \infty$, and consumers purchase from their local firm.

(iii) When $\sigma = 0$ and $\lambda = 1$, the set of pure strategy equilibria can be characterized as follows: $p_1 \in [0, v]$. If $p_1 \in [0, v)$, $p_2 = p_1 - c$, $r_2^* = p_1 = p_2 + c < v$. If $p_1 = v$, $p_2 \in [v - c, \infty)$, and $r_2^* = \min\{v, \infty\}$.

Proof.

Case (i) follows from the standard Bertrand argument.

Case (ii): Claim 3.1 of Proposition 3.1 still holds, so the lower bound for both firms has to be the same. However, from Equation 3.4, we know that the firm distributions must be degenerate. So it must be the case that $p_1 = p_2 = p$. Suppose $p = v$. Given consumer (correct) beliefs, $\rho_1 = \rho_2 = \rho = c + v > v$, so the reservation price is $r^* = \infty$, which implies that consumers never search. Consider any other price $p < v$. Then $\rho = c + p$ and both firms can profitably deviate to $\min\{v, p + c\}$.

Case (iii): We restrict attention to pure strategy equilibria (see footnote 5). Consider the strategies $p_1 \in [0, v)$ and $p_2 = p_1 - c$. Given consumer (correct) beliefs, $\rho_2^* = p_1 = p_2 + c < v$, so $r_2^* = \rho_2$. Following the same reasoning as in Corollary 3.1, γ must equal to 1, so consumers never search in equilibrium. Consider any $p_2 > p_1 - c$. Then $p_1 < \rho_2 = p_2 + c$ and firm 1 can profitably deviate to $p_2 + c$. Now suppose that $p_2 < p_1 - c$. But then $p_1 > \rho_2$ so all consumers leave firm 1 and never come back. Thus, firm 1 can profitably deviate to $p_2 + c$. Finally, when $p_1 = v$, firm 2 may charge any price $p_2 > p_1 - c$. Given consumer (correct) beliefs, $r_2^* = \min\{v, \infty\}$ so firm 1 has no incentive to deviate. $\square$

Chapter 4

Kin-Targeted Altruism with Noise

4.1 Introduction

Can repetition increase cooperation when players' preferences exhibit kin-targeted altruism and kin recognition is noisy? As suggested by kin selection models [Hamilton, 1964], players are altruistic towards relatives when such behavior increases inclusive fitness. Inclusive fitness, also known as Hamilton's Rule, is defined as an average of an individual's own survival probability and that of her kin's, weighted by the degree of relatedness between them. Favoring only kin and not those who are not related requires kin recognition mechanisms, which rely on using noisy signals to discriminate between kin and non-kin. Kin-targeted altruism is said to occur when players condition their strategies on these signals, choosing to cooperate only with those who exhibit the signal associated to kin. We are interested in how repeating the game for one more period can enhance cooperation resulting from pure altruism generated by uncertainty about relatedness.

We will study a prisoners' dilemma played between two people who are randomly matched. The state of the world refers to whether players are related or not related. The probability of being related to one's opponent is common knowledge, but the realized state is not known to either player. Before the game starts, players observe a noisy private signal about the state of the world. We are interested in the case where kin-targeted altruism is the unique Nash equilibrium of the one shot game. If, after observing the signals, the prisoners' dilemma is played for two periods instead of one, given that the signals are private, players' strategies can be used to reveal or to conceal information. Even if the probability of being related to one's opponent is not very high, repetition generates an incentive to cooperate, either to conceal or to reveal information.

Depending on the payoffs of the game and the degree of noise, we distinguish two cases. In both of them, a player who observes the relatedness signal acts like a behavioral type: either a grim trigger or a blind cooperator. In the first case, if the grim trigger punishment is strong enough, cooperation increases because players who observe the unrelatedness signal prefer to conceal their signal in order to avoid the grim trigger punishment (and defect against a cooperating opponent in the second period). However, if the grim trigger punishment is not strong enough, repetition generates the opposite result. In the second case, cooperation always increases, because knowing that at least one player observed the relatedness signal is sufficient for both players to want to cooperate.

The remainder of this paper is organized as follows. Section 4.2 sets up the basic model. Section 4.3 analyzes the equilibrium of the one-period game. Section 4.4 analyzes the equilibrium of the two-period game, and Section 4.5 concludes.

