Essays in Political Economy

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Part I

The Politician-Candidate Model

The Politician-Candidate Model
Campaign Promises, Identity and Salience

by Filip Lazarić[0]

Abstract

In a voting game with spatial competition and endogenous entry I look at the effect partially credible promises have on equilibrium existence, and the equilibrium qualities. The typical assumption in the citizen-candidate model is that campaign promises are cheap talk, and therefore rational equilibria have candidates running truthfully. This is relaxed by assuming that voters are naive (i.e. they believe campaign promises), and politicians may have to keep their promises if they are salient enough. Voter naiveté implies that the strategic individuals contemplating entry are politicians, and not voters, making this a *politician-candidate model*. In particular, the paper looks at whether politicians run honestly, whether an institutional structure can be imposed to ensure only honest politicians are willing to enter, and finally, what is the effect of salience. I find that campaign promises increase the multiplicity of equilibria by allowing candidates to run dishonestly. There are two opposing forces at work: lying provides a benefit to candidates whose ideology is not in line with the public, however allowing these dishonest candidates to form equilibrium configurations is also costly for challengers. That is, letting a bad candidate run is costly for political competitors. I find that the multiplicity can be reduced, however, generally one cannot certainly guarantee only honest candidates. In fact, one of the main results of the paper is that for all pooling equilibria, the probability that all of the entrants are honest is zero. That is, in equilibrium it is practically impossible that candidates are honest. The reason is that when institutional arrangements are picked such that honest candidates can form an equilibrium and the incentives for dishonest candidates are minimized, a continuum of dishonest candidate configurations will also be possible. Finally, salience disciplines the politicians by increasing the cost of lying, however the equilibrium effect of salience is not clear cut. The main policy implications are that retrospective voting (i.e. punishing bad behavior) cannot discipline dishonest entry. Instead, society should invest in institutions that help voters with prospective voting (i.e. choosing the candidate with the correct motivations), such as free media, fact-checking institutions (e.g. Politifact), and similar institutions that could help judge the quality of candidates and their promises.

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1 Introduction

I'm the one who will not raise taxes. My opponent now says he'll raise them as a last resort, or a third resort. But when a politician talks like that, you know that's one resort he'll be checking into. My opponent, my opponent won't rule out raising taxes. But I will. And The Congress will push me to raise taxes and I'll say no. And they'll push, and I'll say no, and they'll push again, and I'll say, to them, read my lips: no new taxes.
George H.W. Bush, Nomination Acceptance Speech at 1988 Republican National Convention (August 18, 1988) [Harris and Bailey, 2014]

During his 1988 presidential campaign, George Bush made the following promise: *"Read my lips: no new taxes."* Bush was elected president and served one term (1989-1993). He tried to keep his promise, but the 101st Congress (1989-1991, majority Democratic) wanted to reduce the national deficit by increasing taxes. Bush ended up signing the Omnibus Budget Reconciliation Act of 1990, one of the most successful budget reconciliation bills (at reducing the deficit) enacted into law,[1] however it increased taxes.[2] During the bid for re-election the fact that Bush reneged on his promise was intensively used by his opponents (Pat Buchanan during primaries[3], and Bill Clinton[4]). Bush was not re-elected, and the broken promise may have had a significant impact. This example demonstrates that promises do get broken, that candidates cannot always anticipate what promise they will have to break, and that breaking promises is costly for politicians.

This articles takes the view that at the campaign stage, George Bush could not have known with certainty whether he would keep his promise. From table 1 we can see that Bush was the Vice President in 1981-1989, where the Senate majority was Republican in all but the last term (1987-1989). This is also the term where Bush campaigned for the Presidency. George Bush could have expected Republicans to win the Senate in 1989, which may have allowed him to avoid tax increases. However, the Democrats won the 101st Congress and pushed a revenue improving bill, where Bush compromised and allowed some tax increases.

It is unclear whether Bush's true preference (when campaigning) was to avoid increasing taxes at all costs, or whether he preferred decreasing the deficit and enhancing revenue by modest tax increases. Revealed preferences suggests Bush implemented his preferred policy, which implies

[1] http://www.crfb.org/blogs/remember-remember-omnibus-budget-reconciliation-act-1990

[2] "Today I am signing H.R. 5835, the "Omnibus Budget Reconciliation Act of 1990," the centerpiece of the largest deficit reduction package in history and an important measure for ensuring America's long-term economic growth. This Act is the result of long, hard work by the Administration and the Congress. No one got everything he or she wanted, but the end product is a compromise that merits enactment.", from http://www.presidency.ucsb.edu/ws/index.php?pid=19000.

[3] "Why am I running?" he asked. "Because we Republicans can no longer say it is all the liberals' fault. It was not some liberal Democrat who declared, 'Read my lips no new taxes,' then broke his word to cut a seedy backroom budget deal with the big spenders on Capitol Hill.", from https://www.nytimes.com/1991/12/11/us/buchanan-urging-new-nationalism-joins-92-race.html

[4] During the final presidential debate Clinton said: "he [Bush] is the person who raised taxes on the middle class after saying he wouldn't. And just this year, Mr. Bush vetoed a tax increase on the wealthy that gave middle class tax relief. He vetoed middle class tax relief this year." to which Bush replied: "I think everybody's paying too much taxes. He refers to one tax increase. Let me remind you it was a Democratic tax increase, and I didn't want to do it and I went along with it. And I said I make a mistake. If I make a mistake, I admit it. That's quite different than some. But I think that's the American way.", from http://www.debates.org/index.php?page=october-19-1992-debate-transcript, and audio at https://bush41library.tamu.edu/files/audio/Third%20Presidential%20Debate%20-%20East%20Lansing,%20Michigan%20-%2019%20October%201992.mp3

President	Ronald Reagan (R)	—‖—	—‖—	—‖—	George Bush (R)	—‖—
Senate	53	55	53	45	45	42
Senate President[5]	George Bush (R)	—‖—	—‖—	—‖—	Dan Quayle (R)	—‖—
House	44	38	42	41	42	38
House Speaker	Tip O'Neill (D)	—‖—	—‖—	Jim Wright (D)	Tom Foley (D)	—‖—
Congress	97th	98th	99th	100th	101st	102nd
Years	1981-1983	1983-1985	1985-1987	1987-1989	1989-1991	1991-1993

Table 1: Percent Republican seat share, where "—‖—" means same as previous term

his campaign promise was disingenuous and pandered to the public. Regardless of his preference, the broken promise was costly since free speech allowed his political competitors and the media to hurt Bush's reputation and his re-election chances.[6]

The previous example elucidates the issue of accountability with regards to political campaign promises. The law seems to have taken the position not to force accountability on political promises, but rather that free speech is the stick disciplining falsities. One of the main legal difficulties with campaign promises is that any government punishment may be partial and can diminish the freedom of speech. In one of the earliest famous cases where the US Supreme Court voiced their opinion about dishonesty in campaign promises (*Brown v. Hertlage*, 1982), it took the position that the lie was not made with malicious intent, and was therefore constitutional (protected by free speech). For more details about this case, see Appendix A. As shown in [Sencer, 1991], a similar judicial unwillingness has been present in other cases. Furthermore, from [Norris, 1997] it seems clear that there are no legal provisions which would punish politicians breaking promises. The most recent important case where the Supreme Court voiced their opinion, *United States v. Alvarez* (2012), stated in their final opinion:

> "Permitting the Government to decree this speech [at a board meeting candidate lied he received the Medal of Honor] to be a criminal offense, whether shouted from the rooftops or made in a barely audible whisper, would endorse government authority to compile a list of subjects about which false statements are punishable. That governmental power has no clear limiting principle. All this suffices to show that how the Act conflicts with free speech principles. [...] Counter speech has long been the First Amendment's preferred method for responding to falsity. [...] The Nation well knows that one of the costs of the First Amendment is that it protects the speech we detest as well as the speech we embrace. Though few might find respondent's statements anything but contemptible, his right to make those statements is protected by the Constitution's guarantee of freedom of speech and expression."

The Supreme Court illuminated the reason for the limited reach of the law, with respect to campaign promises. The counter to falsities is free speech.[7] The obligation placed on politicians and the media seems quite great, at the very least electoral competition has to be genuine (i.e. without coordination), the media has to be unbiased, and speech has to be completely free. As argued

[6]Due to the realized cost of promise-breaking it seems Bush would have preferred keeping his promise; a belief further reinforced by the fact that during his re-election bid he claimed breaking said promise was a mistake.

[7]Clearly, as demonstrated in the Bush example, it is in each political competitors interest to expose broken promises and other falsities that could increase ones own election chances. Furthermore, journalists expose such falsities due to their own reputation, and due to the fact that exposing a politician to have done things against his electorate is a interesting read for the electorate (i.e. it sells).

in [Hasen, 2013] and [Sellers, 2018], the Supreme Court made lying far more likely excusable in the face of the law, and this case will have an effect on all future decisions the Supreme Court takes about campaign promises. For more details about this case, see Appendix B.

The literature on public projects finds strong empirical evidnece that better informed agents use falsities (which seem to be trusted) to pander to their principal in order to obtain benefits for oneself. In particular, [Flyvbjerg et al., 2002, Wachs, 1990, Wachs, 1989, Flyvbjerg et al., 2005, Flyvbjerg, 2008] find evidence that the cost of public projects is often underestimated, and the demand overestimated, in order to improve the chances of getting the project going. For more details see Appendix C. In light of such empirical evidence, and the clear moral hazard and adverse selection problem involved in the electoral game, it is not surprising that the public has a bad image of politicians. In fact, in 2014 a Rasmussen survey found that only 4% of likely voters think most politicians will keep their promises, while 83% believe they will not (remainder is undecided).[8] Furthermore, the 2012 General Social Survey asked people whether they agreed congressmen try to keep their promises, and 59% of respondents said no.[9] Nevertheless, there is a large body of empirical evidence that politicians keep about two thirds of their promises. The meta-study [Pétry and Collette, 2009] finds that the average share of promises kept by US presidents in the period 1944-1999 is 67%. Furthermore, in the 17 studies that cover Great Britain, Canada, Greece, the US and Netherlands, they find that the average share of promises kept by politicians is 69%. In other words, politicians break their promises roughly a third of the time. When a politician breaks their promise, this is called *policy dissonance* (or policy incongruence), because the voter expected one thing but another thing happened [Imbeau, 2009]. There exist two reasons for breaking promises: the politician misrepresented true intentions (i.e. lied), or the politician made the promise genuinely but something changed. The second reason can be further separated into the politician making a forecasting error (e.g. lack of information at campaign stage), or the circumstances changing[10].

Why do politicians keep about two thirds of their promises? It seems unlikely that two thirds of their promises are in line with their own preferences, making them unwilling to renege. The experimental literature seems to identify two main reasons: promises are not cheap talk (voters react to them, giving good promises more votes; politicians keep promises even when they do not have to), and voters seem to be voting retrospectively (punishing politicians bad behavior, e.g. who break their promises). The evidence that individuals keep their promises when they do not have to, may also suggest that politicians may have some internal cost to breaking promises.

[Corazzini et al., 2014] design a one-shot game experiment where candidates receive money and need to decide how to share with the voters. Their two main findings are that 1) electoral competition (rather than random appointment), and using non-binding campaign promises (compared to no campaigning treatment), increases candidate benevolence (they give more), and 2) voters do not treat promises as cheap talk (better promises get more votes, and politicians who promise more give more). [Feltovich and Giovannoni, 2015] conduct a similar rent-appropriation experiment, however they have a repeated setting. They also find that promises aren't cheap talk (good promises are rewarded and bad promises punished), that broken promises are punished (i.e. voters vote retrospectively rather than prospectively)[11], and that campaign promises

[8] http://www.rasmussenreports.com/public_content/politics/general_politics/november_2014/just_4_say_candidates_keep_their_

[9] https://fivethirtyeight.com/features/trust-us-politicians-keep-most-of-their-promises/

[10] Changing circumstances can change the optimal promise, as well as the underlying preferences

[11] Retrospective voting means to punish bad behavior, while prospective voting means to try and select individuals

are informative about what the politician ends up implementing.

A unique experiment is performed by [Banerjee et al., 2018]. They design a lab in the field experiment, where they modified the dictator game where with probability 0.2 the split chosen by the candidate is implemented, and with complementary probability nature allocates the full endowment randomly. This means that if the voter received zero, he could not tell whether it was nature, or the politicians choice. When candidates can hide their action, roughly a third of the candidates give zero to their partner. In their second treatment, where politicians also had to make non-binding promises, they find: politicians are more generous (28% higher mean giving), a drop in zero giving (from 28% to 12%), and a significant increase in 50:50 giving. In fact, with promises 88% of politicians promise to distribute a positive amount, and 83% of politicians keep their promise. The high degree of promise keeping is striking, since the promises are non-binding and the participants do not know each other (i.e. they are from distant villages). Finally, they compare how local politicians vs. non-politicians behave in this experiment, and find that politicians promise more, and are more likely to keep their promise, than non-politicians. This seems to suggest that politicians are different to non-politicians.[12] Given the experimental evidence that political promises are not cheap talk, it is important to theoretically analyze environments where promises are not cheap talk, and voters trust these promises.

[Fehrler et al., 2016] design an experiment testing a one-shot two stage game where first candidates compete in a primary, the amount of campaigning in primaries can be transparent or not, and then the primary winners compete for office by making non-binding promises. They find that dishonest people over-proportionally self-select into the political race, and this adverse effect can be prevented by making the first stage transparent (making it possible for good candidates to separate). The key mechanism they find is that dishonest individuals stand to gain more from winning the election than honest individuals, and are therefore willing to invest more to become candidates. This arises endogenously in my model, since candidates can lie, but they need to consider political competitors.[13] They also confirm the results that promises are not cheap talk (more votes for better promises, and positive correlation between non-binding promise and what was implemented), and that promises are used for personal benefit (candidates promise a lot). This experiment is particularly relevant, since the current paper looks at who will self-select into the political race, given that voters believe the candidate promises, and promising something different from ones own ideal is costly.

[Fox and Shotts, 2009] write down a theoretical model where they show that if voters want politicians who share their own preferences they pick delegates (to implement those preferences), while if they value competence (or the environment is very uncertain) they prefer trustee representation, which allows the politician more autonomy to optimally act given the circumstances one finds. [Woon, 2012] tests a simplified version of this theoretical model and finds that, contrary to the rational predictions of the model, there seems to be a strong behavioral tendency to use retrospective voting. Furthermore, additional treatments seem to suggest that voters use retrospective voting as a simple heuristic for who to vote for, since it is easier to punish wrongdoing than to figure out who the candidates are. That is, the rational calculations involved may be overbearing on voters, so they fall back on heuristics. A second reason seems to be that voters

who will be best politicians.

[12]It may be that politicians learn to be different, or are different from the outset (selection issue).

[13]For example, when two candidates run the equilibrium message is the median. Then an extremist politician loses more from the opposite extremist winning, than does a median politician.

focus on accountability rather than selection. In fact, [Woon, 2012] seems to find evidence against sequentially rational equilibria, since they require voters to be forward-looking, that is, he finds voter choices to be inconsistent with the perfect Bayesian equilibrium.

Sequential rationality and dynamic consistency play a big role in spatial electoral competition games with endogenous entry. Rationally, office-seeking candidates have an incentive to state promises that pander to the voters, however if they win they would like to renege on all of the promises that were not in line with their true preference. That is, as was pointed out in [Alesina, 1988], there is a dynamic inconsistency in the incentives of the politicians. In the original citizen candidate model this was dealt with by assuming that rational voters and politicians are sequentially rational and the only possible (rational) equilibrium has the candidates running with their true identity. Note that an equilibrium where politicians consistently make promises not in line with their preferences, and voters believe those promises, is time inconsistent and irrational.

In addition to the strong experimental evidence suggesting this is not true, there exists empirical evidence from the revenue forecasting literature which suggest that political predictions are not rational. That is, [Feenberg et al., 1989, Rider, 2002, Campbell and Ghysels, 1995, Ohlsson and Vredin, 1996, Auerbach, 1999] find that revenue forecasts do not use all available information and by improving forecasting methods one can improve forecasting accuracy; and forecasting errors follow economic cycles. For more details see Appendix D. The fact that governmental forecasts using complex statistical tools do not use all available information, and are oftentimes biased following economic cycles, suggest that promises may be used for gain, and that it seems unlikely voters and politicians are sequentially rational. In fact, the experimental evidence seems to point towards behavioral biases, such as intrinsic costs to promise-breaking (or other explanations based on internalized norms); or towards retrospective voting as a simple heuristic which makes breaking promises costly. The current model tries answering the following questions: Given that politicians lie using non-binding campaign promises, and that voters trust these promises, who will self-select into the electoral race? Is there any way to set up institutions in order to minimize the amount of lying politicians do? Can we set up institutions to ensure the only politicians that become candidates maximize voter welfare?

This paper looks at the situation where voters trust the campaign promises, and vote for the politician whose promise is closest to their own ideal. Furthermore, each potential candidate is aware there exists a chance he may not be able to renege on his promise, if he wins. This uncertainty implies that lying[14] is costly. Costly lying does not capture why someone would adhere to a message from which they would prefer reneging, once they won. This is captured through salience. Each candidate has a belief about the probability the winner will be forced to keep his promise. This belief is *salience*. It is each politician's estimate of the likelihood something will happen to force promise keeping. The more salient the issue, the more difficult it is to renege from it.

There is a large literature on salience in political science, in particular, empirically looking at roll-call votes in order to determine whether promises were broken, and why the politicians voted in a particular way. Many papers [Snyder Jr and Groseclose, 2000, Levitt, 1996, Faas et al., 2002, Klüver and Spoon, 2015, Jenkins, 2010, Schwarz et al., 2010], amongst others, find that if the party finds some issue salient, their members in power are unlikely to vote against the party preference; while they are more likely to vote against the party preference on non salient (for the

[14]Promising something different from own preference.

party) issues. Similarly, when the issue is very salient for the constituents, politicians are more likely to vote in line with their constituency preferences. For example, [Hutchings, 1998] finds that US Democratic Congressmen from southern states with a large black community voted in favor of the Civil Rights Act 1990, however less of them voted in favor of the Michel-LeFalce Amendment (1988). Both of these issues were very salient for their black community, however the Civil Rights Act received more media attention, making it more salient. The media attention could not have been anticipated. Finally, [Grossman, 2012] finds that with unexpected (energy) shocks, politicians need to appear to "do something".[15] This seems to support the importance of unforeseen circumstances in policy dissonance, and that unexpected events can affect the salience of issues.

The game of the paper is a modified citizen-candidate model, and it proceeds as follows. The population votes for the candidates who decided to run (i.e. paid the cost of entry C), from the set of potential candidates. I assume that the pool of potential candidates is small enough so their decisions are irrelevant for the voting outcome. Furthermore, for each voter preference there exists a potential candidate whose preference is exactly that (i.e. the set of potential candidates is representative of the voter preferences). Voters believe candidate promises, and sincerely vote for the candidate whose promise is closest to their own preference. Politicians are office motivated, so they use promises strategically (capturing as many votes as possible), which they know they may have to uphold if the content of the promise becomes overly salient. Each politician has a belief about how likely his promise may become salient. The model looks at the case where all politicians have the same belief, that is any promise has the same probability of becoming salient after the election is completed. In other words, each candidate believes that no matter what the content of his promise, it has an equal chance of becoming a problem for him.

This paper looks at how the fact that politicians can credibly lie affects the incentives to enter the political race (self-select into the electoral race). It turns out that the fact that politicians run with messages rather than with ideal points imposes an additional equilibrium condition which simplifies the problem. All the messages stated have to be individually rational, that is, no politician could be in a situation that he would have preferred to run with a different message. This allows us to first look at what the only possible equilibrium messages are, and then look at who are the politicians willing to use those messages. Some of the main findings are that the credibility of promises increases the multiplicity of equilibria (since dishonest candidates are possible),[16] that individuals who are further from the equilibrium messages (worse candidates) have a higher incentive to lie in their promises (as they gain from lying, and can stop their worst competitor from entering),[17] that the tie-breaking rule plays a key role in determining equilibria, and that the probability of randomly picking the equilibrium configuration with only honest candidates is (almost) zero for a large class of equilibria.

The paper proceeds as follows. It first looks at a couple of case studies about campaign promises that candidates made, after which it briefly reviews the relevant papers from the different related strands of literature. Then follows a section on how campaign promises are introduced into the citizen candidate model,the model setup and the 1-,2-,3- and 4-Candidate equilibria, where we look at 1-and 2-Candidate equilibira in most detail. Finally, we define all possible

[15]Their preference is to avoid precise statements, since any definitive action can anger some voters, or turn out to be wrong, but at the same time they cannot appear to do nothing.

[16]Unlike in the original citizen candidate model, where only honest entry was possible.

[17]As in [Fehrler et al., 2016].

equilibrium message configurations for N-Candidate equilibria, and look into the simplest possible N-Candidate equilibrium where all candidates pool at the median.

2 Case Studies

2.1 First female Croatian President - Kolinda Grabar-Kitarović (2015-)

Kolinda Grabar-Kitarović promised that she will remove Tito's bust from the office of the president. She could not have known whether after the election, the promise would become very salient in the public. A month after being in office, Kolinda still did not remove Tito's bust from office, and furthermore made claims on national TV that it's not of utmost priority, but that it will be taken care of.[18] This gave the media the possibility of making headlines, and exert pressure. They made claims that she does not care enough about the promise, with headlines like: "Tito's bust still at residence, as it's 'not a priority for the President'".[19] She could not have foreseen the pressure the media imposed. Soon afterwards she put significant effort into making headlines how she will remove the bust, and within a month the bust, and other belongings of Tito, were sent to a museum.[20]

It is unclear whether she really wanted to keep this promise, or she was forced to keep it. The socialist past is a very divisive issue in Croatia. Nevertheless, it seems that there was party pressure focusing her political statements towards a more extreme (contra-Tito) position during her campaign, and that she reacted to media pressure after the election. It seems reasonable to assume she could not have predicted the media pressure. Finally, it turned out that removing the bust was against her political interests. An independent survey from 2016 found that the largest fraction of the respondents thought that exactly her decision to remove Tito's bust was a mistake, that is 38 percent.[21] For more details about this case see Appendix E.

2.2 UK's Prime Minister 2010-2016 - David Cameron

One of the most famous promises David Cameron made prior to the 2015 general election was that he will hold a in/out EU referendum by 2017.[22] It seems that Cameron's actual preferences may have been soft Euroscepticism[23], who may have preferred to stay in. This is further suggested by

[18] https://www.24sata.hr/news/ako-mi-vlada-ne-odgovori-razmislit-cu-o-drugim-mjerama-408084, https://vijesti.hrt.hr/272309/javna-stvar-kolinda-grabar-kitarovic-hrt1-19-sati

[19] https://www.tportal.hr/vijesti/clanak/titova-bista-jos-na-pantovcaku-jer-nije-prioritet-predsjednici-20150224, https://kamenjar.com/je-li-predsjednica-grabar-kitarovic-napravila-prvu-veliku-grjesku/?fb_comment_id=663960520382330_664347663676949

[20] https://www.index.hr/vijesti/clanak/Kolinda-izbacila-Titovu-bistu-s-Pantovcaka/808080.aspx, http://www.novilist.hr/Vijesti/Hrvatska/Grabar-Kitarovic-ispunila-obecanje-i-delozirala-bistu-Tita-s-Pantovcaka?meta_refresh=true, https://vijesti.rtl.hr/novosti/1550865/sto-je-obecala-to-je-ispunila-grabar-kitarovic-uklonila-titovu-bistu/

[21] https://www.klix.ba/vijesti/regija/gradjani-hrvatske-kolindi-najvise-zamjeraju-micanje-titove-biste-i-sporna-pomilovanja/160213096

[22] https://www.newstatesman.com/politics/staggers/2016/07/david-cameron-made-these-6-promises-what-he-actually-delivered, https://www.bbc.com/news/uk-politics-21148282

[23] https://www.theguardian.com/politics/2016/jun/24/david-cameron-downfall-european-tragedy

recent statements that he does not regret calling for a referendum, but thinks the wrong decision was made.[24]

Already in 2011 it was clear that a large fraction of Cameron's party (Conservatives) supported a referendum on EU membership. A vote was held whether to hold a EU referendum, which was defeated 483 to 111, however more than half of the Conservative party (81 MPs) voted for the referendum. At this time the public sentiment was such that 70% wanted a referendum, 49% said they would vote to exit the EU, while 40% said they would vote to remain.[25] A further pressure Cameron felt was the growing presence of the extreme right party UKIP (UK Independence Party). In 2013 UKIP had it's first major victory in local elections where it won 140 out of about 2300 contested seats, and averaged 25% of the votes in the wards where it was standing.[26] In 2015 UKIP won control of the Thanet Council, the first time the party won control of a local council. Finally, in the 2014 European Elections UKIP won 24 seats in the European Parliament, more than any other party from the UK. It is clear that David Cameron was under tremendous pressure to make this promise during his 2013-2014 campaign.[27]

From one of the pieces in the Guardian[28] the following conversation between Nick Clegg (the leader of the Liberal Democrats) and David Cameron took place:

> "He's so busy wondering how to get through the next few weeks that he could endanger Britain's international position for the next few decades. It's all very very risky", [Nick] Clegg told [David] Laws in 2012, wholly accurately as things were to turn out. When Clegg put this to Cameron later in 2012, Cameron's reply was eloquent and to the point: "You may be right. But what else can I do? My backbenchers are unbelievably Eurosceptic and UKIP are breathing down my neck."

Even though Cameron promised to hold the referendum, he was working hard to get exemptions from the EU, to have as much sovereignty as possible, in order to appease his voters, and fellow politicians.[29] During the period 2014-2016 the UK was re-negotiating its terms of EU membership with EU legislators, however the EU did not want to give many concessions. In fact, Cameron went as far as to claim that if the EU was willing to give in to UK's demands for more control over migration, he could have avoided Brexit altogether.[30] This seems to suggest that Cameron was hoping his work would pay off, and he may be able to avoid the referendum. At the campaigning stage he could not have predicted whether his negotiation efforts would work. Furthermore, at the campaigning stage Cameron could neither predict the exact salience of the Brexit issue for the public, what the election outcome will be, and whether high salience would tie his hands after the election (due to the high cost of reneging on salient issues).

A key feature not mentioned is the unanticipated election results. In the 54th parliament no party received majority (Conservatives 47%, Labour 40%, Liberal Democrats 9%, and smaller parties), where Liberal Democrats and Labour opposed the referendum during the campaign.

[24]https://www.telegraph.co.uk/news/2018/04/18/david-cameron-says-doesnt-regret-eu-referendum/

[25]https://www.theguardian.com/politics/2011/oct/24/david-cameron-tory-rebellion-europe

[26]https://www.bbc.com/news/uk-politics-22382098

[27]https://www.theguardian.com/politics/2016/feb/07/british-euroscepticism-a-brief-history

[28]https://www.theguardian.com/politics/2016/jun/24/david-cameron-downfall-european-tragedy

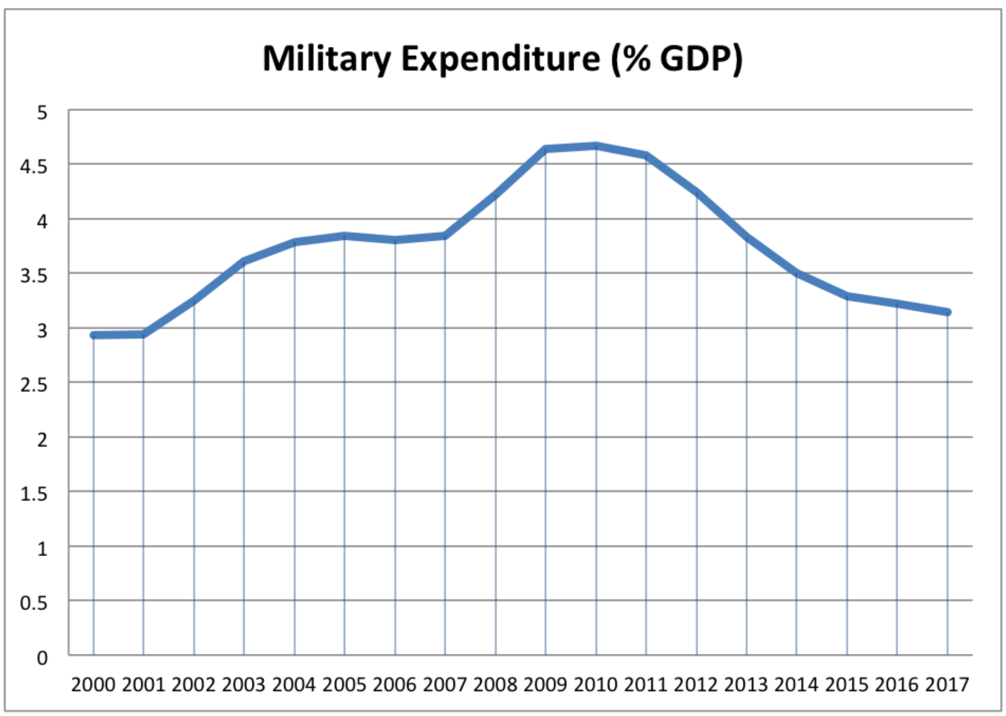
[29]https://www.theguardian.com/politics/2013/jan/23/david-cameron-uk-exemption-eu

[30]https://www.ft.com/content/3901dd48-3cee-11e6-9f2c-36b487ebd80a, https://www.theguardian.com/world/2016/jun/28/cameron-eu-leaders-uk-control-immigration

The 54th government was formed by a coalition between the Conservatives and the Liberal Democrats. The 55th parliament, elected in 2015, saw unexpected gains for the Conservatives, who managed to get majority (51%) and large losses for the Liberal Democrats (1%). The losses the Liberal Democrats experienced were somewhat expected (since they coalesced with the party their electorate significantly disliked), however the Conservative's big win was unexpected. It seems possible that at the time of making his promise, Cameron believed there existed a chance his party would again have to coalesce with the Liberal Democrats who would block the referendum from ever taking place. The unexpected election results meant he had to stick to his promise.

2.3 US President 2009-2017 - Barack Obama

One of Obama's 2008 presidential campaign pledges was: "Barack Obama supports plans to increase the size of the Army by 65,000 troops and the Marines by 27,000 troops. Increasing our end strength will help units retrain and re-equip properly between deployments and decrease the strain on military families."[31] At the time Obama was elected a plan was already in place to increase the size of the army. He simply promised that he would not obstruct this plan. He kept his promise, and did not obstruct the plan. The following data from the World Bank[32] show the military expenditure as a percentage of GDP for the US, over the period 2000-2017:



As the above chart clearly shows, during 2009-2011 there was a minimal fall, or stagnation, of the military budget. From 2011 onwards, there was a clear fall in military expenditure. The

[31]https://www.politifact.com/truth-o-meter/promises/obameter/promise/135/increase-the-size-of-the-army-and-marine-corps/

[32]https://data.worldbank.org/indicator/MS.MIL.XPND.GD.ZS?end=2017&locations=US&start=2000&view=chart

main reason for this drop is that Obama was trying to pull the military out of Iraq, and sequestration.[33] The fact that throughout Obama's presidency sequestration did not stop, and military expenditure continued to fall, suggests that Obama's underlying preference was to curtail US military expenditure. This suggests that Obama inherited wars from previous presidents, and had to fulfill plans that were already in action. During his first term he seems to have wanted to quickly increase presence in Afghanistan, so he can get troops out quicker.[34] This could explain the sustained high numbers, while his preference could explain the later fall. Even though the military is a salient issue in the US, at the campaign stage Obama could not have predicted (with certainty) whether it would remain a very salient issue. In particular, a big win in Afghanistan would have allowed him to renege on this promise. Nevertheless, the big win never happened, and he had to keep his promise.