4.2 The model

Players are randomly chosen from the population and matched to play a Bayesian game. The set of players is $N = \{1, 2\}$. The set of actions available to each player is $A_1 = A_2 = \{C, D\}$.

The state space, R, refers to whether the players are related or not. $r \in R$ is the degree of relatedness between two players, which is the probability of finding an exact copy of the players' genes in the other individual. Given this definition, r takes values between 0 and 1. For instance, if player 2 is a copy of player 1, then $r = 1$; if they share half their genes (full siblings), $r = 0.5$; if they are not related at all, $r = 0$. For simplicity, our model will assume that $R = \{0, 1\}$. This means that players are either copies of each other, or completely unrelated individuals.

Players' payoffs are determined using Hamilton [1964]'s Rule from biology, also known as inclusive fitness. That requires taking the genes' rather than the individual's total payoffs whenever players are related. The explanation is that players' genes are the ones playing instead of the individuals. If the same gene is present in both individuals, we should take into account both players' payoffs.

Table 4.1: Payoffs to both players in a Prisoners Dilemma

		Player 2	
		C	D
Player 1	C	b, b	$-c, b+c$
	D	$b+c, -c$	$0, 0$

First, we will define individual payoffs, given by u_i, which can be interpreted as monetary payoffs (i.e. the amount of money that each player receives after playing the game). These are the payoffs of the original prisoners' dilemma being played, shown in Table 4.1. More formally, $u_i : A_i \times A_j \to \Re$. For instance,

$u_1(D, C) = b + c$.

Both $b, c > 0$. This representation is convenient because all the information about the game that is of interest to us can be captured via the ratio $\frac{b}{c}$. A player whose opponent cooperates gets b. A player who defects on the other player gets c. If player 1 decides to cooperate with player 2 instead of defecting, he benefits player 2 in b dollars at a cost of c dollars that he could have kept for himself. Therefore, $\frac{b}{c}$ captures how costly it is to help one's opponent in the original game.

Given this, we can now define inclusive fitness, given by $U_i : A_i \times A_j \times R \to \Re$ in the following way: $U_i(a_i, a_j, r) = u_i(a_i, a_j) + ru_j(a_i, a_j)$. That is, a player's individual payoff is augmented by his opponent's payoff weighted by the degree of relatedness between them. Thus, the payoffs shown in Table 4.1 represent inclusive fitness when the state of the world is $r = 0$. That is, when players meet a non relative, the game they play is the original additively separable prisoners dilemma. When they meet a relative ($r = 1$), their payoffs are augmented by their opponent's payoff, according to Hamilton's rule. For instance, suppose that players are related, player 2 cooperates and player 1 defects. Then $u_1(D, C) = b + c$, $u_2(C, D) = -c$, so player 1's payoffs are $U_1(D, C, 1) = (b + c) + (1)(-c) = b$ The payoffs for state $r = 1$ are shown in Table 4.2.

Table 4.2: Players' payoffs in state $r = 1$

		Player 2	
		C	D
Player 1	C	$2b, 2b$	b, b
	D	b, b	$0, 0$

The probability that a player is related to his opponent is common knowledge: $P[r = 1] = z$. This is the probability of positive assortative matching. It is a known fact that the frequency of interaction with relatives favors cooperative behavior (See Bergstrom [2003] for references). In general, in a population of cooperators and defectors, when matching is assortative, cooperation can be sustained, since the cost of cooperating may be repaid by higher probabilities of playing against a cooperating opponent. We are not interested in the cases where matching is assortative, though. Instead, we will try to focus on cases where being related is not that likely.

Given the payoffs described above, it is clear that players would want to cooperate if they knew they were facing a relative and to defect otherwise. But, when matched to play, players do not observe the state of the world. Instead, they use signals to try to discriminate between kin and non-kin. A large number of studies of animal populations as well as simulations performed by biologists provide evidence in favor of kin-targeted altruism through kin recognition mechanisms. These mechanisms operate through the recognition of an observable arbitrary trait that individuals have no control over, such as skin or eye color, which must also be heritable. It is often referred to as the green beard effect or the armpit effect (Dawkins [1976]). For instance, if an individual's family tends to have green beards, every time the individual meets someone with a green beard, he thinks that it's very likely to be facing a relative.

However, kin recognition mechanisms can also be deceptive. The fact that individuals have to identify kin based on arbitrary traits generates two types of errors. In some of the cases, people will mistakenly identify individuals who are not kin as kin due to the presence of the trait. Conversely, players might dismiss a relative because the absence of the trait impeded his recognition. Thus, kin recognition mechanisms rely on signals that are not perfectly informative.

To avoid incurring in these types of errors, the signals used for kin recognition should be traits that tend to be present in one's relatives and absent in non-relatives. However, given the multiplicity of characteristics that are present in human beings that people could check for, one cannot make sure that one's opponent is looking for the same trait that one is looking for, and, consequently, cannot be certain that one's own recognition is 100% correct. For example, player 1 could think that player 2 is a relative because he also has a green beard, but player 2 might not agree because their eye color is different. Therefore, we assume that the signal observed by each player is private instead of public.

More formally, each player observes a signal $y_i \in Y_i = \{0, 1\}$. For simplicity, we assume that the distribution of the realized pair of signals $y \in Y_1 \times Y_2$ conditional on the state of the world is common knowledge. See Table 4.3.

Table 4.3: Conditional distribution of y

On state 0	On state 1
$\Pr[(0,0)\|r=0] = 1-2n$	$\Pr[(0,0)\|r=1] = 0$
$\Pr[(1,0)\|r=0] = n$	$\Pr[(1,0)\|r=1] = m$
$\Pr[(0,1)\|r=0] = n$	$\Pr[(0,1)\|r=1] = m$
$\Pr[(1,1)\|r=0] = 0$	$\Pr[(1,1)\|r=1] = 1-2m$

Both $m, n < 1/4$. This means that $y_i = 0$ is the unrelatedness signal, and $y_i = 1$ is the relatedness signal. Also, at least one of the two parameters m or n has to be strictly positive (otherwise there would be no noise). For simplicity, we assume that $\Pr[(0,0)|r = 1] = 0$ and $\Pr[(1,1)|r = 0] = 0$. That is, we assume that at least one of the players observes the "right" signal given the state of the world: if the state is unrelatedness, it will not be the case that both players observe the relatedness signal. The results of this paper are robust if these two probabilities are small enough. For example, if players knew that the signal realization is $(1, 1)$, instead of knowing for sure that the state is relatedness, players would consider a very small probability that they might be unrelated.

4.3 The one-period game

Consider the following game: players meet, observe the private signal, and then play a one shot prisoners dilemma.

There are four pure strategies in this game:

- C if $y_i = 1$
  D if $y_i = 0$
- C if $y_i = 0$
  D if $y_i = 1$
- C always
- D always

We are interested in the case where players play kin-targeted altruism: defect when the unrelatedness signal is observed and cooperate when the relatedness signal is observed. Given that both the distribution of the states of the world and that of the signal pairs y conditional on the state, we can calculate the distribution of r conditional on the observed signal, y_i, shown in Table 4.4.

Table 4.4: Conditional distribution of r

On $y_i = 0$	On $y_i = 1$
$\Pr[r = 0 \mid y_i = 0] = \frac{(1-z)(1-n)}{(1-z)(1-n)+zm} = 1 - \alpha$	$\Pr[r = 1 \mid y_i = 1] = \frac{z(1-m)}{z(1-m)+(1-z)n} = \beta$
$\Pr[r = 1 \mid y_i = 0] = \frac{zm}{(1-z)(1-n)+zm} = \alpha$	$\Pr[r = 0 \mid y_i = 1] = \frac{(1-z)n}{z(1-m)+(1-z)n} = 1 - \beta$

Given these distributions, players can calculate expected payoffs to each pure strategy. See Table 4.5.