If Obama broke his promise, he would have to endure the cost of reneging, and may not have been re-elected (like George Bush Sr. wasn't due to his lie: "Read my lips: no new taxes"). He could not have anticipated the results of the efforts in Afghanistan, and even though he wanted to decrease the military, he could not have stopped the plan already in place since Afghanistan did not work out and the salience of the issue remained very high. This paper argues that unanticipated (at the campaign stage) reasons force politicians to keep promises that they would prefer to break.

3 Literature Review

This paper is related to several strands of literature: spatial competition, endogenous entry and strategic information transmission. A more extensive review of some of the related theoretical literature can be found in Appendix F.

Spatial competition has initially been researched with respect to economic competition in industrial organization, where firms compete by positioning at some location (or product differentiation etc.), before the methods were applied to spatial electoral competition. Even though the two are not completely equal, economic spatial competition models have valuable insights for electoral spatial competition. Entry was then endogeneized in these spatial competition models. The citizen-candidate model captures spatial competition and endogenous entry. Since the current model modified the citizen-candidate framework, I will cover the basic setup of the original citizen-candidate model. The information transmission literature developed simultaneously as a separate strand of the literature.

The seminal papers of two firm/party spatial competition are [Hotelling, 1929] and [Downs, 1957], which sprouted a research agenda in politics and political economics about spatial electoral competition. The spatial competition among firms was then generalized to n firms in [Eaton and Lipsey, 1975], and among politicians by [Cox, 1987]. Due to the strategic nature of messages, electoral competition has a similar effect on messages as in these seminal papers. Therefore, some results in this paper are similar to the results in [Cox, 1987]. When moving from two to three candidate situations, note that the third candidate may want to enter strategically (i.e. not for winning, but making his preferred candidate win). Some of the important early articles

[33] https://www.politifact.com/truth-o-meter/article/2015/dec/14/politifact-sheet-our-guide-to-military-spending-/

[34] https://www.nytimes.com/2009/12/06/world/asia/06reconstruct.html

that considered the strategic entry incentives of a third, or more, candidates are [Palfrey, 1984, Osborne, 1993, Feddersen et al., 1990]. Note that the equilibrium in [Feddersen et al., 1990] has all candidates entering at the median. This is supported by beliefs that a message different from the median will result in all of the voters coordinating on a single candidate using the median message. This is different from the usual assumption that voters equally share their votes among all of the candidates using the median message (pooling). The current model explicitly treats both assumptions (i.e. tie-breaking rules), however derives more results for the case similar to the beliefs in [Feddersen et al., 1990].

This paper extends the citizen-candidate framework by allowing candidates to run with messages that differ from their preferences, where political competitors are completely informed while voters are naive and trust the campaign promises. Since campaign promises are messages, the paper is related to the strategic information transmission literature. The seminal paper on information transmission is [Crawford and Sobel, 1982] which showed that, in sender-receiver games, messages can range from completely precise (receiver knows exactly what the sender will do), to completely imprecise ("cheap talk" - the receiver gets no information from the sender message). The most direct experimental test of this paper is [Cai and Wang, 2006], who find that senders send more than they should and receivers trust senders more than they should. This intuition may be important for lying in a political game; i.e. voters may often trust politicians more than they should, and the politicians may oversignal. The information transmission literature has been extended in many directions, where several articles treat the topic of lying, such as [Kartik, 2009, Callander and Wilkie, 2007]. A related model looks at the effect politician character has, [Kartik and McAfee, 2007].

The series of papers, which the authors recently tested in the lab [Grosser and Palfrey, 2008], [Grosser and Palfrey, 2011], [Grosser and Palfrey, 2014] and [Grosser and Palfrey, 2017], is closest to the current model. They write down a citizen candidate model in an imperfect information environment. Even though these are similar to the current paper, they take the dynamic inconsistency problem as given and do not allow for any credibility of candidate messages. More precisely, in the policy implementation stage the winner implements his own ideal point, while in the policy promise stage the announcements are cheap talk (completely uninformative), therefore, in the voting stage voters are indifferent among all candidates, since they can not tell who is a good and who is a bad type. This results in no pure strategy equilibria being possible, i.e. only mixed strategy equilibria are possible (precisely, they look at equilibria in symmetric cutpoint strategies), and therefore this sequence of papers significantly differs to the current paper. The current model exclusively focuses on pure strategy Nash equilibria.

3.1 Citizen Candidate Model

This paper directly extends the citizen candidate model of [Osborne and Slivinski, 1996] by allowing candidates to credibly run with messages. Therefore, we will review this citizen-candidate (CC) framework. The economy is composed of a continuum of citizens with single peaked preferences, which have some intrinsic preference (x) and they decide whether to become candidates at a cost C. The citizens that enter do so with their ideal policy. Once the entry decisions are made, the citizens vote sincerely amongst the candidates.[35] The winner depends on the electoral system in place; the authors consider plurality rule[36] and a runoff system[37]. Sincere voting means that citizens vote for the candidate that is closest to their own preference. Precisely, let $\mathcal{C} = \{c_1, c_2, \ldots, c_N\}$ be the set of N candidates each using a corresponding policy $Z = \{z_1, z_2, \ldots, z_N\}$, then the voter i (with voting function v_i) votes for the candidate that minimizes the distance of his preferred policy (x_i) from all of the candidate policies, $v_i = \{c_j \in \mathcal{C} | \arg\max_{\{z_j \in Z\}} \{-|z - x|\}$.

The winner gets a benefit (B) and implements his preferred policy as the winning policy, $w = x$. This rests on the assumption that the citizen-candidate literature seems to have internalized: voters believe that once a candidate wins and gets into office, he will implement his preferred policy. This dynamic consistency was introduced into the political economics literature by [Alesina, 1988], who looked at the questions: if voters are rational and forward looking, while politicians can misrepresent their true preferences and run with lies, will any messages of the parties be credible? In one-shot interactions politicians cannot credibly commit to their promises, therefore the only possible equilibrium promise, consistent with sequentially rational forward looking voters, is for the candidate to run truthfully.[38] The major departure the current model takes from the citizen-candidate literature is that I assume that politicians can credibly commit to some of their promises.

Returning to the original CC model, the set of policy positions is $\mathbb{R}$, and F is the distribution function of the citizens ideal positions on $\mathbb{R}$ that has a unique median, M. Let the set of ideal policies be $\mathcal{X} = [X_{\min}, X_{\max}]$, so that F has full support on $\mathcal{X}$. The exogenous benefit of winning and cost of entry is assumed positive, $B, C > 0$. If multiple citizens get the same number of votes, a random candidate amongst these is chosen as the winner (the *tie-breaking rule*). If no candidate exists ($\mathcal{C} = \emptyset$) everyone gets utility $-\infty$, i.e. anarchy is infinitely costly. This is a two stage simultaneous entry game, where the basic timing is:

1. Citizens decide whether to enter (E), i.e. become condidates, or not enter (N_e).
2. An election is held, where all[39] citizens sincerely vote for their preferred candidate. The election winner implements his prefferred policy, w.

We can now summarize the preferences in the CC model when there is a single (realized) winner. Let $a_i^e \in \{E, N_e\}$ be the entry decision citizen i takes, and W the set of winners, w is the winning

[35]It is irrelevant whether the same citizens decide whether to become candidates and vote, or if it is two distinct populations with the distribution of ideal points having the same support, because the entrants prefer their own policy the most, and no matter how many entrants, each votes for himself in equilibrium. This is not true in my model.

[36]"Under plurality rule the winner of the election is the candidate who obtains the most votes. If two or more candidates tie for first place, then each wins with equal probability." (CC pg 67)

[37]"If some candidate obtains a majority (more than half the votes), then she is the winner. If no candidate obtains a majority, then the winner is the candidate who obtains a majority in a second election betweeen the two candidates who obtained the most vote in the first round. In both cases ties are dealt with via an equal-probability rule." (CC pg 67)

[38]A final point to note about dynamic inconsistency, is that Alesina identifies "individually rational" conditions, which make the policy proposals credible. Essentially, individual rationality requires the benefits from the policy proposal to outweigh the cost of stating it, making the proposal individually rational. We will see that the equilibrium definition of the current paper has individual rationality constraints.

[39]There is no abstention.

policy, and $x_i \in W \implies x_i = w$:

$$U_i = \begin{cases} \mathbb{1}_{\{a_i^e = E\}} \left(-C + \mathbb{1}_{\{x_i = W\}} B\right) - |w - x_i| & \text{if } \#\mathcal{C} > 0 \\ -\infty & \text{if } \#\mathcal{C} = 0 \end{cases} \tag{1}$$

where $\mathbb{1}$ is the indicator function. In order to enter one has to pay a cost of C, therefore a winning candidate get utility $B - C$ (since $x_i = w \implies |w - x_i| = 0$), a losing candidate $-C - |w - x_i|$, a non-candidate $-|w - x_i|$, and anarchy is infinitely costly. To summarize, the set of players is the set of citizens' $\{\mathcal{X}\}$ ideal positions, the set of actions of each player are $\{E, N_e\}$, and the preferences are specified by the utility function. Note that the second stage is mechanical, i.e. voters sincerely vote for their preferred candidate and the candidates run honestly.

There is another citizen-candidate model written around the same time, [Besley and Coate, 1997]. It is more general, where voters vote strategically. Their model allows for a multidimensional issue space, a discrete number of citizens, utility functions dependent on the policy variable *and* policymaker identity, heterogeneity in policy implementation (which makes candidate identity important beyond the identity itself), sophisticated voting, and a default policy if no one runs. The problem at hand is complex enough, therefore the simpler CC framework seems appropriate to study lying in the CC environment. Furthermore, the [Osborne and Slivinski, 1996] framework imposes functional forms (Euclidean distance $|w - x|$) that seem empirically relevant, and hence more directly testable in the lab.

4 Campaign promises in the Citizen-Candidate framework

4.1 Politicians run with messages

In the original citizen-candidate model each candidate runs with (and implements) their true underlying preference.[40] It seems unlikely that all candidates run with their true preferences.[41] Furthermore, such an assumption is unable to explain two important points: 1) politicians may be strategic with their promises, and 2) campaign promises can be broken. A strategic politician uses promises to capture votes within a competitive environment; this incentive is absent in the original model. Furthermore, the only way the original model could explain a difference between candidate promises and the policy implemented, is that the true preference of the winner changed. This is also present in *the politician-candidate model*. However, this is not explicitly treated in either model.

In order to allow promise breaking I allow the politician to run with a message (m), rather than their true underlying preference (x). The message has the same support as the possible preferences, $x, m \in \mathcal{X}$. Like in the original citizen candidate model, voters vote sincerely, however they only observe the promises made by the candidates and make no inferences about the identity of

[40]Because the equilibria are dynamically consistent. The equilibria in my model are not dynamically consistent, because voters are assumed to be naive. The only dynamically consistent configurations are where all running candidates make honest promises.

[41]One could argue that the underlying preference of the citizen-candidate takes the party and public into account, however the interpretation of the model becomes less clear. For example, do citizens and candidates come from the same pool?

those candidates.[42] That is, the voters are *naive* and sincerely vote according to the promises candidates make. The assumption of non-strategic voters seems to find some experimental support. It may be caused by too much retrospective voting.

4.2 Potential entrants are politicians, different from voters

The implication of this *naivete* is stronger than it appears. The solution concept used in the citizen-candidate model is the complete information pure strategy Nash, where all citizens know the identities and preferences of everyone else. In equilibrium everything is anticipated, and no one has a unilateral profitable deviation. Since the candidates run truthfully, sincere voting is sequentially rational in equilibrium.[43] In my model, this fails, as *naive* sincere voting is not rational when promises can be broken. Therefore, I assume that the pool of potential candidates is different from the pool of voters. Politicians are the subset of the population who find the strategic calculations involved in the electoral race, the easiest. That is, politicians form a measure zero subset of the population, and they form a representative subset of the voters.

The model assumes politicians have complete information. The voters are not players in the entry game, they are simply mechanical voting machines, only politicians know the identity and preferences of each candidate. This circumvents the issue of rationality, since the voters do not have complete information, unlike the potential candidates. Due to the fact that voters and potential candidates are separate populations, where potential candidates have strategic information and voters do not, the interpretation of the model changes from a citizen-candidate to a *politician-candidate* model. This paper focuses purely on the behavior of the measure zero set of politicians, that is it looks at the *politicians* (citizens with highest strategic potential) incentives to enter the election, given their complete information about other politician preferences, and the voting rule of naive voters.

This setup analyzes the incentive for promise breaking and for self-selection into the electoral race: a politician can misrepresent their true preference (i.e. $m \neq x$) and then try and break his promise, or make genuine promises (i.e. $m = x$).[44] In other words, a candidate lies if he proposes a message that is different from his true preference. The cost of lying is described by some function $f : (x, m) \mapsto \mathbb{R}$. Following [Osborne and Slivinski, 1996] I use the absolute value metric to determine the cost of lying, i.e. it is linear in the distance between the candidates ideal and promise (for $m, x \in \mathbb{R}$ the distance is $f(m, x) = |m - x|$).[45]

[42] This seems to be in line with the experimental evidence that voters use heuristics rather than complicated rational calculus.

[43] Policy proposed equals the policy implemented, and voters know that. That is, the policies are dynamically consistent so the equilibria satisfy rationality.

[44] The model does not explicitly treat what happens when an event changes the underlying preference, or optimal policy.

[45] Let $d = m - x$, then the derivative of $f(d) = |m - x|$ is $f' = 1$ for $d > 0$, $f' = -1$ for $d < 0$, and f' not defined for $d = 0$. For $d \in \mathbb{R}$ we can rewrite $f(d) = |d|$ as $f(d) = \sqrt{d^2}$, then differentiate to $f'(d) = \frac{d}{\sqrt{d^2}} = \frac{d}{|d|}$. We can also think of this intuitively, as d goes from $-\infty$ to 0 the function is decreasing by 1, and as d goes from 0 to ∞ is increasing by 1. At $d = 0$ the derivative does not exist, because the function has a kink (therefore f' has a jump at $d = 0$).

4.3 Salience

Making promises that are not in line with true preferences (which implies that the politician wants to renege) is costlier the more salient the issue. Formally, I introduce an exogenous probability with which the candidates will be forced to implement the promise they make, $s = 1-\beta \in (0,1)$. Precisely, the random variable determining whether the politician will be able to renege on his promise is Bernoulli distributed with parameter s. That is, the event "the issue became too salient to renege on the promise" is when the s-biased coin toss returns 1 (rather than 0). Therefore, at the campaigning stage the bias of the coin toss (s) is the *belief* about how likely it is that, after the winner is picked, any promise about the issue (being voted on) becomes overly salient. This means that the candidate expects to implement his preferred policy x with probability β instead of the message they ran with (i.e. the winner can get away with a lie a β fraction of times), and expects to be forced to implement own promise m with probability $s = (1-\beta)$ (i.e. the winner is forced to implement their promise a s fraction of times). Intuitively, if $s = 0$ the issue is non-salient[46] so individuals will never be forced to implement their promises (lying is free), while if $s = 1$ the issue is so salient each politician knows he will have to keep his promise. Nevertheless, if $s \in \{0, 1\}$ the candidate would certainly know what will occur, so this will not be treated.

The fact that candidates run with messages, instead of their ideal preference, and that promise-keeping will be forced sometimes where all politicians share the same belief about how likely their promise will have to be kept, implies that this model significantly departs from the original citizen candidate model. In either limit $s = \{0, 1\}$ the model does not collapse to the original citizen-candidate model, because candidates use promises strategically, and voters trust them. One would have to remove this to obtain the original citizen-candidate model.

Since s is a belief politicians hold when campaigning, which they use to calculate own expected utility from entry or non-entry, it enters the equilibrium conditions as a parameter. This implies that one could also treat salience as a behavioral parameter that determines the exact mix the candidate decides to run with. This behavioral parameter interpretation seems in line with the intrinsic cost of promise-breaking some experiments identified. Nevertheless, since changes in the salience of issues may play an important role in what promises candidates end up keeping, this paper follows the stochastic interpretation.

Having a stochastic element determining whether the message will be implemented is a simplifying assumption that intuitively captures salience, which determines the cost of lying for the candidates. This framework captures two realistic features of the political process. First, politicians may *run* with a message different from their true preference, using their promises strategically. Second, politicians in office may not *implement* their ideal policies. The citizen candidate utility from entry (equation 1) then becomes:

$$U_i(\text{entry}) = \mathbb{1}_{\{a_i^e = E\}} \left(-C + \mathbb{1}_{\{x_i = W\}} B\right) - (1-s)|x^* - x| - s|m^* - x|$$

where the starred variables are the policy preference and message of the single winner. Note that with probability β the winning candidate will implement his true preference, i.e. $-\beta|x^* - x| = 0$. Recall that $s = 1-\beta$. I will use these interchangeably.

[46] Alternative, one can suppose that when $\beta = 1$ the institutional framework is so weak that all politicians know nothing can force them to implement their message, while if $\beta = 0$ the institutional framework is so strong that every citizen is aware that whatever message they propose they will also be forced to implement it.

The entire structure is common knowledge, i.e. all politicians know each others preferences (and possible messages), and know the others know it, etc. Therefore, as we shall see, the messages used by candidates in equilibrium will be determined by electoral competition, while self-selection into candidacy will be determined by the exogenously imposed institutional structure (s, m, x, B, C). Note that the second stage of the game is unchanged for voters who now vote sincerely (mechanically) and naively. That is, voters blindly believe the politicians' statements. This could be extended to incorporate informed voters, however the purpose of this paper is to look at candidate self-selection given that all politicians know each one can lie, and voters believe the promises.

5 The *Politician-Candidate* Model

I modify the citizen candidate model in three ways. Potential candidates are a measure zero set covering all possible voter preferences; candidates run with messages that voters find credible; and potential candidates share the belief that with probability $s \in (0,1)$ the election winner has to keep his promise (with complementary probability can renege to own ideal). The potential candidate knows s while the voter trusts the promises (believes $s = 1$) and votes for the candidate whose promise is closest to himself.

5.1 Setup

Suppose there exists a continuum of politicians, each with single-peaked preferences on the set of *policy positions* $\mathcal{X}$, for simplicity let $\mathcal{X} = [0,1]$.[47] Let x_i be politician i's peak preference, and let this be their *identity* or *preference*. Let the distribution of peak preferences on $\mathcal{X}$ be $F \sim$ Uniform $[0,1]$. There is a benefit to winning and a cost of entry. Suppose both are positive, $(B, C > 0)$. An election is held, under plurality rule, with a single winner. The potential candidates are *politicians*, and they run by making campaign promises $m \in \mathcal{X}$. One can interpret this as a promise about their identity. Note that $x, m \in \mathcal{X}$. Finally, note that voters are *naive* (i.e. non-strategic): they believe the candidates promises and sincerely vote for the candidate whose promise is closest to their identity. Suppose the distribution of voter peak preferences is the same as for the politicians, i.e. Uniform $[0,1]$. More precisely, the population of politicians and voters, their identities, is described by a cumulative distribution function $F_k(x) = \int_0^x f_k(x)dx$ over all of the possible identities, $x \in [0,1]$, where $f_k(x)$ is the probability density function, $F_k[1] = 1$, $F_k[0] = 0$, and $k \in \{\text{politician, voter}\}$. Furthermore, let the random variable I_k, describing the population identities (mapping from an abstract set $\bar{\mathcal{X}}$ to the measurable space $\mathcal{X} = [0,1]$), be uniformly distributed on $[0,1]$, that is $f_k(x) = 1$ for $0 \le x \le 1$ and $f_k(x) = 0$ otherwise.

Politicians need to choose whether to enter the political race (i.e. become candidates), $a_i^e \in \{E, N_e\}$, by paying a cost of entry C, and what message (i.e. campaign promise) to run with,

[47] There is a common ordering defined by the inequality $<$ on the policy domain $[0,1]$. The preference order $\succeq_i$ for agent i is single-peaked iff: i) $\forall a, b, x_i^* \in [0,1]$ with $a < b \le x_i^*$ we have $b \succeq_i a$, and ii) $\forall a, b, x_i^* \in [0,1]$ with $x_i^* \le a < b$ we have $a \succeq_i b$, where x_i^* is the unique peak preference of agent i. This simply ensures that the further a policy from the most preferred policy, the less it is preferred, and that there is a unique most preferred policy.

$m_i \in [0,1]$. That is, each candidate i runs with a promise m_i about their identity.[48] Note that politicians are a measure zero subset of the entire population (voters and politicians), therefore their voting decisions are irrelevant. Let $\mathcal{C} = \{c_1, c_2, \ldots, c_N\}$ be the set of N candidates each stating a corresponding promise $Z = \{z_1, z_2, \ldots, z_N\}$, then voter i (with voting function v_i) votes for the candidate that minimizes the distance to his preferred policy (x_i^v), i.e. $v_i = \{c_j \in \mathcal{C} | \arg\max_{\{z_j \in Z\}} \{-|z_j - x_i^v|\}\}$. Voters are mechanical and *naively* vote for the candidate whose promise is closest to their own identity.

The model can be described as a three stage game.[49] *Politicians* (i.e. potential candidates) decide whether to enter, or not enter; candidates pick the message to run with; and then a vote is held. For simplicity we will consider *Plurality Rule*, i.e. whichever candidate gets the most votes wins, and ties are broken by a coin flip. In the first stage (*Candidacy stage*), all politicians $x_i \in \mathcal{X}$ decide whether to enter or not, i.e. $a_i^e \in \{E, N_e\}$. In the second stage (*Message stage*) all candidates $c_i \in \mathcal{C}$ state their message $m_i \in \mathcal{X}$ simultaneously without randomizing their statements. Note that the set of candidates $\mathcal{C} \subseteq \mathcal{X}$ is composed of the identities of the entrants, i.e. $\mathcal{C} = \{c \in \mathcal{X} | c = x_i \text{ where } a_i^e = E\}$. In the final stage (*Voting stage*), each voter j sincerely votes for one of the candidates $v_j = \{c_i \in \mathcal{C}_N | \arg\max_{\{m_i \in \mathcal{C}_N\}} \{-|m_i - x_j|\}$ where $\mathcal{C}_N = \{(c_1, m_1), (c_2, m_2), \ldots (c_N, m_N)\}$ is the extended candidate set, and a single winner with the most votes is picked. If multiple candidates get the same number of votes a coin flip determines the winner, i.e. if K individuals have the same number of votes each of these candidates has $P(win) = \frac{1}{K}$. Each citizen votes sincerely for the candidate whose *message* is closest to their own ideal policy. To complete the setup the *indifference assumption*: if a citizen is indifferent between entering or not, let him enter. The politician (U_i^p) and voter (U_i^v) preferences in this model are as follows:

$$
\begin{aligned}
U_i^p &= \begin{cases} \mathbb{1}_{\{a_i^e = E\}} \left(-C + \mathbb{1}_{\{x_i = W\}} B\right) - \underbrace{(1-s)|x^* - x_i|}_{\text{Identity Cost}} - \underbrace{s|m^* - x_i|}_{\text{Message Cost}} & \text{if } \#\mathcal{C} > 0 \\ -\infty & \text{if } \#\mathcal{C} = 0 \end{cases} \\
U_i^v &= -(1-s)|x^* - x_i| - s|m^* - x_i|
\end{aligned}
$$

where x^* is the underlying preference of the winning candidate, and m^* is the message he won with. Furthermore, as described in the previous section on salience, $s \equiv (1-\beta) \in (0,1)$ is a Bernoulli distributed random variable describing the belief politicians have prior to making a promise. That is, if s realizes to 1 then the issue became overly salient and the winner cannot renege on his message. In the original citizen-candidate model the winning identity (x^*) and the number of entrants ($\#\mathcal{C}$) are the only endogenous parameters determined in equilibrium. In addition to x^* and $\#\mathcal{C}$, the winning message (m^*) is also endogenous in the politician-candidate model. The exogenous parameters of the citizen-candidate model are B, C, x, while the current framework additionally has salience s.

Note that non-entrant politicians get $-(1-s)|x^* - x_i| - s|m^* - x_i|$, the winner gets $B - C - s|m^* - x_i|$, and losers get $-C - (1-s)|x^* - x| - s|m^* - x_i|$. Note that the additional message cost

[48] Note that, as in the original citizen-candidate model, this two stage structure can be reduced to a single stage if the action space is transformed to $\{E \times [0,1], N_e\}$.

[49] The first two stages could be collapsed into a single stage where each politician needs to decide both whether to enter, and what message to use ($a_i^e \in \{E \times \mathcal{X}, N_e\}$), however I believe splitting it into two stages facilitates understanding and intuition.

$-s|m^* - x_i|$ is present in any realization of the utility function, even for winners. The message cost is zero for honest winners, i.e. $(m^* = c^*)$ where c^* is the winning candidate, and for non entrants j whose preferences are in line with the winners message $(m^* = x)$. Furthermore, note that the non entrant politicians and voters utility is maximized when their identity is in line with an honest winner. Since the focus of the article is on the strategic behavior of politicians, U_i will refer to the utility of politician i.

5.2 Equilibrium

The solution concept of this three stage game is the *Subgame Perfect Nash Equilibrium.* Since the Voting stage is mechanical, the solution concept in the Message stage is the *simultaneous pure strategy Nash Equilibrium*, which drives the entry decisions through backward induction. That is, politicians simultaneously[50] choose their entry decisions given equilibrium behavior in the Voting (mechanical) and Message stages. The simultaneous Nash Equilibrium seems like the correct solution concept for this type of political game, as there is a very large number of individuals who may consider entry before elections, and they try to anticipate the behavior of other potential candidates. Only some combinations of candidate identities and messages, *configurations*, will be possible for a combination of model primitives, therefore, we will first look at comparative statics to better understand the politicians incentives. The drawback of this setup with simultaneous message revelation is that it does not allow for incumbency, which a sequential setup could allow; and that in reality the announcement of multiple candidates is rarely truly simultaneous.

Let $\mathcal{C}_N = \{(c_1, m_1), (c_2, m_2), \ldots, (c_N, m_N)\}$ be the configuration of N candidates and their messages, and let $\mathbb{E}_{x_i}(a_i^e|\tilde{\mathcal{C}}_{N-1})$ denote the ex-ante[51] expected value candidate x_i gets from the action $a_i^e \in \{E, N_e\}$ when the configuration of candidates is $\tilde{\mathcal{C}}_{N,i} := \mathcal{C}_N \setminus \{(c_i, m_i)\}$.

Theorem 1. *[Equilibrium] Necessary and sufficient conditions for configuration $\mathcal{C}_N$ to form an Subgame Perfect Nash Equilibrium:*

1. *Each candidate prefers entry, over non-entry ("Entry Condition")*

$$\mathbb{E}_{x_i}(E|\mathcal{C}_N) \geq \mathbb{E}_{x_i}(N_e|\tilde{\mathcal{C}}_{N,i}\}), \quad \forall c_i \in \mathcal{C}_N \tag{2}$$

2. *Every x_{N+1} citizen prefers non-entry, over entry ("Challenger Non-Entry Condition")*

$$\mathbb{E}_{x_{N+1}}(E|\mathcal{C}_{N+1}) < \mathbb{E}_{x_{N+1}}(NE|\tilde{\mathcal{C}}_N), \quad \forall x_{N+1} \in \mathcal{X} \tag{3}$$

3. *No candidate prefers unilateraly deviating to another message ("Individual Rationality")*

$$\mathbb{E}_{x_i}(E|\mathcal{C}_N, m_i) \geq \mathbb{E}_{x_i}(E|\mathcal{C}'_N, m'_i)\ \forall m'_i \neq m_i \tag{4}$$

Proof. See Appendix. □

[50] At the Candidacy stage the solution concept is again the simultanous Nash Equilibrium, conditional on the next stage choices.

[51] Note that when defining the utility function, we defined them ex-post, when a single winner was already realized.

Condition one and two are the same as the equilibrium conditions in the original citizen-candidate model, however condition three is an additional equilibrium requirement of this modified framework.[52] As we will see, the fact that messages have to be in equilibrium will greatly reduce the space within which equilibria exist.

Proposition 2. *A political equilibrium in pure strategies exists*

As in the original citizen-candidate model, a political equilibrium exists. The proof is trivial: since anarchy is infinitely costly one is always willing to enter, and the net benefits (B, C) can be made negative enough so no second candidate is willing to enter. Therefore, we certainly know that one candidate equilibria exist. This model will only consider pure strategies, however it can easily be extended to mixed strategies.

Lemma 3. *In equilibrium*

- *with strategic entry, all entrants who have a positive probability of winning have the same probability of winning*
- *without strategic entry, each candidate has the same probability of winning.*

Proof. See Appendix □

The main intuition is that if there are no strategic entrants, all candidates enter iff they have a chance of winning. The only way several candidates can have a chance of winning is if they all get the same number of votes, and hence have the same probability of winning. Strategic entrants certainly lose, therefore the non-strategic candidates receive the same number of votes in equilibrium. This model considers only office motivate politicians, and thereby ignores strategic entry.

5.3 Notation

Before looking at specific equilibria it is useful to define some more notation. Let the probability candidate i wins with his message, given that there are N candidates, be $P_i^{\mathbf{N}} = P((x_i, m_i) \text{ wins} | x_i \in \mathcal{C}_N)$. Note that $P_i^{\mathbf{N}}$ is completely driven by the number of votes each candidate receives and the number of candidates (entrants). Let $W(\mathcal{C}_N, V(\mathcal{C}_N, M))$ be the set of potential winners, where $V(\mathcal{C}_N, M)$ are the votes received by each candidate given the candidate identities $\mathcal{C}_N$ and message M configuration, and where each candidate $c \in W$ has a positive probability of winning. Proposition 3 implies that each $c \in W$ obtains the same number of votes, so each has an equal chance of winning, that is, for each $c_i \in W \implies P(c_i \text{ wins}) = \frac{1}{\#W}$. In other words, in equilibrium $P_i^{\mathbf{N}}(W)$ is completely driven by the amount of candidates who have a positive probability of winning.

Let $V_i(\mathcal{C}_N)$ be a function defining the number of votes candidate c_i receives given that N candidates entered. When $N > 1$, let the candidates be in ascending order according to their

[52] In his paper on dynamic inconsistency [Alesina, 1988] identifies a similar individual rationality requirement, as imposed in this model.

identities $c_i \leq c_j$, $\forall i, j \in \{1, 2, \ldots, N\}$ such that $i < j$. Finally, suppose $m_i \leq m_j$. Then we know that $V_1(\mathcal{C}_1) = 1$ and when $N > 1$

$$V_i(\mathcal{C}_N) = \begin{cases} \frac{m_1+m_2}{2} & \text{if } i = 1 \\ 1 - \frac{m_{N-1}+m_N}{2} & \text{if } i = N \\ \frac{m_{i+1}+m_i}{2} - \frac{m_{i-1}+m_i}{2} = \frac{m_{i+1}-m_{i-1}}{2} & \text{otherwise} \end{cases}$$

where the fact that votes are split in between the candidate messages is a result of the uniform distribution of voter ideals, and the fact that voters have single-peaked preferences, vote sincerely and naively. Let the profile of candidate vote shares be $V = \{V_1, \ldots, V_N\}$, where each vote share is driven by the message configuration (let M be the message configuration of the candidates).

6 Results

The paper proceed by looking at 1-,2-,3-,4- and N-Candidate equilibria, where the 1- and 2-Candidate cases are analyzed in most detail. All proofs will be relegated to the appendix. Since there are many results, before proceeding to the technical section, it is useful to briefly summarize all of the main results of the paper.