Table 4.5: Player i's expected payoffs conditional on y_i

Expected payoff given $y_i = 0$

		Player 2: C	Player 2: D
Player 1	C	$b + \alpha b$	$\alpha b - (1-\alpha)c$
	D	$b + (1-\alpha)c$	0

Expected payoff given $y_i = 1$

		Player 2: C	Player 2: D
Player 1	C	$b + \beta b$	$\beta b - (1-\beta)c$
	D	$b + (1-\beta)c$	0

Kin targeted altruism is the unique Nash equilibrium of this game if and only if the ratio $\frac{b}{c} \in [\frac{1-\beta}{\beta}, \frac{1-\alpha}{\alpha}]$.[1] When the relatedness signal is observed, $\frac{b}{c} \geq \frac{1-\beta}{\beta}$ guarantees that the gains to cooperation are higher than the likelihood of mistakenly favoring a non-relative. On the other hand, when the unrelatedness signal is observed, $\frac{b}{c} \leq \frac{1-\alpha}{\alpha}$ guarantees that cooperating is too costly because non-relatives are more likely to be correctly identified.

Rearranging the terms, we can restate the conditions stated above as follows: $z \in [\frac{n}{(1-m)\frac{b}{c}+n}, \frac{(1-n)}{m\frac{b}{c}+(1-n)})]$. This can be interpreted in terms of assortativity. If positive assortative matching were too likely, players would blindly cooperate with any opponent, regardless of the observed signal, as explained earlier. On the other hand, if the likelihood of positive assortative matching were too low, players would prefer to defect all the time.

[1]Notice that $\frac{1-\beta}{\beta} = \frac{(1-z)n}{z(1-m)} < \frac{1-\alpha}{\alpha} = \frac{(1-z)(1-n)}{zm}$ always, since $m, n < \frac{1}{4}$.

4.4 The two-period game

Suppose that players meet, observe their signal, and then play the prisoners dilemma for two periods. We want to know if repeating the game for one more period can induce more cooperation in games where kin-targeted altruism is an equilibrium.

Before introducing the two-period game, it is relevant to explore what would happen if players somehow knew the realized signal pair (y_1, y_2). They would prefer to cooperate if $(1,1)$ takes place and to defect if $(0,0)$ takes place, but what happens when the realization is $(1,0)$ or $(0,1)$ is not obvious. Since both cases are computationally equivalent, it is enough to analyze only one. We call the game where both players know that the signal realization is $y = (0,1)$ "artificial" because it might never be played, but has important implications on the equilibrium of the two times repeated game.

4.4.1 The artificial game

Suppose that both players know that the signal realization is $y = (0,1)$ and they will play the game only once. In this case, both players know that one of them observed the wrong signal, but none of them knows who did. We can find the distribution of r conditional on $y = (0,1)$. Let $\gamma = \frac{zm}{zm+(1-z)n} = P[r = 1|y = (1,0)]$. Then, we can write expected payoffs conditional on $y = (0,1)$ as shown in Table 4.6.

This game has a unique Nash equilibrium, which depends on the value of the ratio $\frac{b}{c}$. Defection is the unique Nash equilibrium of the artificial game if and only if $\frac{b}{c} \in [\frac{(1-\beta)}{\beta}, \frac{(1-\gamma)}{\gamma}]$. Conversely, cooperation is the unique Nash equilibrium if and

Table 4.6: Player 1's expected payoffs in the artificial game

		Player 2	
		C	D
Player 1	C	$b+\gamma b$	$\gamma b-(1-\gamma)c$
	D	$b+(1-\gamma)c$	0

only if $\frac{b}{c} \in [\frac{(1-\gamma)}{\gamma}, \frac{(1-\alpha)}{\alpha}]$. We will analyze each of these cases separately.

4.4.2 Main result for the twice repeated game with imperfect information

Proposition: *When players' preferences are altruistic towards kin and kin recognition is noisy,*

if Cooperation is the unique Nash equilibrium of the artificial game

OR

if the grim trigger punishment is strong enough,

then there is strictly more cooperation in the twice repeated game.

The proof follows directly from Claims 4.1 and 4.2, which will be stated in the following subsections. The rest of the section is devoted to explain this result.

Games with defection in the artificial game

In this case, knowing the signal realization is $(0,1)$, cooperating is too costly because favoring a non-relative is too likely. Given this, observing $y_i = 0$ is sufficient for wanting to defect in the twice repeated game. For instance, suppose that player 1 observed $y_1 = 0$. Learning which signal player 2 observed will not make a difference, since defecting is dominant in either case. Conversely, suppose that 1 observed $y_1 = 1$, so he would initially prefer to cooperate. Learning his opponent's signal would make a difference: if $y_2 = 0$, player 1 would rather defect. That is, learning the other player's signal can never persuade a defector to cooperate, but it can dissuade a cooperator from cooperating.