1-Candidate equilibria: The 1-Candidate case is specific due to the assumption that anarchy is infinitely costly, making the entry condition meaningless. That is, the equilibrium configuration (i.e. candidate identity and promise) is completely determined by the non-entry condition, which makes sure that no second candidate is willing to enter, given some (x_1, m_1). Furthermore, since the candidate is running unopposed, in equilibrium no candidate is willing to incur the cost of lying, that is, each candidate runs truthfully. For fixed B, C, an increase in salience increases the amount of possible equilibrium entrant identities. Finally, for the smallest benefits, relative to costs, everyone is willing to run unopposed. As benefits (B) increase, the most extreme entrants ($\{0, 1\}$) are the first ones unable to run unopposed (truthfully), as the benefits keep increasing (for fixed costs C) the set of potential 1-Candidate equilibrium entrants is shrinking symmetrically from the extremes. In fact, for the highest possible relative benefits, which allow for 1-Candidate equilibria to exist, only the median politician is able to run unopposed. The reason for this is intuitive. The challenger (second candidate) who loses the most from non-entry (i.e. *worst challenger*) is an extremist. If the worst challenger is unwilling to enter, we know no other politician will be willing to enter. The worst challenger (ie. the most extreme politician) loses the most from an opposite extremist running (honestly) unopposed, while he loses the least from letting the median candidate run (honestly) unopposed. Note however, that when the only possible 1-Candidate configuration is where the median runs honestly, 2-Candidate equilibria also exist. That is, there is an overlap of equilibria.

2-Candidate equilibria: In the 2-Candidate case all equilibrium conditions are non-trivial. The 2-Candidate equilibrium entry condition is the reverse of the 1-Candidate equilibrium non-entry condition, which drives the fact that there is 1- and 2-Candidate equilibrium overlaps. This same holds true for any N- and $(N + 1)$-Candidate equilibria. Unlike in the citizen-candidate model, the only message configuration that survives individual rationality is both candidates running with the median message. This gives rise to the issue I call the *tie-breaking rule*. If two candidates run with the same message, do they both anticipate to get the same number of votes (*expected votes* case), or does each one of them anticipate to get all of the votes with probability half (*lucky votes* case)? Both of these possibilities result in each of the two entrants winning with probability half, however they have very different equilibrium implications. In fact, there exist no 2-Candidate equilibria in the expected votes case, because a third challenger can enter and certainly win.[53] Equilibria are much easier to sustain in the lucky votes case, since no challenger can enter and certainly win. As the issue becomes very salient, any identities will be able to form 2-Candidate equilibrium configurations. This occurs since all candidates run with the median promise in equilibrium, the challenger also needs to use it (in lucky votes case), therefore the message cost becomes irrelevant for determining the equilibrium identity configurations (which is purely driven by the identity cost). As salience increases, in the limit $s \to 1$ the identity cost becomes negligible. Finally, in equilibrium the further the candidates are from one another, the lower relative[54] benefits they require to be willing to form equilibrium configurations. This occurs for two reasons, two candidates that are far apart lose more by exiting and letting their opponent certainly win, than if the candidates are near in identity. Furthermore, candidates symmetrically opposed around the median form the least costly identity configuration for challengers. That is, the worst challenger (the challenger who requires the least benefits to enter as a third candidate, i.e. an extremist) loses the least by letting two symmetrically opposed candidates around the median (or two median entrants) run unopposed, and loses the most from two same extremists running together. This happens because the worst challenger is an extremist, and symmetrically opposed candidates minimize the average distance from the worst challenger. As we can see, the equilibrium configuration in the 2-Candidate case is driven by the entry force (what the two candidates get by entry vs. non-entry), the non-entry force (what any possible challenger gets from entry vs. non-entry), and by individual rationality (that all candidates need to use the median message). The final result of this section foreshadows the main result of the paper. Out of all possible equilibrium configurations, the configuration with two honest entrants ($x_i = m_i = 0.5$ where $i = 1, 2$) is a singleton, while there exists a continuum of configurations with dishonest entrants. That is, the probability of randomly picking the equilibrium configuration with two honest entrants (out of all the equilibrium configurations possible for any model primitives) is (almost) zero. Finally, note that the median message is optimal for voters, as it minimizes the total message cost to all of the voters. Furthermore, given that there is only a single winner, from an ex-post perspective it is optimal for voters that the two entrants in 2-Candidate equilibria have the median identity and enter honestly. This ensures that the winner will be the candidate minimizing the (message and identity) cost for all of the voters. In fact, this holds true for all equilibria, that is, the best possible outcome for voters is to have N honest entrants all running with the median message, because that makes sure the winner will be the median citizen running honestly.

3-Candidate equilibria: There are no 3-Candidate equilibria in the *expected votes* case, since there exists no message that would satisfy individual rationality. The only possible message

[53] And due to the simplifying assumptions: $\mathcal{X} = [0, 1]$, the cost of lying is linear in distance, and $B, C > 0$. For example, 2-Candidate equilibria are possible in the expected votes case if $B < 0$.

[54] To costs

configuration in the *lucky votes* case is where all three candidates promise the median. As in the 2-Candidate case, the more extreme the candidates are from one another the higher the cost of exit for each of the three entrants; and the further the average distance of the three candidates from the extremes the higher the cost of non-entry for challengers. Finally, given all candidates pool, salience has the same effect as in the 2-Candidate case, that is, as salience increases any identities will be able to form 3-Candidate equilibria.

4-Candidate equilibria: The only possible equilibrium message configuration in the *expected votes* case is where two candidates promise 0.25 and the other two promise 0.75. This demonstrates an important electoral competition force: when candidates pool in groups of two, and split the votes equally, any deviation is disciplined by the other candidate in the group. That is, if one candidate deviated from 0.25 to $0.25 \pm \epsilon$, his closest competitor stating 0.25 would get $0.25 + 0.5\epsilon$ and win. Furthermore, note that a fifth challenger can enter with $m_5 \in (0.25, 0.75)$ and certainly win. This leads to the result that no 4-Candidate equilibria exist in the expected votes case. In the *lucky votes* case, the semi-separating (at 0.25 and 0.75) message configuration remains a possibility, however pooling at the median is an additional equilibrium message configuration. Note that in neither of the lucky votes configurations can a challenger certainly win.[55] Therefore, 4-Candidate equilibria are possible in the lucky votes case. The forces determining the identity are the same as in the previous equilibria, where distance between the four candidates drives their entry/non-exit decisions (i.e. the further they are from one another the higher their cost of exiting), while their distance from the extremes drives the challengers non-entry decisions (i.e. the further the four candidates from an extreme, the higher the incentive for a challenger to enter).

N-Candidate equilibria: In the *expected votes* case message configurations that satisfy individual rationality exist only if there are an even number of candidates which use $\frac{N}{2}$ messages, each with exactly two candidates stating them. That is, there are $\frac{N}{2}$ groups equally splitting the votes, each candidate getting $\frac{1}{N}$ votes ($m_1 = m_2 = \frac{1}{N}, m_3 = m_4 = \frac{3}{N} \dots$). Under the assumption that candidates pick messages that minimize their loss of entry, we can find well defined ranges where the identity of the entrants must remain. That is, candidates will use their closest message that satisfies individual rationality. Note that for $N \geq 6$ no challenger can enter and certainly win, which suggests that equilibria may be possible in the expected votes case for $N \geq 6$, however I have not proven it yet. In the *lucky votes* case, a lot more message configurations satisfy individual rationality, and they exist for any N. There are semi-separating message configurations where each of the two most extreme messages are stated by groups composed of at least two candidates, while the messages in between split the votes equally, however the group sizes (number of candidates stating that message) is arbitrary (one or more entrants). This ensures that none of the extreme entrants has a profitable deviation, and none of the entrants in the middle have profitable deviations. Non-symmetrically sized groups is unlikely to make stable equilibria, as the expected benefits for entrants in different sized groups are different, therefore all messages stated by exactly two candidates seems most stable. The other type of message configuration satisfying individual rationality, in the lucky votes case, has all of the candidates pooling at the

[55]Note that in the semi-separating case the challenger can make one candidate from the further group certainly win, while in the pooling case it will be a random individual from the five (including the challenger) using the median.

median. This is supported because one of the candidates gets all of the votes with probability $\frac{1}{N}$. As foreshadowed in the 2-Candidate case, this is the utility maximizing message configuration for the voters, since the winner will minimize the total message cost (of all voters). Furthermore, the configuration where all of the candidates are honest (i.e. $m_i = x_i = 0.5, \ \forall i \in \{1, \ldots N\}$) is the optimal N-Candidate configuration for the voters. To society's misfortune, the main result of the paper states that the probability of randomly picking this singleton possibility, where all of the candidates are honest median entrants (for any model primitives), out of the continuum of possible configurations with dishonest entrants, is zero. Finally, note that the number of equilibrium entrants (generally) depends on the net benefits (B, C), and that as the issue becomes very salient ($s \to 1$), in any pooling equilibria, any candidate identity configuration can arise in equilibrium. Furthermore, as in the previous pooling equilibria, distance between the candidate identities increases the incentives of the entrants not to exit (from the entry condition), while increasing the distance of the average identity from either extremes increases the incentive of a challenger to enter (from the non-entry condition). Therefore, the three main forces at work generalize to N-Candidate equilibria: 1) the candidate entry decision, and the cost they face from exiting (higher the further apart the candidates are from one another); 2) the challenger non-entry decision, and the cost the worst challenger faces (higher if the average identity of the candidates is far away from an extremist $\{0, 1\}$); and 3) the individual rationality force driving the equilibrium message configurations taking the particular shape identified. Finally, note that electoral competition plays a role both with the endogenous entry/non-entry decisions of the candidates and potential challengers, as well as with what campaign promises are individually rational.

6.1 1-Candidate Equilibria

The 1-Candidate equilibrium requires that at least one candidate wants to enter, that no second candidate wants to enter, and that no entrant prefers some other message. Before stating all of these conditions, looking into individual rationality will simplify the analysis.

Proposition 4. *There exists no 1-Candidate equilibrium where the single entrant runs with a lie.*

Proof. See Appendix □

The implication of this is non trivial. Even though there exist situations where equilibrium conditions 2 and 3 are both satisfied for dishonest single entrants (i.e. the dishonest entrant willing to enter as a sole candidate and no second candidate willing to enter) none of these configurations can exist in equilibrium, since the sole entrant always has a profitable deviation to avoid the cost of lying (i.e. to run honestly) and still certainly win. This means that there exist parameter combinations where conditions 2 and 3 are such that a lying single entrant can be sustained in equilibrium, however the single candidate's selfish objectives (IR) make this configuration unsustainable in equilibrium. We will see that IR will significantly simplify the problem by constricting the message space.

Now we can state the three necessary and sufficient equilibrium conditions in proper functional forms for 1-Candidate equilibria, which need to be jointly satisfied. The three conditions realize to:

1. $B - s|m_1 - x_1| - C \geq -\infty$

2. $P_2^{\mathbf{2}}\left[B - s|m_2 - x_2|\right] - C < (P_1^{\mathbf{2}} - 1)\left[(1-s)|x_1 - x_2| + s|m_1 - x_2|\right], \quad \forall x_2$

3. Each candidate runs truthfully

The three equilibrium conditions ensure that the single entrant wants to run, can run unopposed, and uses his preferred message. Entry Condition (1) is arbitrarily satisfied: since anarchy is infinitely costly we know one candidate is willing to run. Individual Rationality (3) implies that everyone runs honestly: suppose they didn't, once they won they would have preferred to run with their true ideal, and they could have won with it.

The Challenger Non-Entry Condition (2) is going to determine how lying responds to the net benefits. The non entry condition needs to be satisfied for any possible challenger, which implies that if it is satisfied for some x_1 then we know that if x_1 runs, in equilibrium he will run unopposed. The non entry condition (which has to hold for all x_2) in full detail:

$$\begin{aligned} P_2^{\mathbf{2}}\left[B - s|m_2 - x_2|\right] - C - P_1^{\mathbf{2}}\left[(1-s)|x_1 - x_2| + s|m_1 - x_2|\right] \\ < - P_1^{\mathbf{1}}\left[(1-s)|x_1 - x_2| + s|m_1 - x_2|\right] \end{aligned}$$

In equilibrium we know that $P_1^{\mathbf{1}} = 1$, while $P_1^{\mathbf{2}}$, $P_2^{\mathbf{2}}$ will depend on $x_1 = m_1$ and m_2. The entrants message is irrelevant, as we know that the single equilibrium entrant runs honestly. Specifically, a potential challenger (x_2) can propose a message closer to the median and win as long as the entrant is not the median politician. If the single entrant is the median then no message exists which a challenger could use to certainly win. The best the challenger could do is run with the median message and win with probability half. The following lemma simply tells us that a potential challenger will always use a message which gives him the highest probability of winning.

Lemma 5. *Challenger (x_2) always uses cheapest winning (or almost winning) message.*

Proof. See Appendix □

Suppose you were the single entrant: which politician loses the most from letting you win? The individual on the further extreme of the identity space. This is also the individual who requires the least net benefits to enter, given your candidacy. Let this individual be the *worst challenger* ($\tilde{x}$). If the net benefits of entry are so low that the worst challenger is unwilling to enter, then they are too low for any other challenger. This intuition simplifies the analysis of the non entry condition.

Lemma 6. *The worst challenger is an extremist, and if the non-entry condition is satisfied for the worst challenger, it is satisfied for any other type of challenger.*

Proof. See Appendix □

It is interesting to note that this is the worst challenger no matter s. This is because individual rationality constraints what message each single candidate can use (i.e. runs honestly), while electoral competition and the distribution of voter preferences determine what the cheapest winning lie is for any challenger. Given that $s \notin \{0, 1\}$, the furthest extremist (from the candidate) always imposes the highest identity cost, while the message he can use is the same as the worst honest challenger. For example, $x_1 = m_1 = 0.5 \implies$ any challenger needs to use the median message, and the furthest extremists suffers the highest identity cost from letting x_1 run unopposed. It turns out that the worst challenger is an extremist in all of the equilibria considered in this paper. From lemma 6 we know that if the worst challenger will not enter, then no one will. This implies that all we have to do to understand 1-Candidate equilibria is look at when the worst challenger will not want to enter for some x_1. When the single entrant uses a non median message the challenger can always certainly win, therefore the non-entry condition simplifies to:

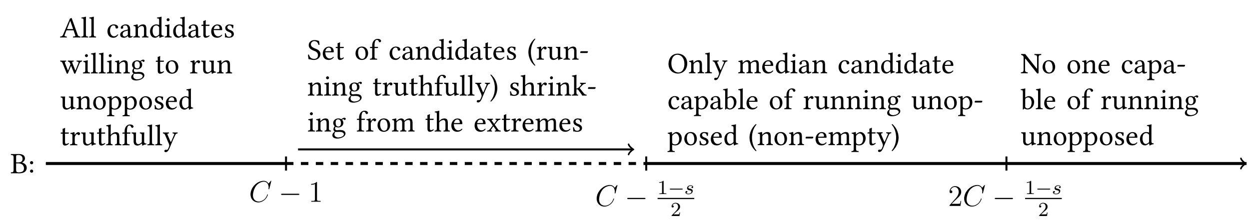
$$
\begin{aligned}
B - C &< s|m_2 - x_2| - |x_1 - x_2| \text{ if } x_1 \neq 0.5 \\
B - 2C &< s|m_2 - x_2| - |x_1 - x_2| \text{ if } x_1 = 0.5
\end{aligned}
\tag{5}
$$

Proposition 7. *For a given candidate x_1, an increase in salience decreases the incentive for challengers to enter.*

Proof. See Appendix. □

We can see that increasing salience increases the cost of lying for the worst challenger, thereby making entry less attractive for the challenger. That is, if the single candidate $\hat{x}_1$ was picked such that the non-entry condition binds (i.e. worst challenger is indifferent between entry or not), an increase in salience breaks indifference and makes it such that the challenger strictly prefers not to enter.

Finally, note that the upper bound on B is lower for any $x_1 \neq 0.5$, than for the median entrant, because there is a discontinuous jump in the probability of winning ($P_1^{\mathbf{1}}$ goes from zero to half) for the single entrant, when the challenger uses his best message available. By increasing the benefits of entry (B) sufficiently, we can ensure that the only candidate who can run unopposed is the median citizen. If any other individual were to enter a second challenger would prefer entry, breaking that equilibrium. Using condition 5 the following diagram summarizes how the 1-Candidate equilibria look for some fixed cost of entry $C > 0$.



For low enough net benefits of entry, i.e. $C - B > 1$, we know that everyone will be willing and capable of running unopposed. As the net benefits increase, the first individuals who will not be capable of running unopposed will be the extremists. The intuition behind this is that the worst challenger loses the most from the single entrant being the opposite extremist, and the least if the single entrant is the median citizen. Therefore, the set of potential entrants is shrinking from the extremes as the net benefits increase, up to the point where only the median citizen will be cabable of running unopposed when $B - C \in [-0.5\beta, C - 0.5\beta]$.

To see that the set of equilibria is shrinking from the extremes as B increases, note that condition 5 for $x_1 = 0.5 + k$ where $k \in (0, 0.5]$ is $B < C + s - (0.5 + k)(1 + s)$. Notice that

${}^{\partial RHS}/_{\partial k} < 0$, which means that as k falls (single entrant becomes less extreme) the inequality becomes looser. In other words, the closer the single entrant to the median, the higher benefits the worst challenger requires to prefer entry over non-entry. Again, the intuition is that the worst challenger loses less by letting someone closer to the median run unopposed.

6.2 2-Candidate Equilibria

The 2-Candidate equilibrium requires that at least two candidates want to enter, that no third candidate wants to enter, and that no entrant prefers some other message. The three conditions realize to:

1. $0.5\,[B-(1-\beta)|m_2-x_2|]-C \geq -0.5\,[\beta|x_1-x_2|+(1-\beta)|m_1-x_2|]$
2. $P_3^{\mathbf{3}}[B-(1-\beta)|m_3-x_3|] < \sum_{i=1}^{2}(P_i^{\mathbf{3}}-0.5)(\beta|x_i-x_3|+(1-\beta)|m_i-x_3|), \quad \forall x_3$
3. In all 2-Candidate equilibria both candidates run with the median message

where in equilibrium $P_1^{\mathbf{2}} = P_2^{\mathbf{2}} = 0.5$, while any unopposed candidate wins certainly $P_1^{\mathbf{1}} = 1$. The probabilities of winning when considering a potential challenger depends on the messages chosen. We will first explain individual rationality, then the entry condition, and finally the non entry condition.

Proposition 8. *In any 2-Candidate equilibrium candidates pool at the median, i.e.* $m_1 = m_2 = 0.5$

Proof. See Appendix □

The proof is intuitive. We know that in any equilibrium all candidates have the same probability of winning. The only messages that satisfy this are separating messages symmetrically opposed around the median, or any pooling message. No symmetrically opposing message around the median can arise in equilibrium, as each candidate can state a message slightly closer to the median and certainly win. No pooling messages different from the median message can exist in equilibrium, as for any other message configuration each candidate has a profitable deviation to another message (i.e. a message slightly closer to the median which guarantees victory).

This differs from the citizen-candidate model, as there they find that the candidates can have symmetrically opposed platforms. It is interesting to note that pooling was impossible in the citizen-candidate model, the only equilibria that exist are two symmetrically opposing (around the median) candidates running truthfully, never two same ones. Here this type of equilibrium fails as such messages are not individually rational. The difference lies in that they assume messages are not credible, so candidates can only run with their true identity (they cannot hide anything), while I relax this assumption by introducing naive voters. Finally, note how individual rationality constricts the set of possible equilibrium messages from a continuum $(m_1, m_2) \in [0,1]\times[0,1]$ to a singleton, the median message configuration $(m_1, m_2) = (0.5, 0.5)$. In the 1-Candidate case it meant that everyone ran honestly in equilibrium.

The 2-Candidate entry condition is essentially the reverse of the 1-Candidate non entry condition, the only difference is that in the 1-Candidate case it has to hold for all challengers and their messages ($\forall x_2, m_2$). We will state the condition in full, for clarity:

$$\begin{aligned} -C + P_2^{\mathbf{2}}[B-(1-\beta)|m_2-x_2|] - P_1^{\mathbf{2}}(\beta|x_1-x_2|+(1-\beta)|m_1-x_2|) \\ \geq -\,P_1^{\mathbf{1}}(\beta|x_1-x_2|+(1-\beta)|m_1-x_2|) \end{aligned} \tag{6}$$

There are two details which were present in the 1-Candidate case, however it made more sense to introduce them now. The left hand side (LHS) of the entry condition (6) describes what entrant x_2 gets from entry, while the right hand side (RHS) is what he obtains from staying out and letting x_1 win certainly. An entrant gets the expected benefit of winning, but pays a cost to enter. On top of B, C, an entrant (e.g. x_2) also incurs two additional types of costs: the *identity* and *message costs* ($|x_1 - x_2|$ and $|m_1 - x_2|$). The identity cost is the cost the candidate incurs from the distance to his competitors true identity, while the message cost is the cost the candidate incurs from the distance to his competitors message.

The entry condition (6) ensures that a citizen x_2 prefers entry (LHS) over non-entry (RHS). If x_2 does not enter, then his competitor x_1 will certainly win, therefore only by entry can x_2 change the election outcome. By entry the second candidate changes the probability with which his competitor wins, precisely, instead of x_1 certainly winning, x_1 wins with probability half in any 2-Candidate equilibrium. Since both candidates have an equal probability of winning (i.e. $\mathcal{C}_2 = W(\mathcal{C}_2)$) and both candidates use the same message, the message cost of entry and non entry is equivalent for any x_2. Intuitively, if x_2 enters he does so with the median message, so either he will win with probability half and implement the median or his competitor will win with probability half and implement the median. If x_2 does not enter his competitor will certainly win and implement the median. Therefore, both actions of x_2 lead to the median message arising certainly, so x_2's entry does not impact the message cost.

On the other hand, the identity cost does change with x_2's entry. By entry x_2 decreases the probability with which x_1 wins ($P_1^{\mathbf{1}} = 1$ to $P_1^{\mathbf{2}} = 0.5$), thereby halving the probability of incurring the full cost of his competitor implementing his true ideal (with probability β). Furthermore, by entering x_2 wins with probability 0.5, in which case he can freely implement his true ideal with probability β. We can summarize this intuition: by chosing entry the second entrant gets (net ex ante) $0.5(B + \beta|x_1 - x_2|) - C$, which is just the LHS - RHS of condition (6). If this is positive, then entry is advantageous.

The reason why no 2-Candidate pooling equilibria exist in neither of the original citizen-candidate model, is that if two candidates pool at the median, a third candidate can enter with identity in $(1/6, 5/6)$ and certainly win. There is an implicit assumption present in both of the original citizen-candidate models, which wasn't explicitly discussed in the original papers. Individuals who run with the same identity will get the expected value of votes assigned to that identity. For example, if two candidates run with the median message, each of them will receive exactly half the votes. This then implies that a third challenger can always enter and win if two candidates pool. Let this case be called the *"Expected Votes"* case.

Even though ties are broken with a coin flip in the original model, the amount of votes received in case of message ties (several candidates running with same policies) is not based on luck.[56] The effect this has is that 2-Candidate pooling equilibria do not exist in CC, since a third candidate can enter and certainly win. However, it need not be true that if two candidates run and both state the same message they each get half the votes. In fact, [Feddersen et al., 1990] support their pooling at the median equilibrium by using out-of-equilibrium beliefs, such that, when any candidate deviates from the median position, all of the voters who prefer the median position coordinate on a single (out of possibly many) candidate using the median position. This makes the median position unbeatable in equilibrium, no matter the number of candidates.

[56] In fact, each candidate gets half of the votes, but the winner of the tie is determined by a random draw.

In this paper we take the same approach to define the opposite of the Expected Votes case: if there are n candidates stating message $\tilde{m}$ then each will receive all votes that belong to message $\tilde{m}$ (i.e. $V_{\tilde{m}}$) with probability $\frac{1}{n}$. Let this be the *"Lucky Votes"* case. We will not consider intermediate cases for now.[57]

Before proceeding to the results, we need to still detail the non entry condition. Again, we will write it in full, for clarity (it has to hold $\forall x_3, m_3$):

$$\begin{aligned} -C + P_3^{\mathbf{3}}[B - (1-\beta)|m_3 - x_3|] - \sum_{i=1}^{2} P_i^{\mathbf{3}}(\beta|x_i - x_3| + (1-\beta)|m_i - x_3|) \\ \geq - \sum_{i=1}^{2} P_i^{\mathbf{2}}(\beta|x_i - x_3| + (1-\beta)|m_i - x_3|) \end{aligned} \tag{7}$$

The LHS is what the challenger (i.e. x_3) gets from entry, while the RHS is what he gets from non-entry. As in the 1-Candidate case the non entry condition needs to hold for all potential challengers and their messages. In order to be able to prove that the worst challenger is an extremist in the expected votes case, given that the two candidates pool at the median, we will first show that when two candidates pool, any challenger will use his cheapest winning lie. The following lemma proves exactly that:

Lemma 9. *If entrants pool (i.e. $w_p = m_1 = m_2$) the challenger will run with the least costly winning lie. Specifically:*

$$w_p \in \begin{cases} \left[0, \frac{1}{3}\right) & \Rightarrow \bar{m}_p^{\mathbf{3}} \in (w_p, 1] \\ \left(\frac{2}{3}, 1\right] & \Rightarrow \bar{m}_p^{\mathbf{3}} \in [0, w_p) \\ \left[\frac{1}{3}, \frac{2}{3}\right] & \Rightarrow \bar{m}_p^{\mathbf{3}} \in \left\{\left(\frac{2}{3} - w_p, w_p\right), \left(w_p, \frac{4}{3} - w_p\right)\right\} \end{cases}$$

where $\bar{m}_p^{\mathbf{3}}$ denotes the message a challenger can use to certainly win, when two candidates are pooling.

Proof. See Appendix. □

This result is more general than required, as we know that in any 2-Candidate equilibrium the only possible message configuration is where both of the candidates pool at the median. In fact, Lemma 9 helps us understand the problem when we do not impose individual rationality. Now we are ready to identify the worst challenger, that is, the challenger who needs the least benefits in order to enter.

Lemma 10. *The worst challenger is an extremist, in both the expected and lucky votes case. That is, if $\frac{x_1+x_2}{2} \geq \frac{1}{2}$ then the worst challenger is $\bar{x}_3 = 0$.*

Proof. See Appendix. □

[57] A simple way to capture the intermediate cases is: let V_i be the total number of voters who will vote for message m_i, let g_i be the number of candidates stating message m_i, then x_i ex ante receives votes $\frac{V_i+\delta}{g_i}$ where $\delta \in [0, V_i(g_i - 1)]$. We can see that the more individuals state the same message, the higher δ can be, capturing the fact that there are more votes to be captured when there is a larger group running with the same message.

Using the identity and message costs we can intuitively understand why the worst challenger is an extremists. We first consider the expected votes case. There are two types of challengers: politicians whose identity is inside the set of messages that can outright win (i.e. people who don't have to lie to win, $x \in (\frac{1}{6}, \frac{5}{6})$) and politicians who are outside this set (i.e. people who have to lie to win, $x \notin (\frac{1}{6}, \frac{5}{6})$). Among the honest challengers, the most extreme honest challengers ($x \in \{\frac{1}{6}+\epsilon, \frac{5}{6}-\epsilon\}$) incur the highest identity and message cost of non entry, that is, they gain the most from the entrants not winning and the median not getting implemented. Therefore, they require the least net benefits to enter among the honest challengers.

On the other hand, politicians who have to lie have extreme identities so they incur a higher identity cost of non entry than any challenger that can win honestly. Since the two entrants pool at the median the cheapest winning lie will be the same for all individuals who have to lie (e.g. if $x_3 \leq \frac{1}{6} \implies m_3 = \frac{1}{6} + \epsilon$). This means that the message gain from entry[58] is the same for all lying challengers, which is also the same for the most extreme honest challenger[59]. Therefore, the identity cost determines who the worst challenger is among the 'liars', which is also the worst challenger globally. In particular, the furthest extremist challenger has the highest identity gain (unique), and the highest message gain (non-unique, all liars and most extreme honest) from entry. The extremist gains the most from entry because he reduces his identity and message cost the most. More intuitively, if two candidates run and their average identity is to the right of the median, the left extremist will be the worst challenger since his identity is furthest from the two entrants' identities and their median message, and therefore he gains the most from winning.

In the lucky votes case the challenger has to run with the median message, as any other message is a losing message. This means that the challenger has no message gain from entry, i.e. both for entry and non entry the median message will certainly be implemented with probability s. However, there is a positive identity gain, and as we saw in the previous paragraph the extremist challengers have the highest identity gain from entry.

As we have seen, in the 2-Candidate case the worst challenger is an extremist for both the expected and lucky votes case, as in the 1-Candidate case. The proof of this was simplified by the fact that we know that in any 2-Candidate pure strategy equilibrium the only message that can arise is pooling at the median. The 1- and 2-Candidate worst challenger proofs are essentially the same, the main difference being that the worst challenger is now obtained by observing the *average identity* of the two entrants. That is, when $\frac{x_1+x_2}{2} \geq 0.5 \implies \bar{x}_3 = 0$, i.e. if the average identity of the two entrants is higher than the median, then the lower extremist is the worst challenger. These conclusions allow us to prove the following.

Proposition 11. *In the Expected Votes case no 2-Candidate equilibria in pure strategies exist.*

Proof. See Appendix. □

For two candidates to be willing to enter at the median, and no third citizen to prefers outright winning, the benefit of winning has to small enough. Precisely, from the combined entry and non entry conditions we know that $\beta(|x_1 - x_2| - x_1 - x_2) - \frac{(1-\beta)}{3} > B$, which implies that the benefit of winning would have to be negative. However, the model assumes that the benefit of winning has to be positive, therefore, when the votes are shared equally amongst candidates using the

[58]The decrease in message cost from entry, over non entry.

[59]That is, $|\frac{1}{2} - x_3| - |\frac{1}{6} + \epsilon - x_3| = \frac{1}{3} - \epsilon$, intuitively the challenger decreases his message cost by a third minus epsilon

same message no 2-Candidate equilibria in pure strategies exist. Now we turn to the lucky votes case.

Proposition 12. *In the Lucky Votes case there exist a lower bound, $\tilde{C} = \beta \max\{\frac{x_1+x_2}{2} - |x_1 - x_2|, \frac{B}{3} + \frac{x_1+x_2}{6}\} > 0$, s.th. for any $C > \tilde{C}$ there exists some $B > 0$ which makes x_1, x_2 entering by pooling at the median an equilibrium in pure strategies.*

Proof. Suppose $\frac{x_1+x_2}{2} \geq 1$. Since $B > 0$, from Non-Entry we know that $C > \beta\frac{x_1+x_2}{6} \geq \frac{1}{3}$

$$\text{Entry:} \quad B \geq 2C - \beta|x_1 - x_2| \tag{8}$$

$$\text{Non-Entry:} \quad B < 3C - \beta\frac{x_1+x_2}{2} \Rightarrow C > \frac{B}{3} + \beta\frac{x_1+x_2}{6} \tag{9}$$

$$\text{Combined 1:} \quad 3C - \beta\frac{x_1+x_2}{2} > B \geq 2C - \beta|x_1 - x_2| \tag{10}$$

$$\text{Non empty interval} \iff C > \beta\left(\frac{x_1+x_2}{2} - |x_1 - x_2|\right)$$

$$\text{Combined 2:} \quad \frac{B}{2} + \beta\frac{|x_1 - x_2|}{2} \geq C > \frac{B}{3} + \beta\frac{x_1+x_2}{6} \tag{11}$$

$$\text{Non empty interval} \iff B > \beta(x_1 + x_2 - 3|x_1 - x_2|)$$

Given some $C > \tilde{C} := \max\{\beta\left(\frac{x_1+x_2}{2} - |x_1 - x_2|\right), \frac{B}{3} + \beta\frac{x_1+x_2}{6}\}$, there always exist some $B > 0$ so the above inequalities are satisfied. □

The main reason why a 2-Candidate equilibrium is possible in the Lucky Votes case is that no challenger can cartainly win. The best a challenger can do is pool with the two entrants at the median and win with probability a third. This implies that we do not have an upper bound on C as in the expected votes case, but rather in equilibrium we can identify a lower bound on the cost of entry.[60] For a high enough cost of entry, there exists a positive benefit of winning so that two candidates are willing to run with the median message, but low enough so that the worst challenger is not willing to enter.

Observe the above proof, in particular the conditions "Combined 1" and "Combined 2". From "Combined 1" we can clearly see that the larger is the cost of entry (i.e. C), the larger is the interval within which the benefit of winning (i.e. B) can reside. Analogously, from "Combined 2" we can clearly see that the larger the benefit of winning (i.e. B) is, the larger the interval within which the cost of running (i.e. C) can reside. The fact that there exists a non-empty interval is a consequence of the worst challenger not being able to certainly win, so his expected benefit of entry is not high enough to remove all equilibria.

As we can see, 2-Candidate equilibria existence depends on the voter tie breaking rule. When the voter tie breaking rule is such that all who state the same message get an average number of votes that belong to that message, any challenger can certainly win, therefore a 2-Candidate equilibrium is not possible. More precisely, given $C > 0$ in the Expected Votes case pure strategy equilibria are only possible for $B < 0$, which is not considered here.[61] If all who state the same message have an equal chance of getting all of the votes of that message, then no challenger can

[60] Ensuring no challenger is willing to enter when two candidates pool together at the median

[61] Having a cost of winning is not considered in this model. E.g. a situation where one is considering to pay a cost of entry into an election whose winner gets to pay the social planner B.

certainly win and 2-Candidate equilibria are possible. More precisely, in the Lucky Votes case there exists an equilibrium lower bound of C which makes sure that when $B > 0$ equilibria exist. Before proceeding to look into the Lucky Votes case in more detail, it is worth emphasizing how salience matters:

Proposition 13. *As salience increases ($\beta \downarrow$) the possible identities of the two equilibrium entrants increases, where in the limit ($s \to 1$) anyone can form a 2-Candidate equilibrium configuration.*

Proof. See Appendix. □

This result is counterintuitive. One would expect a higher cost of lying to imply that only individuals closer to the median could form equilibrium configurations. The reason why this is not the case is that the only message a potential challenger could use is the median. That is, no matter the choices of the two candidates, and the challenger, the median message will certainly arise, which implies that the message gain from entry is zero, both for the two candidates making their entry decisions; and for the challenger making his entry decision. Therefore, only the identity cost matters for pooling equilibria, and the identity cost gains more weight the lower salience is. In other words, as salience increases the identity cost becomes less relevant, where in the limit it becomes irrelevant.

6.2.1 Lucky Votes

Let us consider the Lucky Votes case in more detail. From Proposition 12 we know that 2-Candidate pure strategy equilibria, where x_1, x_2 pool at the median, exist when:

$$C > C^* := \max\left\{\beta\left(\frac{x_1 + x_2}{2} - |x_1 - x_2|\right), \frac{B}{3} + \frac{\beta(x_1 + x_2)}{6}\right\} \tag{12}$$

for some $B > 0$. The left part inside the braces is what ensures there exists some interval where C can exist such that the entry and non-entry conditions are simultaneously satisfied. The right part inside the braces ensures that the cost is high enough so that no third candidate is willing to enter.

$$C^* = \begin{cases} \beta\left(\frac{x_1+x_2}{2} - |x_1 - x_2|\right) & \text{if } \beta(x_1 + x_2) > B + 3\beta|x_1 - x_2| \\ \frac{B}{3} + \frac{\beta(x_1+x_2)}{6} & \text{if } \beta(x_1 + x_2) \leq B + 3\beta|x_1 - x_2| \end{cases}$$

However, from the condition "Combined 2" (11) we know that a non-empty interval where B take values exists iff $\beta(x_1+x_2) \leq B+3\beta|x_1-x_2|$). This implies that the equilibrium lower bound on the cost of entry is $C^* = \hat{C}^* := \frac{B}{3} + \frac{\beta(x_1+x_2)}{6}$ for any values of B. In other words, $C^* = \hat{C}^*$ implies that the condition "Combined 1" (10) has a non-empty interval where B can take values. To gain intuition, suppose $B = 0$, then the only possible entrants satisfy $\beta(x_1+x_2) \leq 3\beta|x_1-x_2|$, i.e. the candidates have to be far enough from one another. Graphically:

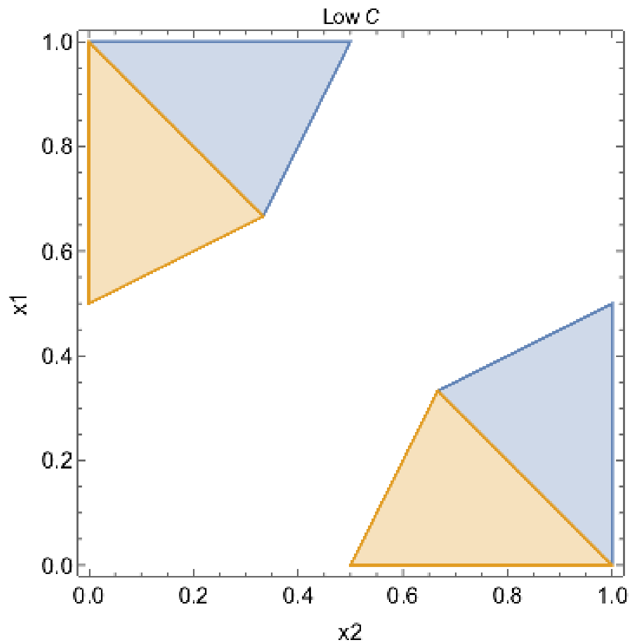


Figure 1: equilibrium identity configurations

To understand why this is the case, first observe the non entry condition (9). Recall that we simplified the problem by assuming that $\frac{x_1+x_2}{2} \geq 0.5$, which covers all the possible relative distances and positions due to symmetry. Given $B = 0$ the non entry condition becomes $C > \beta\frac{x_1+x_2}{6} \geq \frac{\beta}{6}$ which places a lower bound on C. That is, the smallest cost that ensures no other politician is willing to challenge the two entrants is $\beta/6$ for any symmetrically opposed entrants (in identity). Notice that the RHS of the non entry condition is maximized when $x_1 + x_2 = 2$, that is, when the two entrants are the same extremists. This is because the worst challenger is the opposite extremist, and $x_1 + x_2 = 2$ impose the highest cost of non entry for the worst challenger. Furthermore, when $B = 0$ the entry condition (8) becomes $0.5\beta|x_1 - x_2| \geq C$, which imposes an upper bound on C. The closer the two entrants, the less benefit each one of them has from entry. That is, if the two entrants are very far from one another, exit by either is very costly because it would guarantee their competitor certainly wins. In fact, if the two entrants are opposite extremists, each of them loses the most from exit (guaranteeing a win for his competitor). Therefore, candidates that are distant from one another are willing to form an equilibrium (identity) configuration for a higher cost than candidates that are near one another. Precisely, given the lower bound identified from the non entry condition, we know that candidates have to be at least a third distance from one another, i.e. $0.5\beta|x_1-x_2| \geq C \geq \beta/6 \implies |x_1-x_2| \geq 1/3$.

Now we are in a position to identify the three major equilibrium forces in this model:

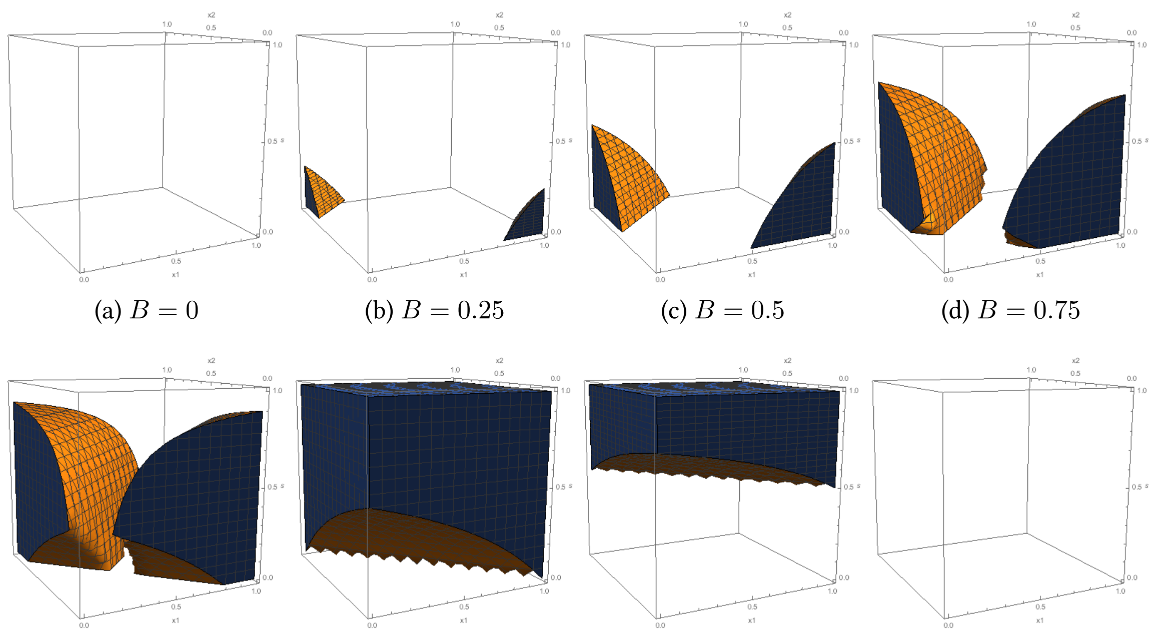
1. *Entry*: the more distant the candidates are from one another the higher their incentive to enter, since exit imposes a higher cost on the candidates

2. *Non Entry*: the more similar, and extreme, the two candidates the higher the incentive for a challenger to enter, since non entry would ensure a bad outcome for the worst challenger

3. *Strategic Promises*: since politicians compete in promises the only possible equilibrium message configuration in the two candidate case is pooling at the median

To further intuition about the model, suppose that $C = 0.5$ and $B \in [0, 1.5]$. The following series of graphs shows which identity configurations can form equilibria, for any salience level. For convenience, I report condition "Combined 1" for $C = 0.5$:

$$1.5 - \beta\frac{x_1 + x_2}{2} > B \geq 1 - \beta|x_1 - x_2|$$



(a) $B = 0$ (b) $B = 0.25$ (c) $B = 0.5$ (d) $B = 0.75$

(e) $B = 0.9$ (f) $B = 1$ (g) $B = 1.25$ (h) $B = 1.5$

Figure 2: equilibrium identity configurations with salience

The horizontal axes are the candidate identities (x_1 and x_2), while the vertical axis is salience (s). As shown in Proposition (13), as salience increases any identity configuration becomes possible in equilibrium. For low B relative to $C = 0.5$ equilibrium configurations are only possible for candidates whose identities are far apart. As argued, this is because distant candidates are willing to tolarate a high cost of entry (relative to the benefit of winning), since non entry allows their distant competitor certain victory. The further the candidates are from one another, the higher the cost of allowing your competitor certain victory. Furthermore, for low B relative to $C = 0.5$ note that equilibria are possible only for low salience. The reason is that when salience is low, the identity cost carries a lot of weight, therefore the identity cost of exit for the candidates is high. That is, low salience makes identity relevant for equilibrium configurations.

Finally, note that when $B = 0$ the only possible equilibrium configuration is $x_1 \neq x_2$ where $x_1, x_2 \in \{0, 1\}$ when $\beta = 1$ (no salience), since I assumed indifferent individuals enter. That is, opposite extremists. However, recall that $\beta \in (0, 1)$, therefore no configuration is possible.

Similarly, when $B = 1.5$ and $\beta = 0$ (full salience), any configuration would be possible, however again $\beta \in (0, 1)$ so no configuration is possible.

We now look at a few questions of interest, in order to better understand the identities of the candidates for different combinations of B, C. For convenience, we will re-state the condition "Combined 1", and due to continuity we will generally ignore the ϵ's that would make each inequality completely precise:

$$3C - \beta\frac{x_1 + x_2}{2} > B \geq 2C - \beta|x_1 - x_2|$$

Can we ensure that both of the entrants are honest?

$$\begin{aligned} x_1 = x_2 = 0.5 &\implies 3C - 0.5\beta > B \geq 2C \\ \text{Entry} &\implies C = \frac{B}{2} \implies B > \beta \\ \text{Non Entry} &\implies C = \frac{\beta}{6} \implies B = 0 \end{aligned} \tag{13}$$

From the Non-Entry condition we know that the smallest C such that two median citizens can run unopposed is $C = \frac{\beta}{6} + \epsilon$. This C also guarantees that any symmetrically opposed citizens can run unopposed (i.e. any $x_1 + x_2 = 1$). The "Combined 1" condition (10) then becomes

$$\underbrace{\frac{\beta}{2}\left(1 - (x_1 + x_2)\right) > B}_{\text{Non Entry}} \underbrace{\geq \beta\left(\frac{1}{3} - d\right)}_{\text{Entry}}$$

where $d = |x_1 - x_2|$ is the distance between the two candidates. Note that the Non Entry condition holds only for $x_1 + x_2 = 1$ configurations, while the Entry condition holds only for $d > \frac{1}{3}$. Therefore, the set of potential 2-Candidate identity configurations is $\{x_1, x_2 | x_1 + x_2 = 1, d > \frac{1}{3}\}$. That is, the two candidates have to be symmetrically opposed around the median, and they have to be at least a third of the policy space away from one another (as then the $B = 0$ suffices for both entrants to be willing to enter). Therefore, for the specified B, C combination it is not possible to have two candidates running honestly. This is due to the fact that we did not ensure the interval where B can exist in condition (13) is non empty for $x_1 = x_2 = 0.5$.

To ensure the interval is non empty the following inequality has to hold: $3C - 0.5\beta > 2C \implies C > 0.5\beta$. Therefore, in order to guarantee that two honest entrants can be an equilibrium configuration the smallest net benefits are $C = 0.5\beta \implies B = \beta$. The equilibrium inequalities then become:

$$\underbrace{\frac{\beta}{2}\left(3 - (x_1 + x_2)\right) > \beta}_{\text{Non Entry}} \underbrace{\geq \beta(1 - d)}_{\text{Entry}}$$

From the entry condition it is clear candidates any distance apart are willing to form equilibrium configurations, however from the non entry condition we see that in equilibrium $x_1 + x_2 = 1$. This means that the only equilibrium configurations possible are where candidates are symmetrically opposed. This has a particularly sad implication. We can re-write the set of entrants as $X^{\mathbf{2}}_{0.5} := \{x_1, x_2 : x_1 = 0.5 + k, x_2 = 0.5 - k, k \in [0, 0.5]\}$, from which we can see that the set of configurations is continuous. This means that the probability of randomly picking exactly $x_1 = x_2 = 0.5$ as the equilibrium realization is zero. Furthermore, the extremists have the highest gain from entry. These two suggest that when $B = \beta$ and $C = 0.5\beta$, the equilibrium that will actually arise is unlikely to be two honest entrants. This leads us to a result that anticipates the main theorem of the paper.

Theorem 14. *Let $X^{\mathbf{2}}_{0.5}$ be the set of possible equilibrium configurations when two honest entrants form an equilibrium configuration. Then, the probability of randomly picking the configuration with honest candidates, out of $X^{\mathbf{2}}_{0.5}$, is zero.*

Proof. Since the realization where candidates are honest is only a point on a line, it is measure zero. □

The main implication of this theorem is that the 2-Candidate identity configuration observed in equilibrium will (almost) certainly be composed of dishonest politicians. Before proceeding to look at equilibria with more candidates we will analyze a few more interesting questions.

Can we have two same extremists as candidates in equilibrium?

$$x_1 = x_2 = 1 \implies 3C - \beta > B \geq 2C$$

From the Non-Entry condition we know that the smallest C such that two same extremists can run unopposed is $C = \frac{\beta}{3} + \epsilon$. This C also guarantees that any citizen configuration with $1 \leq x_1 + x_2 \leq 2$ can run unopposed. The "Combined 1" condition (10) then becomes

$$\underbrace{\frac{\beta}{2}\left(2 - (x_1 + x_2)\right) > B}_{\text{Non Entry}} \geq \underbrace{\beta\left(\frac{2}{3} - d\right)}_{\text{Entry}}$$

As we can see, two same extremists require $B > \frac{2\beta}{3}$ for both of them to be willing to enter (as letting the other equal identity candidate run unopposed is not very costly), however for such a high B two same extremists are not able to run unopposed anymore (since the worst challenger also gains from this $B > 0$). Therefore, we need to identify the smallest C such that two same extremists are willing and capable of forming a 2-Candidate equilibrium from the Entry condition. For any two equal identity candidates to find it profitable to run together the benefit of winning has to be at least twice the size of the cost of entry. Using this in the non entry condition we find that $B > 2\beta \implies C > \beta$. For $C = \beta$ we can write the "Combined 1" condition as:

$$\underbrace{\beta\left(3 - \frac{x_1 + x_2}{2}\right) > B}_{\text{Non Entry}} \geq \underbrace{\beta\left(2 - d\right)}_{\text{Entry}}$$

Since the entry condition is satisfied for any d, and the non entry condition is satisfied for any $x_1 + x_2 \in [1, 2]$ we know that when two same extremists can run in equilibrium, then so can any other candidate configuration. This is intuitive, two same extremists cause the largest amount of losses for the worst challenger, so if we ensured that they can run in equilibrium, so can everyone else.

Can only two opposite extremists be willing to run unopposed?

$$x_1 = 1, x_2 = 0 \implies 3C - 0.5\beta > B \geq 2C - \beta$$

The lowest C and B such that two opposite extremists can pool at the median, and no challenger is willing to enter, is $C = 0.5\beta$ which implies that equilibrium configurations are possible for $B \in [0, \beta)$. Let $B = 0$, then the above equilibrium condition becomes $\beta > 0 \geq 0$ which is clearly satisfied, and only the opposite extremists are willing to form an equilibrium configuration. In fact, this is the only configuration which a policy maker could ensure happens with certainty. This is again a sad implication, the only certainty an institution designer can achieve is that the only individuals willing to form 2-Candidate configurations (and capable of running unopposed) is two individuals who lie the most (extremists), and are on opposite sides of the ideological space (opposite extremist).

Can we ensure that both of the entrants are on opposite sides of the median?

$$x_1 = 1, x_2 = 0.5 \implies 3C - 0.75\beta > B \geq 2C - 0.5\beta$$

From the Non-Entry condition we know that the smallest C such that a median entrant and an extremist can run unopposed is $C = \frac{\beta}{4} + \epsilon$. The "Combined 1" condition (10) then becomes

$$\underbrace{\frac{\beta}{2}\left(1.5 - (x_1 + x_2)\right) >}_{\text{Non Entry}} B \geq \underbrace{\beta\left(\frac{1}{2} - d\right)}_{\text{Entry}}$$

Note that the Entry condition holds only for $d \geq \frac{1}{2}$. Therefore, the set of potential 2-Candidate identity configurations is $\{x_1, x_2 | 1 \leq x_1 + x_2 \leq 1.5, d \geq \frac{1}{2}\}$. That is, the two candidates have to be on opposite sides of the median, and have to be at least half of the policy space away from one another (as then $B = 0$ suffices for both entrants to be willing to enter). The set of entrants also depends on B. For $B = \frac{\beta}{4}$ only symmetrically opposed candidates can enter (who are far enough from one another), since the benefit increases the benefit for the worst challenger making him willing to enter for any configuration with $x_1 + x_2 > 1$. Precisely, the set of possible candidate configurations is $\{x_1, x_2 | x_1 + x_2 = 1, d \geq \frac{1}{4}\}$. As B decreases, the incentive for the worst challenger is falling, so that when $B = 0$ the set of entrants becomes $\{x_1, x_2 | 1 \leq x_1 + x_2 \leq 1.5, d \geq \frac{1}{2}\}$. Therefore, we can find configurations such that the two entrants will be on opposite sides of the median, however their actual identity will remain uncertain, even when we know B. Notice that there are two forces in play as we increase B: (i) the entry condition becomes more lax, allowing for a lower distance between the candidates, and (ii) the non entry condition becomes more constrained, making the only possible configuration closer to symmetry.

Can we ensure only 'best' candidates enter? A question that naturally arises: who are the best candidates for the welfare of the voters? Since this is a voting game where the welfare of voters depends on the candidates identity and message, a reasonable welfare function is the sum of the distances of the candidates identities and messages to each voter. The best candidate would then minimize the distance to the whole voter base. We will see that this is equivalent to finding the candidate configuration that minimizes the maxium distance to each of the voters.

Definition 15 ('Best' Candidates)**.** *The candidate configuration whose preferences and messages minimizes the total distance from all voters.*

In 1-Candidate equilibria a median politician running truthfully is the best candidate, both ex-ante and ex-post.[62] In this model we assumed the voter preferences are uniformly distributed on the $[0, 1]$ interval, i.e. $F \sim U[0, 1]$. For the uniform the mean equals the median, therefore we can easily find the median. To ensure that we minimize the distances (i.e. find the median $x^* = \frac{1}{2}$) we simply find the mean $E(x) = \int_0^1 x dF(x) = \frac{1}{2}$. In 1-Candidate equilibria the best candidate is the median citizen running truthfully, no matter the distribution of voter preferences (as long as a unique median exists). We saw that it is possible to ensure that the only 1-Candidate equilibrium that can arise has the median entering truthfully.

In 2-Candidate equilibria finding the best candidate configuration is slightly more tricky, from an ex-ante perspective. First, note that separation (in identity[63]) is better than pooling for the voters, as the distance to the extremists (i.e. the maximum distance) is larger for pooling. To minimize the sum of distances, each candidate should get half the votes, and each should position themselves at the median of their own voter base. Precisely, we first split the interval $[0, 1]$ into two equal intervals at the median, and then find the median of each of the two sub-intervals (i.e. of $[0, 0.5]$ and $[0.5, 1]$). Therefore, the best candidates are $x_1 = m_1 = 0.25$ and $x_2 = m_2 = 0.75$, however this is not possible since the candidates have to run with the median message. Therefore, the best candidates are $x_1 = 0.25$ and $x_2 = 0.75$ with $m_1 = m_2 = 0.5$. Note that this candidate configuration minimizes the maximum distance from all voters, as the identity distance from the extremists (or the median) is 0.25, while it would be higher for any other candidate configuration.

From the previous analysis of cases we learnt that when two opposite extremists are capable of forming an equilibrium configuration, then so are all symmetrically opposed individuals at least a distance of a third away from each other. The best candidates are a half distance from one another, and since B is bounded by zero there exist no equilibrium configurations that restict the set of entrants more than the two opposite extremists configuration. We cannot make sure only the best candidates run in equilibrium.

Finding the best candidate configuration in 2-Candidate equilibria from an ex-post perspective is simple. Note that there is always only a single winner, which means that we need to find a single candidate that minimizes the total cost from all the voters. This is the same as in the 1-Candidate case, that is, the candidate that minimizes the distance to all of the voters is the candidate whose identity is the median. Furthermore, note that the median message is also the message the minimizes the distance to all of the voters. Therefore, the honest median entrant (i.e. $x = m = 0.5$) is the best candidate ex-post, irrespective of individual rationality or other equilibrium concerns. This statement is more powerful than it seems at first. It means that, for voters, the equilibrium message politicians pick is optimal, and having two honest entrants is optimal, because it makes sure that the candidate picked will be the best possible candidate.

[62]With non-uniform voter preferences it would be trickier to find the median. It can be shown that the median is the minimizer of the mean absolute error with respect to some random variable X (i.e. $\arg\min_c E|X - c| =$ median). In particular, x^* is a sample median iff for a set of real numbers S it minimizes the sum of absolute deviations, i.e. $\arg\min_c \sum_{s \in S} |s - c|$ where c is the median. Since for the uniform distribution the mean equals the median, we know that if x^* is the median, then $E(|x^* - X|) = 0 = \int_0^1 |x^* - x| dF(x)$, which gives us the same result. We will use a basic version of k-medians clustering for equilibria with more candidates, as the median minimizes the error over all clusters with respect to the 1-norm distance metric.

[63]In messages as well, however we know that in equilibrium both run with the median message.

Overlapping 1 and 2-Candidate Equilibria? I The 1-Candidate equilibrium condition (i.e. non entry condition that has to hold for all potential challengers) is the reverse of the 2-Candidate entry condition.[64] We will consider two examples, which will show that there is a continuum of overlapping 1 and 2-Candidate equilibria. Suppose that $B = 2C - 0.5\beta + \epsilon$ where $C > 0.25\beta - 0.5\epsilon$, which implies that the only 1-Candidate equilibrium that is possible is the median running honestly. For simplicity suppose that $C = 0.25\beta - \epsilon$ which implies that $B = \epsilon$. This results in the following 2-Candidate equilibrium conditions: $\beta(0.5 - |x_1 - x_2|) + \epsilon \leq 0 < 0.5\beta(1.5 - x_1 - x_2) + 2\epsilon$. From this we can see that any two candidate configuration where $1 \leq x_1 + x_2 \leq 1.5$ and $|x_1 - x_2| \leq 0.5$ are possible equilibria. That is, when the only possible 1-Candidate equilibrium configuration is $x_1 = m_1 = 0.5$, there exists a continuum of possible 2-Candidate equilibrium configurations.

Overlapping 1 and 2-Candidate Equilibria? II For the second example first note that for $x_1 > 0.5$ we have that $B < C + s(1 - x_1) - x_1$. Suppose that $B = C + s(1 - x_1) - x_1 - \epsilon$, then the set of potential 1-Candidate equilibrium entrants is $[1 - x_1, x_1]$. For this defined B, we know that $\frac{\partial B}{\partial x_1} = -(1+s) < 0$ which means that as the single entrant becomes more extreme the benefit should also fall. The intuition is that for more extreme citizens a smaller B is necessary for the worst challenger not to be willing to enter,[65] therefore the set of entrants is around the median. If $C > 1 - x_1(2 - \beta)$ then two opposite extremists are not willing to enter,[66] which means that there is no overlap between the two equilibria, i.e. $x_1 > \frac{1-C}{2-\beta}$. Therefore, whenever possible let $1 \geq x_1^n > \frac{1-C}{2-\beta} > x_1^y > 0.5$, then if $x_1 = x_1^n$ there is no overlap and if $x_1 = x_1^y$ then there is an overlap. Finally, note that $\frac{1-C}{2-\beta} > 0.5 \iff 0.5\beta > C$ and $1 > \frac{1-C}{2-\beta} \iff C > \beta - 1$, where only the upper bound is relevant (since $\beta - 1 < 0$). This means that for $0.5\beta > C$ we are in a situation where an overlap is possible for moderate single entrants, however not for more extreme single entrants.

The paper proceeds by looking at 3-Candidate, 4-Candidate and N-Candidate equilibria. The paper will not look into the identity of equilibrium candidate configurations for these cases, but it will identify the possible equilibria. Looking into the actual identity works the same way as for the 2-Candidate case. The way to analyze identity is by identifying the worst challenger, and then looking at the entry and non entry conditions. Identifying the smallest possible cost and looking at the different values B can take, and which individuals would be willing to run for those net benefits. As in the original citizen candidate models the net benefits play a major role in defining how many candidates can be supported in equilibrium, the major addition is that the candidates need to identify individually rational equilibrium messages, and the identity and message costs

[64] For convenience I will state the 1 and 2-Candidate equilibrium conditions here:

$$\text{1-Cand:} \quad B < \begin{cases} C - (2x_1 - 1) - \beta(1 - x_1) & \text{if } x_1 > 0.5 \\ 2C - 0.5\beta & \text{if } x_1 = 0.5 \end{cases}$$

$$\text{2-Cand:} \quad 2C - \beta|x_1 - x_2| \leq B < 3C - \beta\frac{x_1 + x_2}{2}$$

[65] Because the worst challenger loses more from more extreme single entrants.

[66] Since two extremists are willing to enter for the lowest benefits, the following condition ensures that the most extreme 2-candidate configuration does not want to enter: $2C - \beta > B := C - (2x_1 - 1) - \beta(1 - x_1) \iff C > 1 - x_1(2 - \beta)$

will determine which candidates will need the least net benefits to be willing to run. In the 2-Candidate case these were two opposite extremists: from the entry condition we know that they require the least benefits to be willing to run together (as letting the other win certainly hurts the other extremists a lot), while from the non entry condition we know that any two symmetrically opposed candidates hurt the worst challenger the least (making the worst challenger require the highest net benefits to be willing to enter). In 2-Candidate equilibria the candidate configuration that requires the highest net benefits are two same extremists (as they lose the least by letting the other same extremists certainly win, but together they hurt the worst challenger the most).

6.3 3-Candidate Equilibria

The 3-Candidate equilibrium in pure strategies requires that each of the three candidates wants to enter, that no fourth candidate wants to enter, and that no entrant prefers some other message. The three conditions realize to:

1. $1/3\,[B-(1-\beta)|m_3-x_3|]-C \geq -1/6\sum_{i=1}^{3}[\beta|x_i-x_3|+(1-\beta)|m_i-x_3|]$
2. $P_4^{\mathbf{4}}[B-(1-\beta)|m_4-x_4|]-C < \sum_{i=1}^{3}(P_i^{\mathbf{4}}-1/3)(\beta|x_i-x_4|+(1-\beta)|m_i-x_4|), \quad \forall x_4, m_4$
3. Cases:
   - Expected Votes: No message configuration satisfies IR
   - Lucky Votes: Only pooling at median satisfies IR

where $P_i^{\mathbf{4}}$ is the probability that candidate i wins when there are four candidates. These probabilities of winning , when considering a potential challenger, again depend on the messages chosen. The entry and non-entry condition are analogous to the previous two examples, therefore we will not consider them in detail. The interesting result is that no message configuration can survive individual rationality in the Expected votes case, that is no message combination exists such that no candidate prefers deviating to another message. The following result states and proves this formally:

Proposition 16. *In the Expected Votes case no message configuration exists such that a 3-Candidate equilibrium in pure strategies exists, and the only possible equilibrium message configuration in the Lucky Votes case is all three candidates pooling at the median.*

Proof. See Appendix. □

The proof rests upon showing that for every type of message configuration and case (except pooling at median in the Lucky Votes case), there always exists at least one candidate who can change his message by ϵ and guarantee victory.

Three candidates running with the median message is a possible equilibrium outcome in the Lucky Votes case, because no challenger can certainly win. The best a challenger can do is pool with the three entrants, and win with probability a quarter. If the challenger were to run with any other message, one of the three entrants would get more than half the votes, while the challenger would get less than half. Before looking at 4-Candidate equilibria, we will formally show that the worst challenger is an extremists for any number of candidates pooling at the median.

Lemma 17. *For any Lucky Votes N-Candidate equilibrium where all candidates pool at the median, the worst challenger is an extremists, i.e.* $x_{N+1} = 0 \iff \frac{1}{N}\sum_{i=1}^{N} x_i \geq \frac{1}{2}$ *or* $x_{N+1} = 1 \iff \frac{1}{N}\sum_{i=1}^{N} x_i < \frac{1}{2}$.

Proof. See Appendix. □

The above proposition formally showed that the worst challenger is always an extremist, if the entrants pool at the median in the Lucky Votes case. Suppose that $\frac{1}{N}\sum_{i=1}^{N} x_i \geq \frac{1}{2}$, then the equilibrium conditions can be simplified to:

$$4C - \beta\frac{\sum_{i=1}^{3} x_i}{3} > B \geq 3C - \beta\frac{\sum_{i=1}^{2}|x_i - x_3|}{2} \iff C > \beta\left(\frac{\sum_{i=1}^{3} x_i}{3} - \frac{\sum_{i=1}^{2}|x_i - x_3|}{2}\right)$$

Here we identified the lower bound of C from the combined equilibrium conditions, however we also have to take into account that the non entry condition itself imposes a further restriction on C, which gives us:

$$C > \max\left\{\beta\left(\frac{\sum_{i=1}^{3} x_i}{3} - \frac{\sum_{i=1}^{2}|x_i - x_3|}{2}\right), \frac{3B + \beta\sum_{i=1}^{3} x_i}{12}\right\}$$

In order to analyse the candidate identity in 3-Candidate equilibria, we have to solve a similar excercise as for 2-Candidate equilibria. We will not do so, as it will suffice to know that there exist B, C combinations such that 3-Candidate equilibria exist in the Lucky votes case where all three pool at the median.