Thus, a separating equilibrium cannot increase cooperation in the twice repeated game. In the previous example, suppose that $y_2 = 1$. If player 1 (male) were given the choice to reveal or not reveal which signal he observed to player 2 (female), he would prefer not to reveal it. The reasoning is as follows. Given his signal, player 1's expected payoff is always higher when he defects instead of cooperating. If he keeps his information to himself, he knows that player 2 will cooperate, but if he chooses to reveal it, she will prefer to defect as well. Since defecting yields a higher payoff when his opponent cooperates, player 1 would be better off not revealing his signal. Hence, cooperation can only increase in the twice repeated game if players always cooperate in the first period.

Note that a player who observes $y_i = 1$ acts like a grim trigger behavioral type: he cooperates as long as he doesn't find out that $y_j = 0$. Define $\lambda = P[y_j = 0 | y_i = 1] = \frac{zm+(1-z)n}{zm+(1-z)(1-n)}$. Given that this is a positive number, a player who observes the unrelatedness signal would only cooperate during the first period to conceal his signal from the player who observed the relatedness signal and avoid the grim trigger punishment. That is, the grim trigger punishment must provide the right incentives for cooperation.

Definition: *We say that the grim trigger punishment is strong enough iff*

$$\frac{b}{c} \geq \frac{1-\lambda}{\alpha+\lambda}.$$

When a player observes the unrelatedness signal, he benefits from cooperating

(i) when the opponent is related (with probability α), and (ii) when the opponent thinks that he is related (with probability λ). His cost of cooperating is given by the times that he doesn't defect against a non-relative (with probability $1-\alpha$). The above condition means that the benefit of cooperating in the first period and waiting until the second period to defect is greater than the benefit of defecting from the very begining and being punished in the second period.

Claim 4.1. *When defection is the unique equilibrium of the artificial game and the grim trigger punishment is strong enough, the following strategy is a Perfect Bayesian Equilibrium of the original twice repeated game:*

- *Play C in the first period regardless of the signal observed.*
- *If $y_i = 1$ and C were observed, play C in the second period. Play D in the second period otherwise.*

Proof. Checking for perfection of the second period actions is immediate from the one-shot game, since no information is revealed at the end of the first period. We need to check for perfection of the first period actions. Assume that if a player observes defection in the first period, he attributes it to his opponent having observed the unrelatedness signal. Given this, and given that there is defection in the artificial game, it is easy to check incentives for the case when $y_i = 1$ is observed.

When $y_i = 0$ is observed, we need to verify that player i prefers to cooperate in the first period. Calculate expected payoffs according to first period action:

	C	D
1^{st} per	$\lambda[b+\gamma b]+(1-\lambda)b$	$\lambda[b+(1-\gamma)c]+(1-\lambda)[b+c]$
2^{nd} per	$\lambda[b+(1-\gamma)c]$	0

Given that the grim trigger punishment is strong enough, the payoff of cooperating in the first period is higher than that of defecting. □

Games with cooperation in the artificial game

In this case, knowing the signal realization is $(0, 1)$, cooperating is attractive because favoring a relative is more likely. Given this, observing $y_i = 1$ is sufficient for wanting to cooperate, and player i acts like a blind cooperator in the twice repeated game. Thus, learning the other player's signal can never dissuade a cooperator from cooperating, but it can persuade a defector to cooperate.

Claim 4.2. *When cooperation is the unique equilibrium of the artificial game, the following strategy is a Perfect Bayesian Equilibrium of the original twice repeated game:*

- *If $y_i = 1$, play C in both periods.*
- *If $y_i = 0$, play C with probability ψ in the first period*
  If C is observed, play C with probability ϕ in the second period
  Otherwise, play D in the second period.

Proof. Given that there is cooperation in the artificial game, incentives for the player observing $y_i = 1$ are easy to check because cooperation is always dominant.