We will only note two facts arising from the equilibrium conditions. From the non entry condition we know that: (i) the configuration most damaging for the worst challenger is three same extremists (i.e. $4C - \beta > B$), and (ii) the least damaging configuration for the worst challenger is any configuration such that $x_1 + x_2 + x_3 = 1.5$ (e.g. $4C - 0.5\beta > B$). Furthermore, from the entry condition we know that (i) the configuration where each of the three entrants requires the most net benefits to enter is three same extremists (i.e. $B \geq 3C$), and (ii) the configuration where the least benefits are required to be willing to enter are $c_1 = 0,\ c_2 = 1,\ c_3 \in [0, 1]$ (i.e. $B \geq 3C - \beta$). Finally, note that the combined equilibrium conditions for $x_1 = x_2 = x_3 = 1$ and $(c_1, c_2, c_3) = (0, 0.5, 1)$, respectively, are

$$4C - \beta > B \geq 3C$$
$$4C - 0.5\beta > B \geq 3C - \beta$$

6.4 4-Candidate Equilibria

The 4-Candidate equilibrium requires that each of the four politicians wants to enter, that no fifth politician wants to enter, and that no entrant prefers some other message. Let $i, j, l, k \in \{1, 2, 3, 4\}$ where $i \neq j \neq l \neq k$. The three conditions realize to:

1. $1/4\,[B - (1-\beta)|m_4 - x_4|] - C \geq -1/12\sum_{i=1}^{4}[\beta|x_i - x_4| + (1-\beta)|m_i - x_4|]$
2. $P_5^{\mathbf{5}}[B - (1-\beta)|m_5 - x_5|] < \sum_{i=1}^{4}(P_i^{\mathbf{5}} - 1/4)(\beta|x_i - x_5| + (1-\beta)|m_i - x_5|), \quad \forall x_5, m_5$

3. Cases
   - Expected Votes: only semi-separation with $m_i, m_j = \frac{1}{4}$ and $m_l, m_k = \frac{3}{4}$
   - Lucky Votes: semi-separation with $m_i, m_j = \frac{1}{4}$ and $m_l, m_k = \frac{3}{4}$, and pooling at median

The entry and non-entry conditions are analogous to the previous cases, therefore we will again only focus on individual rationality. Unlike in the 3-Candidate situation, there do exist messages that are individually rational in the Expected Votes case.

Proposition 18. *In the Expected Votes case the only possible message configuration is* $m_i, m_j = \frac{1}{4}$ *and* $m_l, m_k = \frac{3}{4}$.

Proof. See Appendix. □

First note that it is impossible to have single candidates at the extremes, as they can unilaterally deviate towards more moderate messages, and ensure they gain more than $\frac{1}{4}$ of the votes. Therefore, separation is impossible. Analogously, pooling is impossible, as each candidate can deviate by ϵ and gain more than $\frac{1}{4}$ of the votes. If three candidates pool, while one states a unique message, the pooling candidates can gain more than $\frac{1}{4}$ of the votes by ϵ deviating in the correct direction, and the unique extremists also has a profitable deviation (as in separation). The only possible configuration is semi-separation, where there are two groups of two candidates each pooling at a quarter and at three quarters (i.e. $m_i = m_j = \frac{1}{4}$, and $m_l = m_k = \frac{3}{4}$).

It is worth commenting the fact that they are groups of two. By having a second politician use the same message, no member of these groups of two can find it profitable to deviate, since any deviation will give themselves less than a quarter of the votes, while the remaining group member will get more than a quarter of the votes. Therefore, groups of two discipline deviations.

Lemma 19. *The worst challenger in the Expected votes case is the further extermist, i.e. if* $\sum_{i=1}^{4} x_i \geq 2 \implies \tilde{x}_5 = 0$. *He will use his cheapest winning lie, i.e.* $m_5 = 0.25 + \epsilon$.

Proof. See Appendix. □

Given that we know the possible equilibrium messages in the 4-Candidate Expected Votes case and the worst challenger, let us check whether an equilibrium is possible in the Expected votes case. The combined equilibrium conditions such that the entry and non entry conditions are simultaneously satisfied is:

$$\overbrace{C - \beta\left(\frac{\sum_{i=1}^{4} x_i}{4}\right) - 0.25(1-\beta) >}^{\text{The worst challenger uses cheapest certainly winning lie}} B \geq$$

$$\underbrace{4C + \beta\left(\sum_{i=1}^{4} |x_i - x_4| - 4|x_3 - x_4|\right) + (1-\beta)\left(\sum_{i=1}^{4} |m_i - x_4| - 4|m_3 - x_4|\right)}_{\text{Exit makes other candidate using same message certainly win}}$$

Note that, since the worst challenger can certainly win the cost of entry has to be small in order for this configuration to be possible in equilibrium.

Proposition 20. *In the Expected votes case, there exist no 4-Candidate equilibria in pure strategies.*

Proof. See Appendix. □

The previous proposition proved that no 4-Candidate equilibria are possible in the expected votes case. This is because a challenger can enter and certainly win. Now we look into the Lucky votes case.

Proposition 21. *Let $i, j, l, k \in \{1, 2, 3, 4\}$ where $i \neq j \neq l \neq k$. In the Lucky Votes case the only possible message configurations are:*

1. $m_i, m_j = \frac{1}{4}$ *and* $m_l, m_k = \frac{3}{4}$*, and*
2. $m_i = m_j = m_l = m_k = \frac{1}{2}$.

Proof. See Appendix. □

The fact that the candidates receive votes based on luck changes the setup significantly. Again candidates run in groups of two (i.e. $m_i = m_j = \frac{1}{4}$ and $m_l = m_k = \frac{3}{4}$) where each candidate gets the same amount of votes ex ante, each individual of each group can at most get the same amount of votes (i.e. half the votes). Unlike in the Expected Votes case, a deviation makes a candidate from the other group certainly win. The entry condition also differs to the Expected votes case, since exit by any candidate makes the other individual stating the same message win with probability 0.5, while each individual in the other group has a probability 0.25 of winning. The non-entry condition also differs, since no challenger can enter and certainly win (nor win with positive probability). At most the challenger can make one of the two groups certainly win. We will not look into the details of these equilibria, however they likely exist.

Additionally, all four candidates can pool at the median. The reason why this is a possible configuration in the lucky votes case is that each candidate gets all of the votes with probability a quarter. If anyone ϵ deviates in any direction, they will get $\frac{1}{2} - \epsilon$ votes certainly while one of the remaining three candidates at the median will get $\frac{1}{2} + \epsilon$ votes certainly. Again, a challenger cannot enter and certainly win, at most he can tie with the other candidates. We will see that these equilibria exist.

It is interesting to note an important reason why separation is impossible in either case. When all candidates run with a unique message, such that they all win with the same probability, then the extremists can always deviate towards the median in order to certainly win. We will now look at some basic results for N-Candidate equilibria.

6.5 N-Candidate Equilibria

The general entry and non entry conditions are as follows:

$$P_N^N\left[B-(1-\beta)|m_N-x_N|\right]-C-\sum_{i=1}^{N-1}P_i^N\left[\beta|x_i-x_N|+(1-\beta)|m_i-x_N|\right]$$
$$\geq -\sum_{i=1}^{N-1}P_i^{N-1}\left[\beta|x_i-x_N|+(1-\beta)|m_i-x_N|\right],\ \forall x_N\in\mathcal{C}_N \tag{14}$$

$$P_{N+1}^{N+1}\left[B-(1-\beta)|m_{N+1}-x_{N+1}|\right]-C-\sum_{i=1}^{N}P_i^{N+1}\left[\beta|x_i-x_{N+1}|+(1-\beta)|m_i-x_{N+1}|\right]$$
$$< -\sum_{i=1}^{N}P_i^{N}\left[\beta|x_i-x_{N+1}|+(1-\beta)|m_i-x_{N+1}|\right],\ \forall x_{N+1},m_{N+1} \tag{15}$$

Where P_i^N is the probability candidate i wins in the configuration with N candidates. Note that the entry condition has to hold for each candidate in that specific configuration, and that the non-entry condition needs to hold for any potential challenger x_{N+1}, and any of his messages. From these we can get a general N-Candidate combined equilibrium condition:

$$P_N^N\left[B-(1-\beta)|m_N-x_N|\right]-\sum_{i=1}^{N-1}\left(P_i^N-P_i^{N-1}\right)\left[\beta|x_i-x_N|+(1-\beta)|m_i-x_N|\right]\geq C>$$
$$P_{N+1}^{N+1}\left[B-(1-\beta)|m_{N+1}-x_{N+1}|\right]-\sum_{i=1}^{N}\left(P_i^{N+1}-P_i^{N}\right)\left[\beta|x_i-x_{N+1}|+(1-\beta)|m_i-x_{N+1}|\right] \tag{16}$$

6.5.1 Equilibrium Messages

Through the previous specific equilibria analyzed, we saw that individual rationality constricts the message space, which simplifies the problem. Recall that politicians are office motivated (no strategic entry). We first consider the Exected Votes case. The following theorem identifies the only messages that satisfy individual rationality in the Expected votes case.

Theorem 22. *[Expected Votes] If politicians are office motivated and their messages satisfy Individual Rationality, then for all candidates in a N-Candidate equilibrium, where $N>1$:*

1. *Equilibrium messages exist only when N is even*
2. *Let $j\in\{i|\forall i \text{ where } i \text{ odd}\}=\{1,3,5,\ldots,N-1\}$ and $z=\frac{j+1}{N}$ for all j. Each message $m_j=m_{j+1}=\frac{j}{N}$ has a group of two candidates $g_z=\{x_j,x_{j+1}\}$ stating it.*
    - *Each group gets $v_z=\frac{2}{N}$ vote share from voters $V_z=[\frac{j-1}{N},\frac{j+1}{N}]$*

Proof. See Appendix. □

This result can be further strengthened if we make an assumption on the behavior of the candidates. Suppose the candidates are utility of entry maximizers, i.e. they pick messages that maximize their utility of entry (*Assumption "UEM"*). Any configuration of messages that satisfies individual rationality allots an equal probability of winning to all candidates. Therefore, each potential entrant prefers configurations where he is using his closest winning message, that is, under UEM each entrant prefers to use the equilibrium message closest to his ideal point. This is true because lying is costly, and closer messages decrease the cost of entry (increasing the utility of entry). We can strengthen the previous theorem with a reasonable result.

Proposition 23. *Under UEM, the identity of the candidates willing to use message m_j (from Theorem 22) is inside the set $I_z = [\frac{j-1}{N}, \frac{j+1}{N}]$*

Proof. See Appendix. □

The above strengthened theorem identifies the only possible message configuration available in the Expected votes case when there are $N > 1$ candidates, as well as the interval within which each candidates' true identity resides. It is easiest to see this through some examples. Suppose $N = 2$, then the message configuration is $\{m_1, m_2\} = \{0.5, 0.5\}$ and the messages can be stated by anyone, as we have previously seen. Now suppose that $N = 4$, then the only possible message configuration is $\{m_1, m_2, m_3, m_4\} = \{0.25, 0.25, 0.75, 0.75\}$, where the messages $m_1 = m_2 = 0.25$, and $m_3 = m_4 = 0.75$, can only be stated by someone whose identity is in the set $[0, 0.5]$, and $[0.5, 1]$, respectively. Finally, suppose that $N = 6$, then the only possible message configuration in the Expected Votes case is $\{m_1, m_2, m_3, m_4, m_5, m_6\} = \{\frac{1}{6}, \frac{1}{6}, \frac{3}{6}, \frac{3}{6}, \frac{5}{6}, \frac{5}{6}\}$, where the possible identities are as follows: $x_1, x_2 \in [0, \frac{2}{6}]$, $x_3, x_4 \in [\frac{2}{6}, \frac{4}{6}]$, and $x_5, x_6 \in [\frac{4}{6}, 1]\}$.

Even though the assumption seems reasonable, there is a component that is too strong. Given that all of the candidates are non strategic entrants and satisfy UEM, then a single non strategic entrant who does not have UEM satisfied would like to pick the configuration where he is part of the group furthest from his ideal point (if lying is cheap enough) since he could remove one of the sincere entrants furthest (in identity) from him from the equilibrium, thereby increasing his chances of getting a winning politician closest to his identity. That is, if all candidates are non-strategic entrants and UEM is satisfied for all but a single candidate, that single non strategic candidate has a strong incentive to enter with the furthest possible lie that equilibrium can sustain.

In summary, in the Expected Votes case we learn from individual rationality that fully separating equilibria (i.e. each candidate states a unique message) are impossible for any $N > 1$, as no such message configuration is individually rational. Pooling (all candidates running with the same message) is only possible when $N = 2$, and the candidates pool at the median. For $N > 2$ the only possible message configuration is where each message is stated by exactly two candidates, and the messages split the voter base equally such that each message gets exactly $\frac{2}{N}$ votes. In other words, the interval $[0, 1]$ is split into $\frac{N}{2}$ equal parts of size $\frac{2}{N}$. Note that this result is similar to the results in [Cox, 1987] and [Osborne, 1993]. Finally, note the following result.

Proposition 24. *For configurations that satisfy individual rationality, and $N \geq 6$, no challenger can certainly win in the Expected votes case.*

Proof. See Appendix. □

Given that no 2- and 4-Candidate equilibria were possible in the Expected votes case, due to the fact that the challenger could certainly win, I suspect that for $N \geq 6$ equilibria will exist in the Expected votes case. Nevertheless, I have not yet looked at the Expected votes case, with $N \geq 6$. Now we look at the Lucky Votes case. The following theorem identifies all of the potential equilibrium message configurations.

Theorem 25. *Two types of N-Candidate (pure strategy) equilibrium message configurations satisfy Individual Rationality are:*

1. *For any N: All candidates pool at the median, for any N*
2. *For any $N \geq 4$ we can find a combination of J groups of $g_1, \ldots, g_J$ such that $\sum_{j=1}^{J} g_j = \#\mathcal{C} = N$ and $g_j \geq 2,\ \forall j \in \{1, J\}$. That is, as long as the most extreme messages are stated by at least two candidates, any configuration of group sizes (symmetric iff N not prime, asymmetric for any N) can be supported in equilibrium.*

For semi-separating equilibrium messages, every group receives an equal vote share $\frac{1}{J}$ using messages $m_j = \frac{1+2(j-1)}{2J}$ receiving votes from voters $V_j = [\frac{j-1}{J}, \frac{j}{J}]$. Only one candidate in each group gets the groups whole voter share with probability $\frac{1}{\#g_j}$.

Proof. See Appendix. □

In the Lucky Votes case in any equilibrium message configuration each message receives the same number of votes, since each candidate has a chance of winning the whole voter share of his message. This is what makes deviations unprofitable in the pooling case, because one of the non-deviating players will receive all of the $0.5 + \epsilon$ votes (deviating player gets $0.5 - \epsilon$). This is also the reason why groups larger than two can be supported in equilibrium.

Furthermore, from the above result we learn several things. For any $N > 1$, no fully separating equilibria exist. Pooling is possible for any equilibrium number of candidates. As long as the most extreme messages are being stated by at least two candidates each, any configuration of group sizes can be supported in equilibrium. This is a bit strange as the next example illustrates. Suppose $N = 13$ and $\#g_1 = 10,\ \#g_2 = 3$ with messages $m_1 = \frac{1}{4},\ m_2 = \frac{3}{4}$. Ex post one of the individuals from each groups gets half of the votes, and ex ante each individual expects they have a positive probability of winning half the votes, however the expected benefits for members of the two groups differ.

Even though such asymmetric group sizes satisfy Individual Rationality one still needs to make sure the expected benefit of running in a larger group is high enough to make each of the entrants choose entry (over non entry), and no extra challenger being willing to enter. Nevertheless, symmetric groups sizes seem more likely to be possible equilibrium configurations, as then all candidates would have the same ex-ante probability of winning. Finally, the above result also tells us that all the groups having the same number of candidates (symmetric group sizes) is only possible when the number of candidates is not a prime number. Symmetric group sizes seem more likely to be possible equilibrium configurations, due to the difference in expected utility between the groups when the group sizes are asymmetric. Finally, note that if more than a single individual is occupying some of the possible equilibrium positions, by exiting the probability of his message being picked remains the same, however through entry he can increase the probability of his identity being represented in equilibrium.

Since the analytically simplest N-Candidate message configuration is the pooling equilibrium in the Lucky votes case, we will look into whether these equilibria are possible.

6.5.2 Pooling equilibria (Lucky votes)

In pooling equilibria all candidates use the same message, i.e. $\forall x_i \in \mathcal{C}_N,\ m_i = 0.5$. The probability of winning for any candidate in a N candidate equilibirum is $P_i^N = \frac{1}{N}\ \forall x_i \in \mathcal{C}_N$. Finally, no challenger can certainly win, the best he can do is propose the median message and win with probability $P_{n+1}^{N+1} = \frac{1}{N+1}$, which also makes all other candidates have the same probability of winning as the challenger. Given that all candidates are using the same message, the N-Candidate entry and non entry conditions can be simplified, and then combined, as follows.

$$\underbrace{\frac{1}{N}\left(B+\beta\frac{\sum_{i=1}^{N-1}|x_i-x_N|}{N-1}\right)\geq}_{\text{Entry condition}} C \underbrace{> \frac{1}{N+1}\left(B+\beta\frac{\sum_{i=1}^{N}|x_i-x_{N+1}|}{N}\right)}_{\text{Non entry condition}}$$

To understand why the message component disappears in both the entry and non entry condition, note that if an extra candidate enters or not is irrelevant for what message is going to be proposed. Looking at the general non entry condition (14), from the LHS of equation we see that the median message will be certainly implemented if any challenger enters, while from the RHS we see that the median message will also be certainly implemented whatever the challenger choice. Both of the sides of the non entry condition will have $(1-\beta)|0.5-x_{N+1}|$, which then cancel out. The intuition is that no matter what the challenger does, the median message certainly arises. This same logic holds for the entry condition. Finally, note that this holds for any pooling message, however from individual rationality we know that the only possible pooling message is the median message. In fact, this same logic holds for all message configurations where an extra entrant cannot affect what message will arise in equilibrium.

Lemma 26. *The worst challenger for any pooling N-Candidate equilibrium is the furthest extremist from the average identity, i.e. if $\frac{\sum_{i=1}^{N}x_i}{N} \geq \frac{1}{2} \implies \tilde{x}_{N+1} = 0$ where each $x_i \in \mathcal{C}_N$.*

Proof. See Appendix. □

This further simplifies the equilibrium conditions. Suppose $\frac{\sum_{i=1}^{N}x_i}{N} \geq \frac{1}{2}$, then:

$$\frac{B}{N}+\beta\frac{\sum_{i=1}^{N-1}|x_i-x_N|}{N(N-1)} \geq C > \frac{B}{N+1}+\beta\frac{\sum_{i=1}^{N}x_i}{N(N+1)},\ \forall x_N \in \mathcal{C}_N \tag{17}$$

It is clear that for high enough benefits and costs, there always exist equilibria where N candidates pool at the median. Furthermore, the RHS (Non Entry condition) is always greater than zero, which ensures that $C > 0$ (given $B > 0$). Notice that as we increase B the LHS inequality (Entry condition) grows at a rate of $\frac{1}{N}$, while the RHS inequality (Non Entry condition) grows at a slower rate $\frac{1}{N+1}$, which means that by increasing B we increase the interval within which C can take values for an N-Candidate pooling (at median) equilibrium to be possible. Therefore, as we decrease B the combined equilibrium condition becomes more constrained, and we can find the lowest $B = \tilde{B}$ such that there exists a non-empty interval for C to exist in:[67]

$$B > \tilde{B}(N) := \beta \left(\sum_{i=1}^{N} x_i - \frac{N+1}{N-1} \sum_{i=1}^{N-1} |x_i - x_N| \right)$$

If we increase the number of candidates by one, i.e. $N' = N + 1$, then the lower bound increases by $\beta \left[x_{N'} + \frac{2}{N(N-1)} (\sum_{i=1}^{N-1} |x_i - x_N| - \sum_{i=1}^{N'-1} |x_i - x_{N'}|) \right] > 0$. That is, increasing the number of candidates by one implies that the smallest value B can take also has to increase. This is similar to the results of the original citizen candidate models, because the number of candidates depends on the net benefits of entry. However, the main difference is that now there exists a message and identity cost that one needs to keep track of, which influences who will be willing and capable of running in equilibrium. We can summarize this in the following proposition:

Proposition 27. *The number of equilibrium entrants depends on the net benefits* (B, C)*, such that more candidates are willing to enter when* B *is higher relative to* C*, given* C *is high enough for no challenger to be willing to enter.*

Proof. See Appendix. □

Before concluding, let's briefly look at the identity effect in pooling equilibria. Suppose that $B = 0$, then the combined equilibrium condition becomes: $\beta \frac{\sum_{i=1}^{N-1} |x_i - x_N|}{N-1} \geq NC > \beta \frac{\sum_{i=1}^{N} x_i}{N+1}$. As $\beta \to 0$ the equilibrium conditions cannot be jointly satisfied. Intuitively, since the median message certainly arises (making the message cost irrelevant for determining the equilibrium), as $s \to 1$ the identity cost becomes irrelevant. Given $B = 0$ this results in no pooling equilibrium being possible. Furthermore, note that $\sum_{i=1}^{N} x_i \geq \sum_{i=1}^{N-1} |x_i - x_N|$ for all x_N, since the first term is the distance to the worst challenger and there is one more addition taking place. This leads to the following result:

Proposition 28. *As the issue becomes very salient* $(s \to 1)$*, any candidate configuration can arise in equilibrium, given* B, C *are such that a N-Candidate pooling equilibrium exists.*

Proof. See Appendix. □

Furthermore, note the dependence of the equilibrium condition (17) on the number of candidates. Let $\hat{B}(N)$ be the value of B such that $\hat{B}(N) \geq NC > \hat{B}(N) - \epsilon$ for all N. We can re-write the equilibrium condition as:

$$\hat{B}(N) + \beta \frac{\sum_{i=1}^{N-1} |x_i - x_N|}{N-1} \geq NC > \frac{N}{N+1} \hat{B}(N) + \beta \frac{\sum_{i=1}^{N} x_i}{N+1}, \ \forall x_N \in \mathcal{C}_N \tag{18}$$

[67] To be fully precise, the lower bound for equilibrium existence also needs to account for the assumption that $B, C > 0$, so the lower bound on B is:

$$B > \max \left\{ \beta \left(\sum_{i=1}^{N} x_i - \frac{N+1}{N-1} \sum_{i=1}^{N-1} |x_i - x_N| \right), 0 \right\}$$

Proposition 29. *For all $s \in (0,1)$, as $N \to \infty$ any identity configuration is possible in equilibrium, given B, C are such that a N-Candidate pooling equilibrium exists.*

Proof. See Appendix. □

It is interesting that as the number of candidates increases to infinity, any identity configuration becomes possible. Intuitively, when N is very large, a new entrant contributes less to the identity cost than the unit increase in the number of candidates. Finally, we can approach the main theorem of the paper. Suppose all candidates were honest, i.e. $x_i = x_j = 0.5$ for all $i \neq j \in \{1, 2, \dots N\}$, then condition (17) can be expressed as:

$$\frac{B}{N} \geq C > \frac{1}{N+1}\left(B + \frac{\beta}{2}\right) \tag{19}$$

Theorem 30. *Let $X^{\mathbf{N}}_{0.5}$ be the set of possible equilibrium configurations when N honest candidates form an equilibrium configuration. Then, the probability of randomly picking the configuration with only honest entrants, out of $X^{\mathbf{N}}_{0.5}$, is zero.*

Proof. Let $\bar{C} = \frac{B}{N}$, which ensures N honest candidates are willing to jointly run, and not challenger is willing to oppose them. Simultaneously, all configurations for whom $\bar{C} > \frac{B}{N+1} + \beta\frac{\sum_{i=1}^{N} x_i}{N(N+1)}$ holds are also possible. That is, when the configuration with only honest entrants is possible in equilibrium, so are any configurations where the sum of distances from the worst challengers is kept the same. Precisely, all configurations such that $\sum_{i=1}^{N} x_i = \frac{N}{2}$, which can be achieved by taking any pair of candidates and moving them symmetrically in opposite directions, keeping the sum fixed. Since there exists a continuum of such alternative configurations, but only a single configuration where all of the candidates are honest, we know that the measure of equilibrium configurations with honest candidates is (almost) zero. □

This result has particularly negative implications. If the institution designer imposed the benefits of winning and cost of entry such that N candidates are willing to enter honestly, the probability of observing such an outcome is (almost) zero. More precisely, when N candidates are willing to run honestly, and are capable of running unopposed, the configuration with N honest entrants is only one of infinite possible configurations where candidates enter dishonestly.

7 Conclusion

This paper looked at the effect credible promises and salience have on politicians decisions to become candidates, that is, on the spatial electoral competition game with endogenous entry and lying. Due to the fact that the paper combines several strands of previous literatures we briefly looked at the relevant papers from the strategic information transmission literature, the spatial electoral games literature and the endogenous entry games. As this paper most directly extends the [Osborne and Slivinski, 1996] citizen candidate model, we looked into it before looking at the modification imposed to analyze credible promises. This paper places candidates in an environment where issues have salience which may force them to implement their promises with a certain probability, while they can renege on their message and implement their ideal point with the complementary probability. To allow for credible promises, I assumed that voters were naive and trusted the promises candidates ran with, even though politicians could sometimes renege on them. This implied that the pool of potential entrants (politicians) had complete information of the game, while the voters were naive, i.e. the pool of politicians was different from the pool of voters. This changed the model from a citizen-candidate to a politician-candidate model.

In addition to the equilibrium conditions of the original citizen candidate framework, strategic information transmission required the promises to be individually rational, i.e. that no one has a message through which they can certainly win. This implies that office motivate politicians use their promises strategically, in order to get as many votes as possible. This allowed us to simplify the analysis by constricting the message space. In other words, knowing the only possible equilibrium messages, we could identify the cost of lying for each possible politician identity. Therefore, after identifying the equilibrium messages, I was in a position to look at the identity of politicians who would self-select into the electoral race, in equilibrium.

The paper looked at the 1-and 2-Candidate equilibria in detail, the 3-and 4-Candidate equilibria in less detail, and finally turned to N-Candidate equilibria, where I identified all messages that are possible in equilibrium and looked into pooling equilibria. The paper finds that 1-Candidate equilibria are a special case, due to the assumption that anarchy (no one running) is infinitely costly. Therefore, in 1-Candidate equilibria there exists a non-empty region where it is possible to ensure the only possible 1-Candidate equilibrium has the median politician running honestly. This is because, when the net benefits are small everyone is willing to enter and no one is willing to be a second entrant. Furthermore, when the single entrant is an extremist, the worst challenger loses the most from letting him win, therefore, as the net benefits increase, the set of potential 1-Candidate configurations is shrinking from the extremes towards the median (whom winning is least costly for any potential challenger).

In equilibria with more than a single entrant, the entry condition stops being trivial, and the combined equilibrium conditions define a region within which the net benefits have to reside. This means that, unlike in the 1-Candidate example, the net benefits are bound above and below. Therefore, in 2-Candidate equilibria there is a strong pull towards the extremes. Again, the non entry condition is such that a potential challenger loses more from two same extremists than from two median entrants. Therefore, he will require smaller net benefits to challenge extremists than to challenger median entrants. However, two same citizens have the smallest incentive to enter together, since non-entry would make their same ideal point win (with probability β). Therefore, the entry condition forms an incentive to oppose the other candidate, that is, the smallest net benefits are required for two opposite extremists to be willing to run, while the biggest for two equal extremists. Due to these opposing pulls, 2-Candidate equilibria are such that for the smallest net benefits the only candidate configuration possible is two opposite extremists. However, as the net benefits increase, a multiplicity of equilibria arises where configurations closer to the median are possible. The main result of this section is that the probability of observing two honest entrants is zero. This result anticipates the main result of the paper. For N-Candidate pooling (at the median) equilibria, the probability of observing N candidates using honest promises is zero. This is a particularly troublesome result, as it implies that it is very likely our candidates are dishonest.

Finally, a major driving force of the possible equilibira is the way votes are assigned. The paper looks at two extreme possibilities: if all candidates state the same message, each receives the same number of votes, and therefore has an equal chance of winning (Expected Votes); and, if all candidate state the same message, each candidate receives all of the votes with the same probability (Lucky Votes). This seemingly small change has a strong effect on equilibria, as the Lucky Votes case makes most deviations unprofitable. In particular, pooling at the median can only be supported as an equilibrium in the Lucky Votes case. That is, when all candidates state the same median message in the Lucky votes case, no candidate can profitably ϵ-deviate, as once the election results are realized one of the pooling candidates would have received more than half of the votes, and the deviator less than half. This allows for a much richer set of equilibria than in the Expected votes case.

A *Brown v. Hertlage* (1982)

One of the earliest famous cases where the US Supreme Court addressed the issue of campaign promises is *Brown v. Hertlage* (1982). In a race for Jefferson County Commissioner the winning candidate promised to lower his salary by $3000 per year, however the candidate was unaware the Kentucky Corrupt Practices Act prohibits candidates from making promises in consideration of a vote. His opponent (the losing candidate) asked for the election to be void, and the Kentucky Court of Appeals indeed declared the office vacant and ordered a new election. The US Supreme Court reversed this on grounds that the promise was not made with malicious intent, nor reckless disregard for the truth; and on grounds that taking the Kentucky Act in such meaning would violate the First Amendment and limit the robust political debate. Here the candidate made a promise he intended to keep, but once in office realized he might be in violation of state law and therefore retracted his initial promise. What follows is the Supreme Court's opinion.

"It's of course true that states have a legitimate interest in preserving the integrity of their electoral processes, but when a State seeks to uphold that interest by restricting speech, limitations on state authority imposed by the First Amendment are manifestly implicated. When a State seeks to restrict directly as this Kentucky Act does, the offer of ideas by a candidate to the voters, the First Amendment requires that the restriction be demonstrably supported not only by a legitimate state interest, but a compelling one, and that the restriction operate without unnecessarily circumscribing protected expression. The Kentucky statute as applied to petitioner's promise violates the First Amendment under that analysis. The Court of Appeals analogized petitioner's promise to a bribe. However too, that may be as a matter of Kentucky law, there's no constitutional basis upon which petitioner's pledge to reduce his salary might be equated with a candidate's promise to pay voters for their support from his own pocketbook, like a promise to lower taxes, to increase efficiency in government, or indeed, to increase taxes to provide some group with a desired public service, petitioner's promise to reduce his salary cannot be deemed beyond the reach of the protection of the First Amendment or considered as inviting the kind of corrupt arrangement, the appearance of which I state may have a compelling interest in avoiding. There's no showing that petitioner made the promise other than in good faith and without knowledge of its falsity, or that he made it with reckless disregard, whether it was false or not. Moreover, petitioner retracted the promise promptly after discovering that it might have violated the Act. Under all the circumstances, nullifying petitioner's election victory was inconsistent with the atmosphere of robust political debate protected by the First Amendment. We therefore reverse the judgment of the Kentucky Court of Appeals and remand for proceedings not inconsistent with our opinion." (Supreme Court opinion from https://www.oyez.org/cases/1981/80-1285;

https://supreme.justia.com/cases/federal/us/456/45/#tab-opinion-1954508)

B *United States v. Alvarez* (2012)

The most recent case, and possibly most important, is *United States v. Alvarez* (2012). A candidate (Xavier Alvarez[68]), known for openly lying, at a board meeting lied that he received the Congressional Medal of Honor. This violated a federal criminal statute (Stolen Valor Act 2005). Alvarez was convicted under the statute, after which the US Court of Appeals (9th Circuit) reversed, finding the Act to be in violation of the First Amendment. In an unrelated case the Court of Appeals (10th Circuit) found the Stolen Valor Act 2005 to be constitutional, so a conflict arose the Supreme Court had to resolve. It found the Stolen Valor Act infringes upon the First Amendment: Alvarezs' lie most likely enhanced the value of the Medal of Honor, since once his lie was exposed he was ridiculed and publicly asked to resign, and the government failed to provide any evidence (beyond general appeals to common sense) suggesting the public's general perception of the Medal of Honor having diminished. Furthermore, it argued that counter speech is the First Amendment's preferred method of dealing with falsities, and that permitting the government the power to decree this speech to be a criminal offense has no clear limiting principle, which clearly shows the Act's conflict with free speech.[69] In this case, the Supreme Court took one of

[68]Was elected to the Three Valley Water District Board in California, and at a board meeting made the false statement in question.

[69]"Lying was his habit. Xavier Alvarez is the respondent here. He lied when he said he played hockey for the Detroit Red Wings and that he once married a starlet from Mexico, but when he lied in announcing he held the Congressional Medal of Honor, respondent ventured on to new ground, for that lie violates a federal criminal statute, the Stolen Valor Act of 2005. Respondent was elected to the Three Valley Water District Board in California. At a board meeting, he introduced himself by claiming that he'd been a marine for 25 years, had been wounded in combat and who won the Congressional Medal – Medal of Honor, and none of these statements were true. The Stolen Valor Act, a federal statute, provides that whoever falsely claims to have won the Congressional Medal of Honor can be fined or imprisoned for up to one year. Alvarez was convicted under the statute, but the United States Court of Appeals for the Ninth Circuit reversed. It found the statute invalid under the First Amendment. After we granted certiorari, the United States Court of Appeals for the Tenth Circuit in an unrelated case found that the Act was constitutional so now, there's a conflict in the circuits. It's right and proper that Congress, over a century ago, established an award so the Nation can hold in its highest respect and esteem those who, in performing the supreme and noble duty of contributing to the defense of the rights and honor of this Nation, have acted with extraordinary valor. Fundamental constitutional principles, however, require that laws enacted to recognize the brave must be consistent with the precepts of the Constitution for which they fought. As a general matter, this Court has permitted content-based restrictions only when they are confined to one of the few historic and traditional categories of expression, defamation, obscenity and fraud are among these few categories of punishable speech. Absent from those few categories, where the law does allow content-based restriction of the speech, is any general exception to the First Amendment for false statements. A federal criminal statute does prohibit lying to a government official, but statutes of that sort are inapplicable here. This Court has not endorsed the categorical rule that false statements receive no First Amendment protection. By its plain terms, the Stolen Valor Act applies to speech made at anytime, in any place, to any person and it does so entirely without regard to whether the lie was made for the purpose of material gain. Permitting the Government to decree this speech to be a criminal offense, whether shouted from the rooftops or made in a barely audible whisper, would endorse government authority to compile a list of subjects about which false statements are punishable. That governmental power has no clear limiting principle. All this suffices to show that how the Act conflicts with free speech principles. But even when examined in its own narrow sphere of operation, it cannot survive. In assessing content-based restrictions on protected speech, we've applied the most exacting scrutiny. The Government has a legitimate and even compelling interest in protecting the integrity of its system of military honors, especially with regards the Congressional Medal of Honor, and the opinion recites the

the clearest positions on the importance of free speech, as demonstrated in their final opinion: "*The Nation well knows that one of the costs of the First Amendment is that it protects the speech we detest as well as the speech we embrace. Though few might find respondent's statements anything but contemptible, his right to make those statements is protected by the Constitution's guarantee of freedom of speech and expression.*" As argued in [Hasen, 2013] and [Sellers, 2018], the Supreme Court made lying far more likely excusable in the face of the law, and this case will have an effect on all future decision the Supreme Court takes about campaign promises.

C Evidence from Public Projects

There exists evidence that the cost on public projects is often underestimated. In fact, [Flyvbjerg et al., 2002] find that out of 258 transportation infrastructure projects worth 90 billion US dollars, with overwhelming statistical significance the cost estimates used for deciding whether to build the projects are systematically underestimated. In fact, for a randomly selected project there exists a 86% likelihood the actual cost will be larger, and only 14% of them being smaller or equal. Furthermore, they find that actual costs are on average 28% higher than estimated costs (sd=39). Similarly to a series of papers on a smaller sample about the same topic ([Wachs, 1990], [Wachs, 1989]), they conclude that economic and political reasons may dominate, in particular that deception and lying are used for gain as tactics to get the project going and to make them appear profitable. Similarly, [Flyvbjerg et al., 2005] find that forecasters generally do a poor job of estimating demand for transportation infrastructure projects, with systematic overestimation of demand. Furthermore, over the 30-year period forecasts have not improved. Again, they identify political and economic reasons (power and profit) as the main reasons for this overestimation. This pattern is further confirmed in [Flyvbjerg, 2008]. This suggest that better informed agents in an asymmetric information environment may benefit from deceiving the principal, and they attempt to do so.

D Evidence from Revenue Forecasts

The literature that empirically analyzes the difference in revenue forecasts and revenue realizations seems to have found two results: revenue forecasts do not use all available information and

history of – of the Medal. ... The restriction on speech must be necessary to achieve the Government's interest. There must be a direct causal link between the restriction imposed and injury to be prevented. The Government has failed to demonstrate this link. Beyond general appeals to common sense, the Government provides no evidence suggesting that the public's general perception of military medals is diminished by false claims like those Alvarez made. In fact, the contrary appears true. Counter speech has long been the First Amendment's preferred method for responding to falsity. In this case, the record demonstrates that even before the FBI began its investigation, respondent was perceived as an impostor. Once his lie was exposed, he was ridiculed online and his resignation was called for publicly. The outrage over respondent's lie of anything served to reenforce the public's respect for the Medal and its true recipients. The Nation well knows that one of the costs of the First Amendment is that it protects the speech we detest as well as the speech we embrace. Though few might find respondent's statements anything but contemptible, his right to make those statements is protected by the Constitution's guarantee of freedom of speech and expression. The Stolen Valor Act infringes upon speech protected by the First Amendment." (Supreme Court opinion from https://www.oyez.org/cases/2011/11-210; https://supreme.justia.com/cases/federal/us/567/709/)

by improving forecasting methods one can improve forecasting accuracy[70]; and forecasting errors follow the economic cycles. Politicians may want to understate their revenue to calm demand for public spending, or overstate it if they want to push for public spending. For example, using the same regression-based approach [Feenberg et al., 1989] and [Rider, 2002] find a downward bias in revenue forecasts, indicative of time inconsistency. Similarly, using non-parametric methods [Campbell and Ghysels, 1995] find support for biased forecasts. Regarding the relation of the bias to the economic cycle, for example [Paleologou, 2005] find that expenditure is procyclical in the UK, while [Ohlsson and Vredin, 1996] find evidence that fiscal policy is countercyclical in Sweden. Finally, [Auerbach, 1999] finds evidence that in the pre-Clinton era (1986-1993) the forecast revisions were indicative of overstatement[71], while during the Clinton era (1993-1999) the forecasts revisions are indicative of understatement. This may be in line with the tax increases Bill Clinton signed, that is, by understating the revenue a tax hike is more easily justified.

E Case Study: Kolinda Grabar-Kitarović further details

Croatia is a highly divided society, a good example of which is the opinion about Josip Broz Tito. Tito was a key leader in the antifascist battle in 1945. He united the Croatian partisans to overthrow the fascist Croatian state (NDH, 1941-1945) lead by Ante Pavelic, who were Nazi sympathizers. Tito managed to unite enough differing factions, and people, to help end fascism in Yugoslavia (which included Croatia). Furthermore, Tito managed to liberate socialist Yugoslavia from the Soviet's hold, and created one of the most successful independent socialist experiments. For this he is celebrated. Nevertheless, as the party leader of *the* party, and the highest ranked military figure, he was the de facto autocratic leader of Yugoslavia from 1945 until his death in 1980. During this time (and during the war 1941 - 1945), Tito was also famous for horrific acts. Two prominent examples are the massacres of Italians called "Foibe" in the period 1943 - 1945, the political prison/labour camp (pretty much the equivalent of a concentration camp) "Goli Otok" where dissidents were sent in the period 1949 - 1989. Even though he tried creating a federation of equal republics (with a rotating presidency amongst them), political opposition to his view of Yugoslavia did not exist, at least not for very long. For this, he is hated. Clearly, there are other good and bad things Tito did.

During the 2014 Presidential campaign, the incumbent President, Ivo Josipovic, took the opposing stance; that due to a vast array of reasons, he would not remove Tito's bust from the office of the President (as he didn't in his previous term).[72] On January 11th, Kolinda Grabar-Kitarovic won the elections and became the first female president. In one of her first appearances on national TV, during the campaign, when asked about renaming of a square called "The Square of Marshall Tito", she gave a non-answer by saying that it's a tough question. Generally, she was criticised for not being interesting at all in that interview.[73] Furthermore, there seems to be evidence that Kolinda's stance on Tito was too soft during the nascent of her campaign, for the

[70] All of the cited articles in this paragraph find this result.

[71] They overstated the revenue forecasts, and therefore had to revise them downwards)

[72] https://www.index.hr/vijesti/clanak/Josipovic-Svi-su-razumjeli-poruku-iza-uklanjanja-Titove-biste/808726.aspx, http://hr.n1info.com/a21305/Vijesti/Suceljavanje-Josipovic-napadao-a-Kitarovic-se-branila.html

[73] https://www.index.hr/vijesti/clanak/Kolinda-bez-ijednog-stava-Jedina-zanimljiva-bila-je-muha-na-njenoj-glavi/772704.aspx, http://predsjednicki-izbori.com/vijesti/predsjednistvo-hdz-a-predsjednicka-kampanja-grabar-kitarovic-je-blijeda/

HDZ party[74], which she was a member of before being elected president. The HDZ party leader Tomislav Karamarko stated that she needs to be firmer about Tito, unequivocally calling him a criminal.[75] Karamarko has lead a politics of trying to remove pro-Tito and pro-Communist sentiments that permeate Croatian society who oftentimes think of Tito as one of the best politicians from Croatian history. This stance is visible from statements such as: "I am an anti-fascist, and we all are anti-communist and anti-totalitarian, and it is our [HDZ party] duty to create a society that will be emancipated from any totalitarian type of politics."[76] If we couple this with the fact that at the inaugural speech Kolinda gave, Karamarko was right next to her (closer than her family), and gave a speech celebrating the first Croatian president Franjo Tudman, it seems that the leader of HDZ may have had a strong influence on Kolinda.

Another known promise was that she will move the office of the president to a cheaper, more central location in the capital Zagreb (promise broken). It was one of her major promises. As she took office, she tried moving the office, however it turned out that no Government building in the center of Zagreb satisfied the security protocols necessary for a presidential residence, and furthermore, any that may have satisfied the security protocols would require moving some govenrmental office at great cost. She never moved the presidential residence.[77] This seems to suggest that either she wasn't aware that it was impossible to move the residence, or she purposefully lied that she wanted to move residence. It is not implausible that she was not aware of the great cost moving the residence would involve, and the salience of the issue post-election was not high enough for her to be willing to keep her promise.

F Extended Theoretical-Literature Review

F.1 Spatial competition

The seminal papers of two firm/party spatial competition are [Hotelling, 1929] and [Downs, 1957], which sprouted a research agenda in politics and political economics about spatial electoral competition. One of their main results is the principle of minimum differentiation/median voter theorem: when two parties compete for voters, the party positions converge to the median. Any other message can be beat by going closer to the median. The spatial competition amongst firms was then generalized to n firms in [Eaton and Lipsey, 1975], who showed that the principle of minimum differentiation is largely dependent on feature of the competition, most importantly on the restriction to exactly two parties competing. Note how the basic spatial competition model changes in non-trivial ways. First, positioning oneself next to another candidate (or the median) does not unabmbigously increase the probability of winning. Second, it is no longer the dominant strategy for citizens to sincerely vote for their preferred candidate, that is there exists situations when the citizen will vote strategically to make sure their less preferred candidate does not win.

[74]The two main parties in Croatia are HDZ on the right, and SDP on the left.

[75]http://predsjednicki-izbori.com/vijesti/karamarko-je-nezadovoljan-s-kolindom-ce-obaviti-razgovor-ona-mora-tita-nazivati-zlocincem/, or https://www.jutarnji.hr/vijesti/hrvatska/karamarko-je-nezadovoljan-s-kolindom-ce-obaviti-razgovor-ona-mora-tita-nazivati-zlocincem/570016/

[76]https://www.nacionalno.hr/prof-zdravko-tomac-tomislav-karamarko-bio-je-u-pravu/

[77]https://www.index.hr/vijesti/clanak/godina-dana-kraljice-balkana-neispunjena-obecanja-u-sjeni-druzenja-s-fasistima-i-kriminalcima/876234.aspx, https://www.telegram.hr/politika-kriminal/slucaj-visoka-kako-je-kolinda-grabar-kitarovic-prekrsila-svoje-najvece-obecanje-iz-predizborne-kampanje/

The main theorem in [Eaton and Lipsey, 1975] finds that the necessary and sufficient conditions for an equilibrium with n firms is that (i) no firm's market is smaller than any other firm's half market[78] and (ii) each peripheral firm[79] is paired[80] with a neighbor. The proof is intuitive, (i) if the half market was bigger, the losing firm could capture that half market and unambiguously win, and (ii) if it weren't paired the peripheral firm could move away from the boundary and unambiguously win. Furthermore, note that none of the equilibria have more than two candidates bunched at a single location, and candidates are relatively dispersed. This was already conjectured by EH Chamberlain ([Cox, 1990], pg 182) in 1933, and formally shown in [Denzau et al., 1985]. The logic is that whenever two candidates get too close, any middle candidate gets squeezed, and therefore finds it profitable to move. This prevents convergence of more than two candidates, and ensures some dispersal.

The most relevant contributions from the basic spatial electoral competition literature (without entry), for the current paper are [Cox, 1987] and [Cox, 1990]. The purpose of these papers is to study the effect of different voting systems on equilibrium candidate strategies. More precisely, Cox identifies equilibrium candidate configurations for different voting systems when candidate positions are chosen simultaneously on a unidimensional policy space, the distribution of ideal points is uniform on $[0, 1]$, voters have single peaked symmetric preferences and vote sincerely deterministically (not probabilistically) for $n \geq 3$ candidates who compete for $k \geq 1$ positions. We will only outline the main results on single member elections ($k = 1$) with plurality rule, as this describes one of the building blocks of the current model.

[Cox, 1987] finds that for arbitrary distributions of preferences the same result about bunching (no more than two at any position), that peripheral candidates are paired, and also demonstrates that multicandidate equilibria are non-centrist (that is, some candidate(s) are positioned outside $(Q[\frac{1}{n}], Q[\frac{n-1}{n}])$, where $Q[\alpha]$ is the αth percentile of the distribution of voter ideal points).[81] Furthermore, by focusing on the uniform distribution Cox finds that there are no Nash equilibria when there is an odd number of candidates, and when there is an even number of candidates the unique Nash equilibrium has two candidates at each of the $\frac{n}{2}$ points $(\frac{1}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n})$. I find a similar set of results for what promises the candidates will be willing to make in equilibrium, however in the current model these are only necessary conditions. Furthermore, the current model looks at equilibrium conditions with the same tie-breaking rule as [Cox, 1987], and for a different tie-breaking rule.

An excellent survey of the spatial electoral competition literature, where the entry decision is largely treated exogenously, is [Shepsle, 1991]. A survey which focuses (not exclusively) on the median voter theorem (i.e. convergence in two candidate elections), and the conditions necessary for it to hold, and when it fails, is [Osborne, 1995].

[78] Market is the set of consumers purchasing that firms product, while half market is the customers on the left or right side of the firm location, before the threshold defining the customers who buy from some other firm. The long market would be the side of the market with more customers in it, while the short side of the market has less customers in it.

[79] A peripheral firm has the boundary of the product space on one side, while an internal firm is surrounded by other firms.

[80] Paired in this context means that receives the firms occupy essentially the same position

[81] Note that [Eaton and Lipsey, 1975] find that for arbitrary distributions the number of equilibrium candidates is bounded by the number of modes of the distribution of voter preferences, that is, let r be the number of modes, then a necessary equilibrium condition is $n \leq 2r$.

F.2 Endogenous Entry

When moving from two to three candidate situations, note that the third candidate may want to enter strategically (i.e. not for winning, but making his preferred candidate win). One of the first articles to consider the strategic entry incentives of a third candidate is [Palfrey, 1984]. In Palfrey's model the third candidate chooses his platform after observing the simultaneous choice of the two incumbents. The main result is that in equilibrium the two incumbents choose divergent platforms, but not too divergent, while the entrant loses.[82]

A related generalization is [Osborne, 1993], who studies a simultaneous move candidate entry game, and a sequential entry game. The main result of the section on simultaneous moves is that pure strategy Nash equilibria are possible for almost no distribution function of voter ideal points.[83] The main result of the sequential entry game is that when there are three potential candidates, only one decides to enter. The logic is intuitive, when one candidate enters, a second entrant can always be made to lose by a third candidate, therefore the second (and consequently third) potential candidates decide not to enter at all.[84]

A related important paper on endogenous entry is [Feddersen et al., 1990], where voters vote strategically (for the candidate that maximizes their expected utility, given other voters' expected strategies). One of their main results is that the number of candidates in equilibrium will be driven by the expected net benefits of holding office. This intuitively follows from the fact that in equilibrium each candidate has the same probability of winning, so an extra candidate drives the expected benefits of winning down. Note that when all candidates have the same probability of winning in equilibrium, each candidate is pivotal. This implies that each voter votes sincerely for their preferred candidate, because they would not want to pivot the election outcome towards a less preferred candidate.

The main theorem of their paper defines three equilibrium qualities: (i) all candidates enter at the median voter's position, (ii) more than 1 candidate enters, but less than the net benefit-cost ratio, and (iii) all voters vote sincerely. This result rests on two specific features. First, concavity of the voter utility function, where no equilibrium with three or more policies can exist, since any non-median voter prefers to get the median certainly than the expected value of the three policies. Note that strategic voting makes a very strong implicit assumption: voters coordinate on which among the many candidates using the median position to vote for, such that each candidate gets exactly the same number of votes. This does not arise out of expected votes[85], but through an artificial requirement that each candidate using the same policy gets exactly the same number of votes. Second, the equilibrium is supported by specific out-of-equilibrium beliefs. If any candidate deviates to a non-median message, all voters who don't prefer the deviators position coordinate on voting for exactly one of the voters running with the median message. This makes the deviation itself non-profitable, making the equilibrium consistent. As we will see,

[82] Note that the third candidates objective is to maximize votes, and not to win, therefore he can enter and lose. If his objective was to win, then he would prefer non-entry. Therefore, when candidate enter with the objective of winning, in equilibrium they will all have the same probability of winning.

[83] As we have seen in the previous section detailing the results of Cox, pure strategy Nash equilibria do exist for certain distributions, specifically the uniform distribution in Cox.

[84] They employ the subgame perfect equilibrium, which implies that if two candidates enter they need to employ equilibrium policies (i.e. propose the same position, or symmetrically opposed around the median), which is also the reason why a third candidate can always make the second entrant lose.

[85] That is, each voter votes for each of the candidates using the same policy with equal probability.

this will feature in the current paper as the second type of tie-breaking rule considered.

F.3 Strategic Information Transmission

I have extended the CC framework in order to analyse what effect the fact that citizens know that candidates can credibly lie has on the quality of candidates, i.e. citizens know that candidates can say they will implement some policy other than the one that they will actually implement. In order to do this, the paper partly incorporates the information transmission literature into the CC framework. The seminal article on information transmission is [Crawford and Sobel, 1982], where they show that for an arbitrary finite message space there exists a partition where senders would want to send messages in well defined intervals. More intuitively, they show that in a sender-receiver game, the sender will want to send well defined messages which the receiver can interpret (for particular parameters). These messages can range from completely precise (the receiver knows exactly what the sender is saying), to completely imprecise ("cheap talk" - the receiver gets no information from the sender message). The most direct test of the [Crawford and Sobel, 1982] model is done in [Cai and Wang, 2006]. They essentially find that senders send more than they should and receivers trust senders more than they should. This intuition may be important for lying in a political game; i.e. voters may often trust politicians more than they should, and the politicians may oversignal.

The information transmission literature has been extended in many directions, and there have been several interesting articles that cover the topic of lying. [Kartik, 2009] looks at the effect of having individuals with different costs of lying, and finds that better candidates will separate from the worse ones (who will have to pool). Another interesting article by the same author is [Kartik and McAfee, 2007] that looks at the effect of politicians having character. Finally an interesting article is [Callander and Wilkie, 2007], which allow some candidates the possibility of lying, while having others as honest types. They find that the honest types discipline the bad types' statements. The main difference of my approach to this strand of literature is that I allow all politicians to lie, i.e. the possible politicians do not differ in the respect of having or not having the possibility of lying, instead, the politicians only differ in their underlying preferences. As we will see, this makes the cost of lying arise endogenously in equilibrium.

G Proofs

Theorem 1. *[Equilibrium] Necessary and sufficient conditions for configuration $\mathcal{C}_N$ to form an Subgame Perfect Nash Equilibrium:*

1. *Each candidate prefers entry, over non-entry ("Entry Condition")*

$$\mathbb{E}_{x_i}(E|\mathcal{C}_N) \geq \mathbb{E}_{x_i}(N_e|\tilde{\mathcal{C}}_{N,i}\}), \quad \forall c_i \in \mathcal{C}_N \tag{2}$$

2. *Every x_{N+1} citizen prefers non-entry, over entry ("Challenger Non-Entry Condition")*

$$\mathbb{E}_{x_{N+1}}(E|\mathcal{C}_{N+1}) < \mathbb{E}_{x_{N+1}}(NE|\tilde{\mathcal{C}}_N), \quad \forall x_{N+1} \in \mathcal{X} \tag{3}$$

3. *No candidate prefers unilateraly deviating to another message ("Individual Rationality")*

$$\mathbb{E}_{x_i}(E|\mathcal{C}_N, m_i) \geq \mathbb{E}_{x_i}(E|\mathcal{C}'_N, m'_i) \; \forall m'_i \neq m_i \tag{4}$$

Proof. In the third stage voting is mechanical. In the second stage IR ensures that no candidate has a profitable deviation, therefore the promises are a nash equilibrium of the second stage (given the mechanical voting in the third stage). Finally, using backward induction in the first stage the entry condition ensures no candidate has a unilateral profitable deviation (i.e. each entrant prefers to enter than to exit), while the non-entry condition ensures that no further candidate finds it profitable to enter (i.e. no third politician finds it profitable to enter). Therefore, at each stage of the game a Nash Equilibrium is being played, and no player has a profitable deviation. □

Proposition 2. *A political equilibrium in pure strategies exists*

Proof. Anarchy (no candidates) is infinitely costly, so everyone prefers entry to anarchy, and the net benefits (B, C) can be made negative enough so that no second candidate is willing to run. □

Lemma 3. *In equilibrium*

- *with strategic entry, all entrants who have a positive probability of winning have the same probability of winning*
- *without strategic entry, each candidate has the same probability of winning.*

Proof. With no strategic entrants, each candidate enters to win, so will enter if only if he has a positive probability of winning. Since all have to share the same number of votes for there to be multiple candidates with a positive probability of winning, they all have the same probability of winning. With strategic entrants, the strategic entrants certainly lose, however every non-strategic entrant (potential winner) again needs to get the same number of votes to be willing to enter in equilibrium. □

Proposition 4. *There exists no 1-Candidate equilibrium where the single entrant runs with a lie.*

Proof. $P_1^{\mathbf{1}} = 1 \; \forall x_1, m_1 \implies V_1(\mathcal{C}_1) = 1 \; \forall m_1$, which then implies

$$\langle B - |x_1 - m_1| - C \rangle_{\{m_1 = x_1\}} = B - C > B - |x_1 - m_1| - C \; \forall m_1 \neq x_1$$

Whenever a candidates is running unopposed with a lie, in the message stage, changing his message to an honest one ($m_1 = x_1$) is always a profitable deviation, since the vote share of the single entrant does not change with the message choice of the sole entrant. □

Lemma 5. *Challenger (x_2) always uses cheapest winning (or almost winning) message.*

Proof. Suppose he did not. Then he would not change the election outcome, but incur the cost of entry. If $x_1 > 0.5$ then $m_2 \in (1 - x_1, x_1)$, while if $x_1 = 0.5$ then $m_2 = 0.5$. Note that when $x_1 > 0.5$ then $m_2 \notin \{1 - x_1, x_1\}$, since x_2 has an ϵ deviation towards median which discontinuously increases the probability of winning from 0.5 to certainty. This discontinuity plays a major role in all equilibria. □

Lemma 6. *The worst challenger is an extremist, and if the non-entry condition is satisfied for the worst challenger, it is satisfied for any other type of challenger.*

Proof. The proof rests upon the fact that the further we go from the identity of the entrant, the loss from non-entry keeps increasing, the gain from entry keeps increasing, and the fact that $\beta \in (0, 1)$.

- $x_1 = 0.5 \Rightarrow B - 2C < -\beta|0.5 - x_2|$ and $\frac{dRHS}{dx_2}\Big|_{\{x_2=0.5\}} = -\beta < 0$
  Moving x_2 from 0.5 to extremes decreases the upper bound $B - 2C$ at rate $\beta > 0$ further constraining the above inequality.

- $x_1 > 0.5 \Rightarrow B - C < (1-\beta)|m_2 - x_2| - |x_1 - x_2|$
  - $x_2 \in (1 - x_1, x_1) \Rightarrow$ challenger can enter honestly and win.
  - $x_1 > 0.5 \Rightarrow x_2$ uses message $|m_2 - 0.5| < x_1 - 0.5$
  - Worst honest challenger: $x_2' = 1 - x_1 + \epsilon \Rightarrow RHS = 1 - 2x_1 + \epsilon$
  - A challenger closer to 0 by k (i.e. $x_2'' = 1 - x_1 + \epsilon - k$) running with cheapest winning lie:
  $$RHS(x_2'') = RHS(x_2') + (1-\beta)k - k = RHS(x_2) - \beta k$$
  which is maximized at $\max\{k\} = 1 - x_1 + \epsilon$ when $x_2'' = 0$. In other words, the effect of changing the worst honest challenger to a slightly worse (identity wise) dishonest type changes the RHS by $(1-\beta)k - k = sk - k = -k(1-s) < 0$. Since the probability of being forced to implement the lie is $0 \leq s \leq 1$, we know that $sk < k$. In fact, for $0 \leq l < 1$ we know that the worst challenger will always be the opposite most extreme dishonest candidate using his cheapest winning lie. When $s = 1$ the candidate will always be forced to implement his message, only then will the worst honest challenger be an equally bad challenger as any dishonest challenger (on opposite side of the median) using his cheapest winning lie. Since the model only looks at $s \in (0, 1)$, the proof is completed.

- The worst challenger most constrains the Non-Entry condition, which implies that if it is satisfied for the worst challenger, it is also satisfied for any other potential challenger. That is, when the worst challenger is unwilling to enter, no one is willing to enter.

□

Proposition 7. *For a given candidate x_1, an increase in salience decreases the incentive for challengers to enter.*

Proof. From condition 5 we see that $\partial RHS/\partial s = |m_2 - x_2| > 0$, which implies that as salience increases the inequality 5 is relaxed. Ceteris paribus, this means that a given single entrant x_1 is more likely to be able to run unopposed, because the increase in salience increased the cost of running for the worst challenger, thereby decreasing his incentive to enter. □

Proposition 8. *In any 2-Candidate equilibrium candidates pool at the median, i.e. $m_1 = m_2 = 0.5$*

Proof. Citizens have available actions: $\{E \times [0, 1], N_e\}$

- Neither message can certainly win (since sure loser would prefer non-entry in that case)
  - $P_1^2 = P_2^2 = \frac{1}{2} \Rightarrow (m_1, m_2) = \begin{cases} (0.5 - k, 0.5 + k) & \text{where } k \in [0, 0.5] \\ (l, l) & \text{where } l \in [0, 1] \end{cases}$

- No symmetric message configuration can arise in eq'm

– Each candidate has ϵ deviation towards median, which makes him certainly win (rather than with prob 0.5)

- No pooling with $m \neq 0.5$ can arise in eq'm

  – Each entrant has ϵ-deviation towards median to certain victory

$\square$

Lemma 9. *If entrants pool (i.e. $w_p = m_1 = m_2$) the challenger will run with the least costly winning lie. Specifically:*

$$w_p \in \begin{cases} \left[0, \frac{1}{3}\right) & \Rightarrow \bar{m}_p^{\mathbf{3}} \in (w_p, 1] \\ \left(\frac{2}{3}, 1\right] & \Rightarrow \bar{m}_p^{\mathbf{3}} \in [0, w_p) \\ \left[\frac{1}{3}, \frac{2}{3}\right] & \Rightarrow \bar{m}_p^{\mathbf{3}} \in \left\{\left(\frac{2}{3} - w_p, w_p\right), \left(w_p, \frac{4}{3} - w_p\right)\right\} \end{cases}$$

where $\bar{m}_p^{\mathbf{3}}$ denotes the message a challenger can use to certainly win, when two candidates are pooling.

Proof. If the candidates pool (i.e. $w_1, w_2 \in \underline{w}_p^{\mathbf{2}}$), the challenger ($x_3$) will never pool with them since he can move by ϵ and win certainly. This ϵ move increases his benefit of entry by:

$$B - (1-\beta)|(w_1 \pm \epsilon) - x_3| - \frac{1}{3}\left[B - (1-\beta)|w_1 - x_3|\right] =_{\{\epsilon \to 0\}} \frac{2}{3}B$$

Since $B > 0$ we know that the candidate prefers to infinitesimally lie to get the discontinuous jump in the probability of winning. Furthermore, when the candidates pool the challenger cannot strategically enter (i.e. he cannot make his preferred candidate win). Therefore, when $w_1 = w_2$ the challenger will always run with a message that makes him certainly win.

To get exact expressions for the sets, the challenger needs to get more votes than either of the candidates. If $w_p \leq \frac{1}{2}$. If $w_p \in \left[0, \frac{1}{3}\right)$ then the set of winning messages for the challenger is $w_3(w_p) \in (w_p, 1]$, since for any message in this interval gives more votes to challenger than either of the candidates. If $w_p \in \left[\frac{1}{3}, \frac{1}{2}\right]$ then $w_3(w_p) \in \left\{\left(\frac{2}{3} - w_p, w_p\right), \left(w_p, \frac{4}{3} - w_p\right)\right\}$, where the boundary values of the winning sets of w_3 are obtained as follows:

$$\begin{aligned} w_3 < w_p \text{ wins} \quad &\Longleftrightarrow \frac{w_3 + w_p}{2} > \left(1 - \frac{w_3 + w_p}{2}\right)\frac{1}{2} \\ w_3 > w_p \text{ wins} \quad &\Longleftrightarrow \frac{w_3 + w_p}{4} < 1 - \frac{w_3 + w_p}{2} \end{aligned}$$

Note that $\left(\frac{1}{3}, \frac{1}{3}\right) = \emptyset$. Symmetrically, if $w_p \geq \frac{1}{2}$. If $w_p \in \left(\frac{2}{3}, 1\right]$ then the set of winning messages for the challenger is $w_3(w_p) \in [0, w_p)$. If $w_p \in \left[\frac{1}{2}, \frac{2}{3}\right]$ then $w_3(w_p) \in \left\{\left(\frac{2}{3} - w_p, w_p\right), \left(w_p, \frac{4}{3} - w_p\right)\right\}$. $\square$

Lemma 10. *The worst challenger is an extremist, in both the expected and lucky votes case. That is, if $\frac{x_1+x_2}{2} \geq \frac{1}{2}$ then the worst challenger is $\bar{x}_3 = 0$.*

Proof. Expected Votes: A challenger will use his cheapest winning lie (Lemma 9). The wrost challenger using a winning message is an extremist using his cheapest winning lie:

$$B - C + (1-\beta)\left(\left|\frac{1}{2} - x_3\right| - |m_3'(x_3) - x_3|\right) + \beta\frac{\sum_{i=1}^{2}|x_i - x_3|}{2} < 0$$

Wlog SSE $\frac{x_1+x_2}{2} \geq \frac{1}{2}$:

- Worst honest entrant:
$$\arg\max_{x_3:x_3=m_3'}\{LHS\}=\frac{1}{6}+\epsilon$$

- Does a lying entrant require less net benefits than the worst honest entrant, to prefer entry?
$$\frac{dLHS}{dx_3}\Big|_{\frac{1}{6}+\epsilon}=(1-\beta)0+\beta K \quad \text{where } K=\begin{cases}0 & \text{if } x_1\le x_3<x_2 \text{ or } x_2\le x_3<x_1\\ -1 & \text{if } x_3<x_1\le x_2 \text{ or } x_3\le x_2<x_1\end{cases}$$
meaning, the lowest x_3 (i.e. an extremist) makes the non entry condition least likely to hold.