Now, check incentives when $y_i = 0$ is observed. In the second period, after defection is observed, player i knows for sure that the signal realization was $(0, 0)$ and, therefore, the state of the world is unrelatedness, so he prefers to defect. On the other hand, suppose that cooperation was observed in the first period. Let μ be the posterior probability that the opponent observed the relatedness signal. Player i's expected payoffs according to second period action are:

$$\text{Cooperating:} \quad \mu[b+\gamma b] + (1-\mu)[\phi b + (1-\phi)c]$$
$$\text{Defecting:} \quad \mu[b+(1-\gamma)c] + (1-\mu)\phi[b+c]$$

Since player i is mixing, both payoffs must be equal. Solve for μ:

$$\mu = \frac{\lambda}{\lambda + \psi(1-\lambda)} = \frac{c}{\gamma(b+c)}$$

Given this equality, we can pin down the value of ψ:

$$\psi = \frac{\alpha(b+c) - \lambda c}{1-\lambda}$$

Now check for incentives in the first period. Calculate expected payoffs according to first period action:

	C
1^{st} p	$\lambda[b+\gamma b] + (1-\lambda)[\psi b - (1-\psi)c]$
2^{nd} p	$\lambda[b+\gamma b] + (1-\lambda)\psi[\phi b + (1-\phi)c] + (1-\lambda)(1-\psi)[\phi(b+c)]$

	D
1^{st} period	$\lambda[b+(1-\gamma)c] + (1-\lambda)[\psi(b+c)]$
2^{nd} period	$\lambda[b+\gamma b] - (1-\lambda)[\psi c]$

Given that player i mixes, set both payoffs equal to each other and solve for ϕ:

$$\phi = \frac{c - \alpha(b+c)}{(1-\lambda)(b+c)}$$

Given that $\frac{b}{c} \in [\frac{1-\gamma}{\gamma}, \frac{1-\alpha}{\alpha}]$, both ϕ and ψ are positive numbers. $\square$

4.4.3 Can repetition generate the opposite result?

There is one case in which repetition strictly increases defection instead of co-operation. When there is defection in the artificial game and the grim trigger punishment is not strong enough, the effect of repetition is the exact opposite to the one described in the preceding subsections. In this case, players do not have incentives to conceal their signal from their opponents because the rewards of waiting one period to defect are too low. Therefore, in this case, private information is revealed at the end of the first period and players cooperate in the second period only if both of them observed the relatedness signal. In the rest of the cases, players defect in the second period. In other words, there is strictly more defection than without repetition.

Recall that a player who observes the relatedness signal always prefers to cooperate when he doesnt know which signal his poppponent observed. When there is defection in the artificial game, as seen in subsection 3.2.1, a player who observes the unrelatedness signal always prefers to defect in the second period. In the first period, he can cooperate, thus avoiding the grim trigger punishment at the cost of a lower payoff that period, or he can defect, obtaining a lower period, but accepting the punishemnt. However, the grim trigger punishment is not strong enough, so, given his continuation strategy, the payoff of defecting is higher than the payoff of cooperating in the first period, as it can be seen in the proof of Claim 1. Then, the unique Perfect Bayesian Equilibrium of the twice repeated game is the following:

- If $y_i = 0$, play D in both periods.
- If $y_i = 1$, play C in the first period

  If C is observed, play C in the second period

Otherwise, play D in the second period.

4.5 Conclusion

In games where players exhibit altruistic preferences towards relatives but kin recognition is noisy, playing a one shot prisoners' dilemma results in cooperation whenever players observe the relatedness signal. This kind of cooperation can be interpreted as pure altruism, since its only purpose is to favor kin if it is in their common genes' best interest. When the prisoners' dilemma is played for two periods instead of one, given that there is uncertainty about relatedness, other considerations are brought into the game, even if the odds of being related are small.

In the first case, players use their first period actions to dissuade their opponent from defecting. When there is defection in the artificial game and the grim trigger punishment is strong enough, learning that one's opponent observed the unrelatedness signal induces cooperators to defect. Cooperation increases in the first period because players who observe the unrelatedness signal want to conceal it from their opponent. In the second period, play is the same as in the one period game, and kin targeted altruism is played. However, when the grim trigger punishment is not strong enough, repetition generates the opposite effect and defection increases.

In the second case, players use their first period action to persuade their opponent to cooperate. When there is cooperation in the artificial game, a sufficient condition for wanting to cooperate is to know that at least one of the players observed the relatedness signal. Players who observe this signal cooperate to reveal their information to their opponents.

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