Lucky Votes: In equilibrium we know that $m_1=m_2=0.5$, hence $m_3\neq 0.5$ is a losing message which does not change the election outcome, but costs C, hence in equilibrium $m_3=0.5$. We look for the candidate who gains the most from entry:

$$-C+\frac{B-\sum_{i=1}^{3}(\beta|x_i-x_3|+(1-\beta)|m_i-x_3|)}{3}<-\frac{\sum_{i=1}^{2}(\beta|x_i-x_3|+(1-\beta)|m_i-x_3|)}{2}$$
$$-C+\frac{B}{3}+\frac{\sum_{i=1}^{2}(\beta|x_i-x_3|)}{6}<0$$

Wlog SSE $\frac{x_1+x_2}{2}\ge\frac{1}{2}$ and $x_1\le x_2$:

$$\frac{dLHS}{dx_3}=\beta K \quad \text{where } K=\begin{cases}\frac{1}{3} & \text{if } x_1\le x_2\le x_3\\ 0 & \text{if } x_1\le x_3<x_2\\ -\frac{1}{3} & \text{if } x_3<x_1\le x_2\end{cases}$$

$\Rightarrow \bar{x}_3=0$ maximizes LHS □

Proposition 11. *In the Expected Votes case no 2-Candidate equilibria in pure strategies exist.*

Proof. Suppose $x_1+x_2\ge 1$ then we know the worst challenger is $\bar{x}_3=0$:

$$\frac{B}{2}+\beta\frac{|x_1-x_2|}{2}\ge C>B+\frac{1-\beta}{6}+\beta\frac{x_1+x_2}{2}$$
$$\iff \beta(|x_1-x_2|-x_1-x_2)-\frac{(1-\beta)}{3}>B$$

which is impossible, given $x_1+x_2\ge 1$ and $B>0$. □

Proposition 12. *In the Lucky Votes case there exist a lower bound,* $\tilde{C}=\beta\max\{\frac{x_1+x_2}{2}-|x_1-x_2|,\frac{B}{3}+\frac{x_1+x_2}{6}\}>0$, *s.th. for any* $C>\tilde{C}$ *there exists some* $B>0$ *which makes* x_1,x_2 *entering by pooling at the median an equilibrium in pure strategies.*

Proof. Suppose $\frac{x_1+x_2}{2} \geq 1$. Since $B > 0$, from Non-Entry we know that $C > \beta\frac{x_1+x_2}{6} \geq \frac{1}{3}$

$$\begin{aligned}
\text{Entry:} \quad & B \geq 2C - \beta|x_1 - x_2| \\
\text{Non-Entry:} \quad & B < 3C - \beta\frac{x_1 + x_2}{2} \Rightarrow C > \frac{B}{3} + \beta\frac{x_1 + x_2}{6} \\
\text{Combined 1:} \quad & 3C - \beta\frac{x_1 + x_2}{2} > B \geq 2C - \beta|x_1 - x_2| \\
\text{Non empty interval} \iff & C > \beta\left(\frac{x_1 + x_2}{2} - |x_1 - x_2|\right) \\
\text{Combined 2:} \quad & \frac{B}{2} + \beta\frac{|x_1 - x_2|}{2} \geq C > \frac{B}{3} + \beta\frac{x_1 + x_2}{6} \\
\text{Non empty interval} \iff & B > \beta(x_1 + x_2 - 3|x_1 - x_2|)
\end{aligned}$$

Given some $C > \tilde{C} := \max\{\beta\left(\frac{x_1+x_2}{2} - |x_1 - x_2|\right), \frac{B}{3} + \beta\frac{x_1+x_2}{6}\}$, there always exist some $B > 0$ so the above inequalities are satisfied. □

Proposition 13. *As salience increases ($\beta \downarrow$) the possible identities of the two equilibrium entrants increases, where in the limit ($s \to 1$) anyone can form a 2-Candidate equilibrium configuration.*

Proof. As $\beta \to 0$ the message cost becomes irrelevant and the equilibrium is driven purely by B, C. When $\beta = 0$ it is clear that conditions 10 and 11 are satisfied for any identity configuration. □

Theorem 14. *Let $X^{\mathbf{2}}_{0.5}$ be the set of possible equilibrium configurations when two honest entrants form an equilibrium configuration. Then, the probability of randomly picking the configuration with honest candidates, out of $X^{\mathbf{2}}_{0.5}$, is zero.*

Proof. Since the realization where candidates are honest is only a point on a line, it is measure zero. □

Proposition 16. *In the Expected Votes case no message configuration exists such that a 3-Candidate equilibrium in pure strategies exists, and the only possible equilibrium message configuration in the Lucky Votes case is all three candidates pooling at the median.*

Proof. There are three types of message configurations: separation (all candidates state unique message), semi-separation (two candidates pool and one candidate has a unique message) and pooling (all candidates run with the identical message). Recall that in any equilibrium all candidates have the same positive probability of winning, i.e. all three candidates win with probability a third (i.e. ex ante all candidates get the same amount of votes). This has very specific implications for what the set of potential equilibrium messages is. The next items first state the set of potential equilibrium messages, and then check whether any candidate has a profitable deviation. Let $i, j, l \in \{1, 2, 3\}$ where $i \neq j \neq l$.

- Separation: $m_i = \frac{1}{3} - s_1$, $m_j = \frac{1}{3} + s_1 = \frac{2}{3} - s_2$ and $m_l = \frac{2}{3} + s_2$ where $s_1, s_2 \in (0, \frac{1}{3})$
  - Expected and Lucky Votes: Each extremist (i, l) has a ϵ-deviation closer to the moderate entrant, which guarantees certain victory.

- Semi-Separation: $m_i = \frac{1}{3} - s_1$, and $m_j = m_l = \frac{1}{3} + s_1$ where $s_1 \in (0, \frac{1}{3})$ (and alternatively where the messages are symetrically positioned around $\frac{2}{3}$)
  - Expected Votes: Candidate i has a profitable ϵ-deviation towards the two pooling candidates.
  - Lucky Votes: this type of configuration cannot survive in equilibrium, as one of the two candidates pooling gets $\frac{2}{3}$ of the votes ex post, and therefore certainly wins. That is, not all candidates receive the same number of votes. If $m_i = \frac{1}{2} - s_1$, and $m_j = m_l = \frac{1}{2} + s_1$ where $s_1 \in (0, \frac{1}{2})$ then they get the same number of votes, however candidate i has a profitable ϵ-deviation towards the two pooling candidates.
- Pooling: $m_i = m_j = m_l$
  - Expected Votes: each candidate has a profitable ϵ-deviation
  - Lucky Votes: each candidate has a profitable ϵ-deviation, except if $m_i = m_j = m_l = 0.5$ then no candidate has a profitable deviation.

□

Lemma 17. *For any Lucky Votes N-Candidate equilibrium where all candidates pool at the median, the worst challenger is an extremists, i.e.* $x_{N+1} = 0 \iff \frac{1}{N}\sum_{i=1}^{N} x_i \geq \frac{1}{2}$ *or* $x_{N+1} = 1 \iff \frac{1}{N}\sum_{i=1}^{N} x_i < \frac{1}{2}$.

Proof. The N-Candidate non-entry condition is:

$$P_{N+1}^{\mathbf{N+1}}[B-(1-\beta)|m_{N+1}-x_{N+1}|]-C < \sum_{i=1}^{N}(P_i^{\mathbf{N+1}}-\frac{1}{N})(\beta|x_i-x_{N+1}|+(1-\beta)|m_i-x_{N+1}|), \quad \forall x_{N+1}$$

given that $P_i^{\mathbf{N+1}} = \frac{1}{N+1}$ and $m_i = \frac{1}{2}$ for all $i \in \{1, \dots, N+1\}$, this can be simplified to

$$\begin{aligned}
B-(N+1)C <&(1-\beta)|m_{N+1}-x_{N+1}| - \frac{1}{N}\sum_{i=1}^{N}(\beta|x_i-x_{N+1}|+(1-\beta)|m_i-x_{N+1}|) \\
B-(N+1)C <&-\frac{1}{N}\sum_{i=1}^{N}\beta|x_i-x_{N+1}|
\end{aligned}$$

Since the only endogenous cost is the identity cost (distance of the challenger's identity to the candidates' identities), it is clear that if $\frac{1}{N}\sum_{i=1}^{N} x_i \geq \frac{1}{2}$, then the worst challenger is $x_{N+1} = 0$. □

Proposition 18. *In the Expected Votes case the only possible message configuration is* $m_i, m_j = \frac{1}{4}$ *and* $m_l, m_k = \frac{3}{4}$.

Proof. There are three types of message configurations: separation (all candidates state unique message), semi-separation (at least two candidates pool and the other candidates pool are have a unique message) and pooling (all candidates run with the identical message). Recall that in any equilibrium all candidates have the same positive probability of winning, i.e. all three candidates win with probability a quarter. Let $i, j, l, k \in \{1, 2, 3, 4\}$ where $i \neq j \neq l \neq k$.

- Separation: $m_i = \frac{1}{4} - s_1$, $m_j = \frac{1}{4} + s_1 = \frac{1}{2} - s_2$, $m_l = \frac{1}{2} + s_2 = \frac{3}{4} - s_3$ and $m_k = \frac{3}{4} + s_3$ where $s_1, s_2, s_3 \in (0, \frac{1}{4})$
  - Each extremist (i, k) has a ϵ-deviation closer to the moderate entrants, which guarantees victory.
- Semi-Separation 1: $m_i = \frac{1}{4} - s_1$, and $m_j = m_l = m_k = \frac{1}{4} + s_1$ where $s_1 \in (0, \frac{1}{4})$ (and alternatively where the messages are symetrically positioned around $\frac{3}{4}$)
  - Candidate i has a profitable ϵ-deviation towards the three pooling candidates.
- Semi-Separation 2: $m_i = m_j = \frac{1}{2} - s_1$, and $m_l = m_k = \frac{1}{2} + s_1$ where $s_1 \in (0, \frac{1}{2})$
  - Each candidate has a profitable ϵ-deviation for every $s_1 \neq \frac{1}{4}$ (towards the median if $s_1 > \frac{1}{4}$, or towards extremes if $s_1 < \frac{1}{4}$). At $s_1 = \frac{1}{4}$ any ϵ-deviation will make the other candidate using the same message certainly win. If making his best closest (in messages) competitor not better than the expected benefit of staying in the four candidate configuration, then the ϵ-deviation is not profitable.
- Pooling: $m_i = m_j = m_l = m_k$
  - Each candidate has a profitable ϵ-deviation

□

Lemma 19. *The worst challenger in the Expected votes case is the further extermist, i.e. if* $\sum_{i=1}^{4} x_i \geq 2 \implies \tilde{x}_5 = 0$. *He will use his cheapest winning lie, i.e.* $m_5 = 0.25 + \epsilon$.

Proof. The message cost for the worst challenger is the same no matter the identity of the four entrants, since they all have to use the same messages (which are symmetrically opposed around the median). Then for $\sum_{i=1}^{4} x_i \geq 2$ the identity cost is maximized by the opposite extremist $\tilde{x}_5 = 0$. Since any message in $(0.25, 0.75)$ causes certain victory, by increasing his lie the challenger only increases his cost, therefore he will use the cheapest winning lie. □

Proposition 20. *In the Expected votes case, there exist no 4-Candidate equilibria in pure strategies.*

Proof. We can re-write (subtracting C from all sides) the combined equilibrium entry/non-entry conditions as:

$$\overbrace{-\beta\left(\frac{\sum_{i=1}^{4} x_i}{4}\right) - 0.25(1-\beta)}^{<0} > B - C \geq$$
$$\underbrace{3C + \beta\left(\sum_{i=1}^{4} |x_i - x_4| - 4|x_3 - x_4|\right) + (1-\beta)\left(\sum_{i=1}^{4} |m_i - x_4| - 4|m_3 - x_4|\right)}_{>0}$$

which is impossible. □

Proposition 21. *Let $i, j, l, k \in \{1, 2, 3, 4\}$ where $i \neq j \neq l \neq k$. In the Lucky Votes case the only possible message configurations are:*

1. *$m_i, m_j = \frac{1}{4}$ and $m_l, m_k = \frac{3}{4}$, and*
2. *$m_i = m_j = m_l = m_k = \frac{1}{2}$.*

Proof. There are three types of message configurations: separation (all candidates state unique message), semi-separation (at least two candidates pool and the other candidates pool or have unique messages) and pooling (all candidates run with the identical message). Recall that in any equilibrium all candidates have the same positive probability of winning, i.e. all three candidates win with probability a quarter. Let $i, j, l, k \in \{1, 2, 3, 4\}$ where $i \neq j \neq l \neq k$.

- Separation: $m_i = \frac{1}{4} - s_1$, $m_j = \frac{1}{4} + s_1 = \frac{1}{2} - s_2$, $m_l = \frac{1}{2} + s_2 = \frac{3}{4} - s_3$ and $m_k = \frac{3}{4} + s_3$ where $s_1, s_2, s_3 \in (0, \frac{1}{4})$
  - Each extremist (i, k) has a ϵ-deviation closer to the moderate entrants, which guarantees victory.
- Semi-Separation 1: $m_i = \frac{1}{2} - s_1$, and $m_j = m_l = m_k = \frac{1}{2} + s_1$ where $s_1 \in (0, \frac{1}{4})$ (and alternatively where the messages are symetrically positioned around $\frac{3}{4}$)
  - Candidate i can win by getting closer to the other group.
- Semi-Separation 2: $m_i = m_j = \frac{1}{2} - s_1$, and $m_l = m_k = \frac{1}{2} + s_1$ where $s_1 \in (0, \frac{1}{2})$
  - Each candidate has a profitable ϵ-deviation for every $s_1 \neq \frac{1}{4}$ (towards the median if $s_1 > \frac{1}{4}$, or towards extremes if $s_1 < \frac{1}{4}$). At $s_1 = \frac{1}{4}$ any ϵ-deviation will make the candidate from the opposite group certainly win. Therefore, no candidate will deviate at $s = 0.25$.
- Pooling: $m_i = m_j = m_l = m_k$
  - Each candidate has a profitable ϵ-deviation, for every $m \neq 0.5$

□

Theorem 22. *[Expected Votes] If politicians are office motivated and their messages satisfy Individual Rationality, then for all candidates in a N-Candidate equilibrium, where $N > 1$:*

1. *Equilibrium messages exist only when N is even*
2. *Let $j \in \{i | \forall i$ where i odd$\} = \{1, 3, 5, \ldots, N-1\}$ and $z = \frac{j+1}{N}$ for all j. Each message $m_j = m_{j+1} = \frac{j}{N}$ has a group of two candidates $g_z = \{x_j, x_{j+1}\}$ stating it.*
   - *Each group gets $v_z = \frac{2}{N}$ vote share from voters $V_z = [\frac{j-1}{N}, \frac{j+1}{N}]$*

Proof. For all msgs such that $P_i^{\mathbf{N}} = \frac{1}{N}, \forall i \in \mathcal{C}$:

- Any group larger than two obtains more than $\frac{2}{N}$ votes, even when the message is positioned at the middle of their voter base, each member has a profitable deviation

- Let $\bar{g}$ be a group of $k > 2$ candidates using the same message $m_{\bar{g}}$. This group receives the voter share $\frac{k}{N}$. The message that minimizes the distance to the extremes of this groups' voter base is the median of their voter base, $\bar{m}_{\bar{g}}$. Every candidate using message $\bar{m}_{\bar{g}}$ has a profitable ϵ deviation, towards either extreme of their voter base, that ensures his voter base is larger than $\frac{1}{N}$. If the message used is not the median message, each candidate within that group has a profitable ϵ deviation towards the further extreme of their voter base.
  $\implies$ At most two candidates use the same message

- Full separation: impossible as any single extreme message has profitable deviation towards median to get $v_i > \frac{1}{N}$
  - Let $m_1 < m_2 < m_3 < \cdots < m_N$ such that $m_i = \frac{i-0.5}{N}$, and each message is stated by exactly one entrant. Then x_1 (and x_N) can certainly win by ϵ deviating towards the median, such that $m_1 < m_2$ (and $m_N > m_{N-1}$). Note that $m_i = \frac{i-0.5}{N}$ does not have to be exactly at this position, but symmetrically located around the edges of the voter bases, i.e. $m_1 = \frac{1-s}{N}, m_2 = \frac{1+s}{N}$ etc. However, if it holds for the medians of each candidates voter base, then it will hold for any symmetric messages.
    $\implies$ Most extreme messages stated by a group of 2 candidates

- Any message next to the most extreme message (being used by 2 candidates), used by only 1 candidate, implies the extreme message is not at the median of it's voter base, but closer to the single entrants message (2nd most extreme message). This means that each of the two candidates using the most extreme messages has a profitable deviation towards the extremes.
  - Let $m_1 < m_2 < m_3 < \cdots < m_{N-2}$ where $m_2, m_3, \ldots, m_{N-3}$ are stated by one candidate each and m_1, m_{N-2} are stated by two candidates each. Then $m_1 = \frac{1.5}{N}$ and $m_2 = \frac{2.5}{N}$, so each candidate using message m_1 can ϵ deviate towards 0 and ensure certain victory.
    $\implies$ The second most extreme messages are stated by a group of 2 candidates

- By induction, this holds true for any message closer to the median than these extreme messages. This can be done from both sides. That is, any single candidate message next to a 2 candidate group will shift the groups message away from it's voter base median, towards the single candidate message, leaving each candidate within the 2 candidate group with a profitable deviation towards the long part of their voter base. ‘
  - Let $m_1 < m_2 < m_3 < \cdots < m_{N-4}$ where $m_3, m_4, \ldots, m_{N-5}$ are stated by one candidate each and m_1, m_{N-2} are stated by two candidates each. Then $m_1 = \frac{0.5}{N}$, $m_2 = \frac{3.5}{N}$ and $m_3 = \frac{4.5}{N}$, so each candidate using message m_1 and m_2 can ϵ deviate towards 0 and ensure certain victory.
  - Let $\tilde{g} \in \{1, 2, \ldots\}$ be the number of groups of candidates stating the same message (suppose each candidate has to be part of a group even if alone). If $\tilde{g} > \frac{N}{2} \implies$ some messages stated by only 1 candidate (whose message is next to a group), who has a profitable deviation towards the group. If $\tilde{g} < \frac{N}{2} \implies$ there must be some groups

with more than two candidates, who then have a profitable deviation. Therefore, $\tilde{g} = \frac{N}{2}$ where each group is composed of exactly two candidates.

- Since all messages have to be stated by exactly two candidates, this kind of message configuration is only possible if there is an even number of candidates.

$\square$

Proposition 23. *Under UEM, the identity of the candidates willing to use message m_j (from Theorem 22) is inside the set $I_z = [\frac{j-1}{N}, \frac{j+1}{N}]$*

Proof. Costly lying and UEM $\Rightarrow$ each candidate uses closest potential equilibrium message available. $\square$

Proposition 24. *For configurations that satisfy individual rationality, and $N \geq 6$, no challenger can certainly win in the Expected votes case.*

Proof. Equilibrium messages need even candidates, in groups of two, each group getting the same vote share. With three groups the challenger can position himself at the extremes or between two groups. By positioning himself at either extremes he loses and makes one group lose, while positioning himself internally he can at most make two groups lose, leaving a single group getting the most votes. $\square$

Theorem 25. *Two types of N-Candidate (pure strategy) equilibrium message configurations satisfy Individual Rationality are:*

1. *For any N: All candidates pool at the median, for any N*
2. *For any $N \geq 4$ we can find a combination of J groups of $g_1, \ldots, g_J$ such that $\sum_{j=1}^{J} g_j = \#\mathcal{C} = N$ and $g_j \geq 2$, $\forall j \in \{1, J\}$. That is, as long as the most extreme messages are stated by at least two candidates, any configuration of group sizes (symmetric iff N not prime, asymmetric for any N) can be supported in equilibrium.*

For semi-separating equilibrium messages, every group receives an equal vote share $\frac{1}{J}$ using messages $m_j = \frac{1+2(j-1)}{2J}$ receiving votes from voters $V_j = [\frac{j-1}{J}, \frac{j}{J}]$. Only one candidate in each group gets the groups whole voter share with probability $\frac{1}{\#g_j}$.

Proof. We will first prove that everyone pooling at the median is always individually rational, that is no one has a profitable unilateral deviation. If there are N candidates pooling at the median each receives all of the votes with probability $\frac{1}{N}$. If anyone ϵ deviates to either side, he receives $\frac{1-\epsilon}{2}$ votes certainly while one of the $N-1$ remaining candidates pooling at the median certainly receives $\frac{1+\epsilon}{2}$ votes. Therefore, the deviating player certainly loses.

If N is a prime number, then it can only be divided by 1 or itself. This implies that there is either a single group (pooling at the median) or everyone states a unique message (full separation), or there are multiple groups of different sizes. As we saw above, pooling at the median is possible for any N, while full separation is impossible for the same reason it is not possible in the Expected Votes case. Precisely, if all candidates run with a unique message and all have the same probability of winning (i.e. each candidate receives $\frac{1}{N}$ votes and has the same probability of winning), each extremist can deviate towards his closest competitor that makes him receive more than $\frac{1}{N}$ votes.

Finally, if N is not a prime number, then it can be divided by some integer k, where $1 < k < N$. This implies that there exists at least one integer $\frac{N}{k} = \hat{g}$ (where $\hat{g}$ is an integer) such that each of the k messages are being stated by exactly $\frac{N}{k}$ candidates. In other words, if N is not a prime, there exists at least one number of groups (i.e. k) such that each of the groups is composed of the same number of candidates. A group is composed of candidates running with the same message.

Finally, when N is not a prime, the groups can be of asymmetric sizes. That is, for any $N \geq 4$ we can find a combination of J groups of $g_1, \ldots, g_J$ such that $\sum_{j=1}^{J} g_j = \#\mathcal{C} = N$ and $g_j \geq 2 \forall j \in \{1, J\}$. That is, the only restriction is that the most extreme messages are at least paired (stated by at least two candidates). Every group receives an equal vote share $\frac{1}{J}$ using messages $m_j = \frac{1+2(j-1)}{2J}$ each gets the votes from voters $V_j = [\frac{2(j-1)}{2J}, \frac{2j}{2J}]$ ex-post, meaning that no one has no profitable deviation. Note that the internal groups can also contain a single candidate who certainly obtains vote share $\frac{1}{J}$, who do not have a profitable deviation. □

Lemma 26. *The worst challenger for any pooling N-Candidate equilibrium is the furthest extremist from the average identity, i.e. if $\frac{\sum_{i=1}^{N} x_i}{N} \geq \frac{1}{2} \implies \tilde{x}_{N+1} = 0$ where each $x_i \in \mathcal{C}_N$.*

Proof. Given that the message component is void in pooling equilibria, i.e. no entrant can change the equilibrium message chosen, only the identity of the candidates determines the worst challenger. When the identity of the candidates is such that the average identity is above the median, then the worst challenger is the opposite extremist. □

Proposition 27. *The number of equilibrium entrants depends on the net benefits (B, C), such that more candidates are willing to enter when B is higher relative to C, given C is high enough for no challenger to be willing to enter.*

Proof. Since $\tilde{B}(N+1) - \tilde{B}(N) > 0$ an increase in the number of candidates is only possible if we increase the benefits, which implies (through the non entry condition) that the cost also has to increase. For increasing B the cost has to increase at a slower rate, i.e. at rate $\frac{1}{N+1}$. □

Proposition 28. *As the issue becomes very salient $(s \to 1)$, any candidate configuration can arise in equilibrium, given B, C are such that a N-Candidate pooling equilibrium exists.*

Proof. In the limit $s \to 1$ the equilibrium condition (17) becomes $\frac{B}{N+1} \geq C > \frac{B}{N}$ where the identity cost becomes irrelevant, and if an equilibrium is possible (i.e. appropriate B, C selected), it will be possible for any candidate configuration. □

Proposition 29. *For all $s \in (0, 1)$, as $N \to \infty$ any identity configuration is possible in equilibrium, given B, C are such that a N-Candidate pooling equilibrium exists.*

Proof. Let $B(N+1) - B(N) = C$. Note that the from analysis we know that $\left|\frac{N}{N+1} - 1\right| = \frac{1}{N+1}$, and for all $\gamma > 0$, $\frac{1}{N+1} < \gamma$ if $N > K$ where $K = \frac{1}{\gamma} - 1$. In other words, as $N \to \infty$ the fraction $\frac{N}{N+1}$ comes arbitrarily close to 1. In fact, for a large enough N we can identify a $\gamma > 0$ arbitrarily close to zero, such that the distance $\left|\frac{N}{N+1} - 1\right|$ is smaller than γ. This implies that as $N \to \infty$ we can write condition (18) as $\hat{B}(N) \geq NC > \hat{B}(N)(1-\gamma)$. To finish the proof, note that an added candidate contributes less to the identity cost than the unit increase in N. □

Theorem 30. *Let $X_{0.5}^{\mathbf{N}}$ be the set of possible equilibrium configurations when N honest candidates form an equilibrium configuration. Then, the probability of randomly picking the configuration with only honest entrants, out of $X_{0.5}^{\mathbf{N}}$, is zero.*

Proof. Let $\bar{C} = \frac{B}{N}$, which ensures N honest candidates are willing to jointly run, and no challenger is willing to oppose them. Simultaneously, all configurations for whom $\bar{C} > \frac{B}{N+1} + \beta\frac{\sum_{i=1}^{N} x_i}{N(N+1)}$ holds are also possible. That is, when the configuration with only honest entrants is possible in equilibrium, so are any configurations where the sum of distances from the worst challengers is kept the same. Precisely, all configurations such that $\sum_{i=1}^{N} x_i = \frac{N}{2}$, which can be achieved by taking any pair of candidates and moving them symmetrically in opposite directions, keeping the sum fixed. Since there exists a continuum of such alternative configurations, but only a single configuration where all of the candidates are honest, we know that the measure of equilibrium configurations with honest candidates is (almost) zero. $\square$

Part II
An Electoral Game with Interest Groups and Emotional Voters who can Protest

An Electoral Game with Interest Groups and Emotional Voters who can Protest

by Filip Lazarić

Abstract

This paper modifies the seminal theoretical voting model that includes special interest groups. It looks at how the contributions special interest groups give to politicians, in order to influence the policy implemented, change when voters are emotional and can protest. Specifically, the paper looks at how confidence and trust feed into the voting decisions of the voters, as well as the contribution decisions of special interest groups. Furthermore, the model gives voters the ability to protest, and special interest group members to strike. Most importantly, the paper looks at the effect unexpected economy wide shocks have on voting and contribution decisions. It finds that emotions and the shock have a non-trivial effect, and the fact that voters can protest serves the special interest groups more than the voters. The main result of the paper finds that the members of special interest groups will strike during recessions, while pessimism induced by the slump decreases striking activity. The empirical section confirms that recessions lead to an increase in strikes, however it finds that pessimism further increases striking activity. Therefore, the appendix developes a theoretical justification as to why pessimism will lead to stronger striking activity.

1 Introduction

The basic electoral game consists of politicians and voters, where the politicians are office motivated and set policies such that they maximize their probability of winning the election. The median voter theorem, which holds in certain situations (e.g. two politicians), defines the decisive voter, that is, the voter whose optimal policy will be endorsed by the parties. [Grossman and Helpman, 1996] (GH from now on) extended this game to include special interest groups (SIGs from now on), which are groups of individuals that prefer policies that are not in line with the decisive voters interest, that lobby in its own favour. GH were interested in the effect of a SIG, and the competition among SIGs, on the policy outcome and welfare implications, as well as the motives a SIG may have. Following [Baron, 1994] GH split voters into informed and uninformed where the informed voters vote for the party offering the best policy, while the uninformed voters vote for the party who campaigns more to them. The different agents interact in the following way: interest groups want to implement policies they find optimal by contributing to parties,[86] the parties want to win majority[87] in the proportional representation parliament by endorsing optimal policies for the informed voters and by using the contributions to campaign to the uninformed voters, and the voters behave as previously defined. The SIGs contributions leave the parties with a fundamental trade-off: run with policies that attract well informed voters or run with policies that benefit the SIGs in order to get higher contributions that can be used to attract uninformed/impressionable voters.

Political economy traditionally focused on rational agents and did not deal with behavioural features that may be relevant. Therefore, the first modification this paper makes to the GH model is to include some behavioural concerns. Even though in the long run the business cycle may not be of interest, in the short and medium run the business cycle can have severe consequences for the real economy and its people. A good example is the great depression and the great recession. The second major and third minor modifications are supposed to allow us to analyse what happens in the presence of such unexpected shocks.

More precisely, this paper builds upon the GH model with two main aims. Firstly, it is looking at the effect of incorporating sentiments in a GH style electoral game, and how this changes the equilibrium. The sentiments that will be incorporated are trust and confidence (particularly optimism/pessimism). It is important to model these concerns as parties seem keenly aware that public *opinion* and *sentiments* matter for their re-election prospects. The second aim of this paper is to look at how the political equilibrium changes in the face of unexpected shocks, such as the subprime mortgage crisis of 2008, particularly when there are also sentiments involved. It is important to look at the implications of unexpected shocks on the political game, as they will most likely have non trivial equilibrium consequences. This leads us to the main research question of the paper: what are the equilibrium consequences of having emotional voters and interest groups in an electoral game that suffers an unexpected shock?

We will see that the theoretical model predicts that sufficiently severe recessions will lead to strikes, and that when pessimism is included a more severe recession is required in order for strikes to occur. This occurs due to pessimism shifting the policy implemented away from the special interest groups. Empirically we will find weak evidence that recessions lead to strikes,

[86] More precisely, SIGs design contribution schedules, specifying the contribution the party will get if it endorses the SIGs policy. This changes the parties incentives in a well defined way.

[87] Each party wants to beat the other one, so that it has a higher probability of being able to implement its policies.

while pessimism increases the number of strikes. As this contradicts the theoretical prediction for pessimism the appendix contains an alternative specification, which better fits the empirical result: sufficiently severe recessions lead to strikes, and pessimism will lead to strikes during less severe recessions.

In order to introduce sentiments a further modification was done to the benchmark GH model. I allowed the voters the ability to protest if the policy proposal by the SIG was unacceptable to them. This actually made the voters worse off, because what was implicitly assumed is that this allowed the SIGs to know exactly the payoffs the voters have from their optimal policies. We introduced protesting in order to introduce trust, and assumed that the higher distrust there is in the government, the higher the likelihood that protests will take place. We will see that trust has a non-trivial effect on the equilibrium, such that more trust will lead to policies that are less favourable to informed voters as well as a lower welfare. On the other hand, the theoretical model predicts that more distrustful societies will have policies that are more in favour of informed voters with a higher associated welfare, and also lower contributions from the SIGs.

I also show that confidence has non-trivial effects on the equilibrium; particularly, there will be a direct and indirect effect. The direct effect simply leads to a shift in popularity from one party to the other, and as such its effects are zero sum. The indirect effect operates through trust, such that pessimism will weakly decrease trust, hence it will make the policies more favourable to informed voters. Finally, we will see that the shock has a significant effect on the equilibrium, and under a fairly reasonable assumption a recession will lead to SIG members striking. Furthermore, with pessimism a stronger recession will be necessary for strikes to occur.

During strikes people will not work, factories may come to a halt, the infrastructure may come to a halt, and many other stoppages may take place that could incur large social costs. Therefore, strikes can easily lead to significant social costs. In the empirical section we check the main result of the theoretical section, does the amount of strikes increase in a recession, and does the effect of a recession increase with pessimism. Due to a limited dataset we find weak evidence that the recession increases strikes, and that pessimism will worsen this effect.

The remainder of the paper is structured as follows. First, the paper briefly review some literature surrounding these topics, then it introduces the benchmark mode, after which every modification is analysed. The final theoretical section is the equilibrium analysis. The final section of the paper is an empirical exercise, after which the paper concludes.

2 Literature Review

The model in this paper is an extension and modification of [Grossman and Helpman, 1996] (GH), which introduced lobbying in an electoral game, thereby showing that politicians will take a weighted average of the opinions of voters and lobbyists into account. This will be shown explicitly below. The direct predecessor of GH is [Baron, 1994], with the same interests but different results. The paper [Grossman and Helpman, 1994] is an application of their aforementioned one to trade policy, where interest groups bid for protection. Here the weighting is exogenously imposed. In both papers, when multiple lobbies are involved truthful equilibria (or compensating equilibria) become important, a concept developed by [Bernheim and Whinston, 1986] in their theoretical analysis of menu auctions. This has been applied to common agency problems, that is, when there is a single agent whose decisions influence the outcome for many principals: in GH the single policy maker and many interest groups. Each SIG provides a menu of policy dependent contribution schedules in order to induce the parties to take a position in the SIGs interest. A slightly different approach with a differentiable function is taken by [Becker, 1983], where the competition among pressure groups is of interest and the political equilibrium depends on group efficiency in pressuring, their size and the deadweight costs of taxation and subsidies. One interesting implication of Becker is that with deadweight costs the taxpayer is at an advantage to the subsidy receivers. More specifically, when deadweight costs of taxation and subsidies are such that one dollar of taxes paid gives less than a dollar revenue, and one dollar of subsidies paid costs more than a dollar; then increasing the marginal deadweight cost will decrease subsidies, because it is cheaper to decrease the tax rate than it is to raise the subsidy. In such a way the taxpayer has an intrinsic advantage over the subsidy seeker in this setup. A good summary of the theory and empirics with special interest groups is [Grossman and Helpman, 2001].

An interesting predecessor of the aforementioned articles are the models that focus purely on taxation. Taxation on its own has a long standing theoretical grounding, where the importance of single peakedness was shown by [Romer, 1974] with linear income taxes and majority voting, which lead to the result that the equilibrium tax might not maximize the poorest individuals welfare. Later [Roberts, 1977] showed that with voting over linear income taxes a solution might still exist even if single peakedness was not satisfied. [Meltzer and Richard, 1981] use this framework to further analyse how the size of the government changes with respect to changes in income, given a decisive voter exists. A nice introduction to some of these topics regarding taxation, and an extension of these models to the topic of franchise extension can be found in [Acemoglu and Robinson, 2005].

There is a large experimental literature on trust, we will not review it here. On a more aggregate level, research has been done on regions or states that differ in their trust level and the development of those territories over time. Particular attention has been paid to social capital, which could be interpreted as a proxy for trust. It has been shown that regions that exhibited more social capital have developed more successfully than regions that had less of it. An example of the literature on social capital is [Helliwell and Putnam, 1995]. Confidence is also a crucial determinant of the economy, and many crises may themselves be caused, or at least aggravated, by a drop in confidence. An example of the literature that puts a large weight on confidence is the book by [Krugman, 2008], where confidence matters in many ways, for instance in self-reinforcing crises. A nice overview of these behavioural characteristics, and others, is in the book by [Akerlof and Shiller, 2010]. The literature review is short because it is crucial to introduce the benchmark GH model appropriately.

3 Model - Benchmark (Grossman-Helpman)

The model of this paper could be considered as two consecutive games, but it does not have to be. The intuitive reason for such an interpretation is due to the shock; in the first time period the times are normal, then an exogenous shock (boom/bust) takes place and the game has to be played again. The effects of the shock will be compared to the equilibrium in normal times. The model is an extension and modification the GH model. The reader is urged to GH and their book for further fine points. For easier comparison, the notation is as close as possible to the original article. For clarity, the benchmark model (GH) is presented first, and then the modifications.

The GH model, as well as the modified one, allow policies to be fixed or pliable. Fixed policies are issues on which the party has a strong stance on, such as ideology, while pliable policies are ones for which the party has no strong preference, therefore it is willing to tailor them such that it receives the most votes. The models focus solely on the effect of pliable policies, where the SIGs are a collection of individuals who want same pliable policies.

The GH model considers two motives for SIG contributions, an electoral motive and an influence motive. A SIG might inherently prefer one candidate over another and would like to improve that candidate's chances of getting into office, this is the *electoral motive*. On the other hand, the SIGs may have particular policies that they would like to implement that differ from the decisive voters policy preference, hence they make contributions in order to influence parties to endorse suboptimal policies for the voters, but more favourable policies for the SIG. In the current model only the influence motive will be treated, as the main purpose of the model is to look at the combined effect of sentiments and shocks, rather than the effect of contributions.

3.1 Voters

Voters can be informed or uninformed, where $\alpha \in (0, 1)$ represents the fraction of uninformed voters. Informed voters are rational utility maximizers who are aware of the welfare implications of the policies proposed and ultimately chosen. They will vote for whichever party offers policies that give them a higher utility. Uninformed voters do not know or cannot asses the parties policies. They may have preferences on fixed issues (such as ideologies), however on pliable policies they do not have an opinion. These voters are impressionable, meaning that parties can use campaign rhetoric or advertising to sway their votes in their favour. If the politicians spend sufficient funds they can get the uninformed voters to support any policy, and think that it is optimal.

More precisely, let a typical informed voter $i \in I$ derive utility $u^i(p^k)$ from the pliable policies p^k endorsed by party $k \in K = \{A, B\}$, and an exogenous preference for party k, defined as β^i_k. The function $u^i(\cdot)$ is continuous and twice differentiable. The voter i will vote for party A if and only if he receives a higher utility from A's pliable policies than from B's fixed policies (and other exogenous characteristics such as charisma), i.e. $u^i(p^A)+\beta^i_A \geq u^i(p^B)+\beta^i_B \Rightarrow u^i(p^A)-u^i(p^B) \geq \beta^i$, where β^i is voter i's measure of superiority (or inferiority) of party B's fixed position.[88] The parties do not know the ex-ante proclivities of any specific voter, but assume that it is drawn from the known distribution $F[\beta] = P[\beta^i \leq \beta]$ where β^i are iid random draws for each informed voter $i \in I$. Furthermore, the preference for fixed and pliable positions are statistically independent, i.e. $\beta \perp p$. Therefore, both parties perceive that voter i votes for the slate of candidates from party A with probability $F[u^i(p^A) - u^i(p^B)]$. The total informed vote for party A (by the Law of Large Numbers with a continuum of informed voters) is $\frac{1}{n_I} \int_{i \in I} F[u^i(p^A) - u^i(p^B)]di$ where n_I is the total number (measure) of informed voters, i.e. $n_I = \int_{i \in I} \mathbb{1}_{\{i \in I\}} di$, and I is the set of informed voters.

The uninformed voters may also have initial leanings for fixed positions, however their votes may be swayed through campaign expenditure, specifically their votes depend on the absolute expenditures of the two parties. In other words, if party A spends more on campaigning than

[88] One can make more general specifications, e.g. $u_j(p^A) - u_j(p^B) \geq \beta^i_j$ where j is a group that the voter i is part of. All individuals within group j have the same preferences over pliable policies, but different over fixed party positions and their exogenous characteristics.

party B, some of those who initially wanted to vote for party B vote for party A instead. Let $H(\cdot)$ be the fraction of uninformed voters that vote for party A, and GH assume that this fraction depends on the difference in the parties' campaigning budgets, i.e. $H(C^A - C^B)$. The following footnote defines a possible micro foundation for this GH assumption.[89]

The seats in parliament are allocated by proportional representation, hence the share of seats party A will capture depends on how many informed and uninformed voters vote for it, that is:

$$s = \frac{1-\alpha}{n_I} \int_{i \in I} F[u^i(p^A) - u^i(p^B)]di + \alpha H(C^A - C^B) \tag{20}$$

where $\frac{1}{n_I} \int_{i \in I} F[u^i(p^A) - u^i(p^B)]di$ is the total informed vote for party A and $H(C^A - C^B)$ is the uninformed vote for party A. Note that $F(\cdot)$ and $H(\cdot)$ are the fractions of informed and uninformed votes respectively, who vote for party A. For informed voters it depends on their utility from pliable policies, while for the uninformed it depends on the contributions spent by both parties.

3.2 Parties

In a two party proportional representation system each party wants to achieve majority in the legislature. This makes it easier to implement preferred party policies, the party can then also have more influence and it can get more jobs for their members. In the GH model there are two parties, A and B. Note that with two parties and proportional representation, maximising seats is equivalent to maximising the expected plurality in the election. In order to maximise their legislative representation they need to offer pliable policies such that they attract the informed voters, however, they also need to attract SIGs in order to get contributions that they can use for influencing the uninformed voters. The share of seats party A receives is denoted by $s \in [0, 1]$, therefore the objective of party A is to maximise s and that of party B is to maximise $1 - s$. The policy-setting procedure is not modelled, but rather it is assumed that each parties probability of implementing their policies increases monotonically with their representation. The set of policies is a convex subset of the finite dimensional euclidean space $\mathcal{P}$. The legislature adopts party A's policy, p^A, with probability $\phi^A = \phi(s)$, and B's, p^B, with $\phi^B = (1 - \phi(s))$. In order to capture the benefit of having a majority in the legislature it is further assumed that whichever party obtains a majority has a more than proportional probability of implementing its policy, i.e. $\phi(\frac{1}{2}) = \frac{1}{2}$ and $\phi'(s) > 0$.

[89]The assumption can be motivated with the same micro motives as the informed voters have. As with the informed voters, let a typical uninformed voter $\bar{u} \in \bar{U}$ derive utility $u^{\bar{u}}(C^k)$ from the contribution level given to party k which it then spends on campaigning, C^k. Let $u^{\bar{u}}(\cdot)$ be continuous and twice differentiable. Therefore they will vote for party A if and only if the contribution level compensates for their fixed preference for party B, i.e. $u^{\bar{u}}(C^A) + \beta^{\bar{u}}_A \geq u^{\bar{u}}(C^B) + \beta^{\bar{u}}_B \Rightarrow u^{\bar{u}}(C^A) - u^{\bar{u}}(C^B) \geq \beta^{\bar{u}}$. Let the function $H(\cdot)$ denote the fraction of uninformed voters that vote for party A. Again, the parties know only the distribution of fixed preferences of the uninformed voters ($\beta^{\bar{u}}$) that is iid across $\bar{u} \in \bar{U}$, and the fixed preferences are statistically independent of the contribution levels, therefore the exact fraction of uninformed voters that vote for party A is $H(u^{\bar{u}}(C^A) - u^{\bar{u}}(C^B))$. With homogenous uninformed voters (or if parties have a significant lack of knowledge about each individual voter) the fraction that votes for party A depends only on aggregate contributions, i.e. $H(u^{\bar{u}}(C^A) - u^{\bar{u}}(C^B)) = H(u(C^A) - u(C^B))$. Finally, if we suppose the utility function is linear, then each voter $\bar{u}$ (the representative voter) will vote for party A with probability $H(C^A - C^B)$, which is also the fraction of uninformed voters who vote for party A.

3.3 Special Interest Group (SIG)

SIGs are collections of voters where the members are individuals with common preferences for pliable policies. They organise themselves to form a group pressuring the government to obtain policies in their favour, which are not in line with the policies that would have otherwise been chosen.[90] Critically, we assume that they form because they have larger benefits than costs of pressuring the government (e.g. overcoming the free rider problem). This model assumes that through contributions (which parties use for campaigning to the uninformed voters) SIGs may push policies towards their own preferred policies. In fact, there are two main reasons for contributing. An *influence motive*, where the SIG hopes to influence the parties policy platforms in their own favour; and an *electoral motive* where they want to improve the election prospects of their preferred candidate (the one with policy platforms closest to their own). Note that within a SIG the members may have different fixed preferences about the candidates, so there may be disagreement about which candidate to support (electoral motive), however all its members will agree on pushing both parties policies towards the SIGs preferred pliable policy platform (influence motive).

Let $W_j(p)$ be the aggregate utility of the members of interest group $j \subseteq J$ from the vector of pliable policies $p \in \mathcal{P}$, where J is the set of SIGs.[91] The preferences within a SIG may be heterogenous, however we assume that the SIG members cooperate fully in their objective to maximise their expected joint welfare from the pliable policies net of contributions made.

The SIG is trying to design a contribution schedule, $C_j(\cdot)$, from which the parties will be able to infer the contributions they will get for different policy choices. It does not seem realistic that interest groups give direct policy conditions to parties, however parties most likely know that the policy platforms they endorse will influence the amount of contributions they receive. Let $C_j^k(p^k)$ be the contribution schedule of SIG j to party $k \in K = \{A, B\}$ for the pliable policy p^k, with well behaved properties: continuous, differentiable when positive, and non-negative[92] everywhere.[93] The SIGs objective function is the expected welfare it gets from pliable policies platforms net of the total contributions made, that is:

$$V_j = \phi(s)W_j(p^A) + (1 - \phi(s))W_j(p^B) - (C^A + C^B) \tag{21}$$

For simplicity, the present paper will focus only on the case of a single lobby ($\#J = 1$), as the purpose of this paper is not to analyse the effect of inter lobby competition. Multiple lobbies is an interesting extension nevertheless. Nevertheless, we will continue to denote the SIGs welfare as $W_j(\cdot)$.

3.4 Political Equilibrium (PE)

The PE is a subgame perfect Nash equilibrium of the two stage, noncooperative, political game, where the SIGs independently and simultaneously announce and commit to a contribution sched-

[90]If the informed voters and SIGs have the same pliable policy preferences, then there is no disagreement between the two and each party will cater to the informed voter preferences. This is an uninteresting case.

[91]Note that if the individuals in group j are homogenous then $W_j(p) = n_j \times u_j(p)$, where n_j is the number of individuals composing SIG j.

[92]The non-negativity captures that SIGs can only contribute to parties and not take from them.

[93]Note that the contribution schedules do not have to depend on pliable policies. This would make the party contribute only to their preferred party for electoral motives.

ule for each party in the first stage, and the party chooses their policy platform in the second stage. An important assumption GH make is that all expectations about next events are accurate, and all promises are kept. One is pointed to GH for the details of the PE of the benchmark model and their equilibrium analysis.

The PE will be precisely defined for the modified model, while here we will intuitively state the four equilibrium conditions: (i) party A chooses the policy that maximises its vote share, given the policy of party B, and the two contribution schedules of the SIG (one for each party), (ii) party B makes this same optimal policy choice, (iii) the contribution schedules are well behaved (continuous and differentiable when positive, and everywhere non-negative), and (iv) no SIG can unilaterally beneficially change their contribution schedule.

Conditions (i) and (ii) ensure Nash equilibrium behaviour among parties in the policy setting stage of the game, while condition (iv) guarantees Nash equilibrium behaviour between the SIGs in the contribution schedule stage of the game. Note that the implicit assumption in the equilibrium definition is that each party can anticipate the contribution schedules offered to the other.

3.5 Functional Forms

GH simplify their analysis by assuming that the random variable describing the voters preference for the parties fixed policy positions is uniformly distributed. They make the random variable describing the exogenous preferences for the fixed policy platforms of party B over party A (i.e. β), of the informed voters, uniformly distributed in a pre specified range. More precisely: $\beta^i \sim U\left(\frac{-2b-1}{2f}, \frac{-2b+1}{2f}\right)$ where $f > 0$ is a parameter reflecting the diversity of ex ante views about the parties. This leads to $F[u^i(p^A) - u^i(p^B)] = \frac{1}{2} + b + f[u^i(p^A) - u^i(p^B)]$ for $[u^i(p^A) - u^i(p^B)] \in \left(\frac{-2b-1}{2f}, \frac{-2b+1}{2f}\right)$. Furthermore, they assume that $H(\cdot)$ is linear and takes the form: $H(C^A - C^B) = \frac{1}{2} + b + h(C^A - C^B)$, where $h > 0$ is a parameter reflecting the productivity of campaign spending.[94] These assumptions make the seat share that party A obtains (equation 20) take the following additively separable (in the variables describing own policy platform and level of campaign spending, and those of its rival) form:

$$s = \frac{1}{2} + b + (1-\alpha)f[W(p^A) - W(p^B)] + \alpha h(C^A - C^B)$$

where $W(p^k) = \frac{1}{n_I}\int_{i\in I} u^i(p^k)di$ is the average welfare of the informed voters when the vector of pliable policies is p^k. Note that with separability each party can make its decision about what contribution offer to accept, and what policy to propose, independently of its knowledge/beliefs about the incentives facing its rival party. Therefore, the equilibrium that arises with perfectly anticipated contribution schedules by both parties, coincides with equilibria where the schedules are privately communicated.

To simplify the notation let $\Lambda = (1-\alpha)f[W(p^A) - W(p^B)] + \alpha h(C^A - C^B)$. Notice that if both parties endorse the same policies ($p^A = p^B$) and spend the same amount on campaigning

[94] As previously specified, the behavior of uninformed voters can also be micro founded. Here we could assume that $\beta^u \sim U\left(\frac{-2b-1}{2h}, \frac{-2b+1}{2h}\right)$ with $h > 0$, which leads to $H(C^A - C^B) = \frac{1}{2} + b + h(C^A - C^B)$ for $(C^A - C^B) \in \left(\frac{-2b-1}{2h}, \frac{-2b+1}{2h}\right)$. In this paper the analytically simpler approach will be taken, as defined in the text.

($C^A = C^B$), Λ will equal zero and party A's vote share will be $s = \frac{1}{2} + b$. Therefore b is the ex ante voter preference where A is preferred if $b > 0$, and party B when $b < 0$, and both parties are equally preferred when $b = 0$. Given that incumbents are usually at an advantage a natural interpretation of b is as an *incumbency advantage.*

3.6 Participation Constraint

With only a single SIG the problem can be viewed as one of direct control. If it provides sufficiently high contributions, it should be able to implement any pair of policies it desires. Since each party can reject the SIGs offer, the contribution schedules have to be such that neither party wants to reject them. This implies that the SIG needs to offer the parties at least as much as they would obtain without any contributions. That is, when the SIG is deciding how much contributions to make, it needs to offer the politicians at least as much as they would get from setting optimal policies for the informed voters without any campaigning. The *participation constraint* ensures this. Let p^* denote the optimal policy for the informed voter such that $p^* = arg\max_{p\in\mathcal{P}} W(p) = \{p : \nabla W(p) = 0\}$. Therefore, the minimum contributions necessary to induce politicians to endorse policies different from the informed voters optimal ones, that is the participation constraint, for both parties is:[95]

$$C^k \geq \frac{(1-\alpha)f}{\alpha h}[W(p^*) - W(p^k)] \tag{22}$$

This guarantees that by accepting contributions from SIGs each party gets at least as much utility as they would have gotten had they endorsed the optimal policy for informed voters, p^*.

4 Model - Modifications

Now we will introduce the three modifications to the model: sentiments (confidence, optimism/pessimism, and trust), the exogenous shock, and the reserves of the SIG.

4.1 Sentiments

4.1.1 Confidence

As this is an electoral game, there is no consumption nor other macroeconomic variables that are most directly affected by confidence, therefore an alternative approach to incorporating confidence is necessary. It seems reasonable to assume that in a recession the incumbent will suffer a stronger hit to their popularity than the challenger, because once the voters learn that the economy is down the most obvious culprit will usually be the government in place at the time. It is an exception, rather than the rule, that voters are very well informed about the causes of the recession and that blame is put on previous governments. Therefore, a natural way of integrating confidence considerations in the model is by allowing it to affect the incumbency bias in the functional assumptions previously made.

[95] Then the SIG needs to offer party A enough to satisfy the following inequality: $\frac{1}{2} + b + \Lambda \geq \frac{1}{2} + b + (1-\alpha)f[W(p^*) - W(p^B)] + \alpha h(0 - C^B)$

In this model confidence, whether the voters are optimistic or pessimistic, will be interpreted as the general feeling about the state of the economy. During recessions they will be pessimistic, and during booms optimistic. Let θ be the parameter determining the confidence of the population, and let $\theta < 1$ denote pessimism and $\theta > 1$ optimism. The confidence parameter will affect the range (both location and range length) over which the preferences for the fixed policies (β) will be uniformly distributed, for both types of voters.[96] This assumption means that a change in confidence will also change the voters fixed preferences, for example it could affect ideological preferences, the preference for charisma, or underlying preferences for redistribution. This leads to the following seat share equation:

$$s = \frac{1}{2} + b\theta + (1-\alpha)f[W(p^A) - W(p^B)] + \alpha h(C^A - C^B)$$

Notice that the incumbency advantage is now denoted by $b\theta$, therefore the sign of b is no longer sufficient in determining which party will be ex ante more popular. Now party A will be more popular if and only if $b\theta > 0$, which means that even if party A was the incumbent ($b > 0$), if a voter were pessimistic enough (or a strong enough negative shock occurs) (i.e. $\theta < 0$), party B who is the challenger will end up being the more popular party. An important point to make is that the confidence parameter will depend on the strength and direction of the exogenous shock, that is $\theta(\gamma)$. The exogenous shock, γ, is introduced below.

4.1.2 Trust

Since trust is assumed to have an immediate effect on the political game the approach taken in this model is different than it usually is. As there are no direct measures where trust can be incorporated, a somewhat roundabout way is taken, that seems reasonable.

The original GH model does not give much power to the voter and the democratic process, it rather gives all of the pressuring power to the SIGs, and treats the voter as a powerless either perfectly informed rational voting machine (informed voter) or as a perfectly impressionable voting machine (uninformed voter). As this paper deals with sentiments, it seems reasonable to assume that when a voter feels they are wronged too much, they will express it. The democratic process gives voters the ability to protest or strike in order to show their dissatisfaction with certain policies. The government generally has two options in response to protests and strikes; give in to the requests of the voters, or it can suppress the protest by force or time ("waiting it out")[97]. The voters can on the other hand intensify the protest and attempt to cause a revolution. These interactions are very complex, therefore this paper will simplify it greatly in order to give only limited power to the voters in the most basic way that will allow us to incorporate trust.

Given that politicians are rational individuals who are maximising their electoral prospects, they will want to avoid protests at all costs, because that will decrease their popularity greatly. As only the informed voters are aware of the policy implications, they will be the driving force behind protests. This means that the informed voters will protest if the policy platforms offered by the two parties give them less expected welfare than they would get from protesting. We

[96]More precisely $\beta^i \sim U\left(\frac{-2b\theta-1}{2f}, \frac{-2b\theta+1}{2f}\right)$ for the informed and $\beta^u \sim U\left(\frac{-2b\theta-1}{2h}, \frac{-2b\theta+1}{2h}\right)$ for the micro-founded uninformed voters.

[97]Usually the main reason the government can use time is because protesting for voters is costly

will assume that if the informed voters find it beneficial to protest, they will protest and it will be successful. If a protest is successful the informed voters optimal policy will be implemented, however a cost of protesting will be incurred. More precisely, let χ denote the cost of protesting, then the informed voters will *not* protest if and only if

$$(1-\alpha)W(p^*) - \chi \leq (1-\alpha)[\phi(s)W(p^{\mathrm{A}}) + (1-\phi(s))W(p^B)]$$

Let this be the "*no protest constraint*" (NPC). This will essentially discipline the SIG and politicians by limiting how far they can set their policies away from the informed voters preferred policy choice. This means that some policy platforms that were possible in the GH model will now no longer be acceptable for the voters.

The no protest constraint allows us to incorporate trust. We will assume that trust concerns do not affect the informed voters. The reason behind this is that the informed voters know their optimal policy, they know the policy platforms proposed by politicians and their implications, and most importantly they know how much these proposed policy platforms deviate from the optimal policy. This implies that informed voters are aware of how strongly the politicians are acting against their interests, therefore these voters are rational and they know why the policies are suboptimal, hence their behaviour is not influenced by trust. On the other hand, uninformed voters do not know the proposed policy platforms or their implications, nor the truly optimal policy, they are simply unaware of the true state of the world so they must rely on feelings more than knowledge. Intuitively stated, when there is some wrong done to the uninformed voter, they feel it rather than precisely know it. The precise mechanism through which this will be captured is through herd behaviour and information spillover from the informed to the uninformed, in the simplest possible manner.

The uninformed voters do not know the policy implications, however they can either trust the government or the informed voters. What we will refer to as trust is the trust in the government, and distrust is trust in the informed voters. We will not specify a mechanism through which information passes from the informed voters to the uninformed, even though this is an interesting extension, but will simply assume that the more dissatisfied the informed voters are, the more likely the no protest constraint is violated, and this dissatisfaction will pass to the uninformed voters if the uninformed voters are distrustful of the government. This could happen through conversations, newspapers or other forms of communication, and the cost parameter could include these costs for the informed. Let τ be the parameter defining the level of trust in the government for the uninformed voters, then the no protest constraint becomes:

$$\tau(1-\alpha)W(p^*) - \chi \leq (1-\alpha)[\phi(s)W(p^{\mathrm{A}}) + (1-\phi(s))W(p^B)]$$

Let informed voters protest if and only if they anticipate that uninformed voters will protest. Therefore, the no protest constraint including trust determines whether voters will protest or not.

If $\tau = 1$ then the uninformed fully trust the government and only the preferences of the informed voters matters for protesting. However, if $\tau > 1$ the uninformed voters are distrustful of the government, and instead trust the opinions of the informed voters who can disseminate their dissatisfaction to the uninformed. This will cause a kind of "herd behaviour" in the uninformed voters who will blindly accept that the informed voters have good reasons to protest and join in the efforts to pressure the government, or even protest if the policy platforms offered are bad enough. Notice that $\tau \in [1, \frac{1}{(1-\alpha)}]$. If it were the case that $\tau < 1$, it would mean that the uninformed voters trust the government so much that they are willing to counter the protesting efforts and dissatisfaction of the informed voters. On the other hand we need the upper bound $\tau^{\max} = \frac{1}{(1-\alpha)}$, as it implies that the uninformed distrust the government to such an extent (and believe the informed voters) that everyone is treated as an informed voter (i.e. $\alpha = 0$) in the no protest constraint. A distrust larger than this would imply that the sentiments were so bad that the welfare effect was larger than if the whole population was informed. Even though it might be possible for emotions to have such an effect that trust is outside of these bounds, we will avoid this possibility in this paper.

If one were to experience consecutive years of pessimism and low confidence, it seems plausible that this would lead to a decrease in the general trust level over time. This relationship would be very slow and gradual, nevertheless the recent confidence level might effect the general trust level of voters towards the politicians. This leads us to the final assumption about trust: there is a direct, weak and slow, link between the confidence level and trust. More precisely, trust depends on confidence, $\tau(\theta)$ and the change is small, such that if confidence increases ($\theta \uparrow$) then trust increases ($\tau \downarrow$), precisely $-1 < \frac{\partial \tau(\theta)}{\partial \theta} < 0$.

4.2 Unexpected Shock

Looking at the effect of an exogenous shock within the previously defined framework is the second major aim of this paper. These are economy wide shocks, therefore they will influence the utility of both types of voters and the amount of contributions, however they will not influence the fixed policy preferences of the voters directly.[98] The intuition behind this assumption is that ones ideology will generally not greatly change due to a boom or bust directly, rather a shift in confidence could be a channel through which these changes in fixed preferences occur. Let us assume that the SIGs and politicians are aware of the effect of the unexpected shocks on fixed preferences, on the utility of informed voters and on contributions. This assumption implies that the underlying statistical independence between the preferences for fixed policies and the preferences for pliable policies for the informed voters remains.[99] This allows the SIGs and politicians to calculate the policy choice from only a known distribution for fixed preferences. Furthermore, assume that the shock is unexpected, and therefore cannot be anticipated. One can assume that the shock is a white noise event, so the shock itself has a uniform prior. Furthermore, one could also model the shock using a stochastic process (e.g. an AR1 with a white noise element, or a Brownian motion), however I significantly simplify this. I assume that the shock cannot be anticipated, however once it occurs everyone is aware of its effects. The model is unchanged, and there is no time, rather there is a pre-shock environment and a post-shock environment. The comparative statics will then compare the pre- and post-shock environments. Therefore, the shock is treated as a parameter, and the post-shock environment looks at the political game that was (visibly) perturbed.

We know from the benchmark model that the preferences for fixed policies are statistically independent from the preferences for pliable policies for the informed voters, i.e. $U^i(p) \perp \beta^i$. When

[98]Note that this is a strong assumption, because it may be the case that strong unexpected shocks influence the underlying fixed preferences and ideologies.

[99]As well as the statistical independence between preferences for fixed policies and for contributions for micro-founded uninformed voters.

an unexpected shock, γ, occurs it will directly influence the utility of informed voters from the pliable policies, $U^i(p,\gamma) \equiv u^i(p^A,\gamma) - u^i(p^B,\gamma)$. On the other hand, it will affect the preference of informed voters for fixed policies indirectly through the changes in confidence, that is $\beta^i(\theta(\gamma))$. Note that β^i is a random variable with a specified distribution, therefore $\beta^i(\theta(\gamma))$ is an imprecise notation stated for simplicity. The precise statement should read: $\beta^i \sim U\left(\frac{-2b\theta(\gamma)-1}{2f}, \frac{-2b\theta(\gamma)+1}{2f}\right)$. Note that these same specifications can be made for uninformed voters, if we take the micro-founded approach.[100]

Since we assumed that the SIGs and politicians know the distribution of the voters preferences for fixed policies without a shock present, we will further assume that they also know how it changes due to an unexpected shock.[101] Let γ denote the shock, then we know that the informed voter will vote for party A if $U^i(p,\gamma) \equiv u^i(p^A,\gamma) - u^i(p^B,\gamma) \geq \beta^i(\theta(\gamma))$, where the persistent independence is denoted by $U^i(p,\gamma) \perp \beta^i(\theta(\gamma))$.[102]

We assume that trust is a long term trait not influences by the business cycle directly, but rather gained through history, which changes very gradually. On the other hand, confidence is treated as a short term trait that is influenced by the business cycle. This is in line with the fact that confidence is measured in consumer surveys on an annual (and even more frequent) basis, while trust is usually obtained from surveys that are taken only every couple of years, such as the World Values Survey (WVS). Therefore, the shock affects confidence directly, however trust only indirectly. Confidence is a function of the shock, $\theta(\gamma)$, while trust changes "slowly" through the effect confidence has on it, $\tau(\theta(\gamma))$, however we assume that these changes are small per period.

Including shocks in the model leads to the following participation constraint (PC), no protest constraint (NPC), welfare of the SIGs and party A's seat share, respectively:

$$C^k \geq \frac{(1-\alpha)f}{\alpha h}[W(p^*,\gamma) - W(p^k,\gamma)] \tag{23}$$

$$\tau(\theta)(1-\alpha)W(p^*,\gamma) - \chi \leq (1-\alpha)[\phi(s(\gamma))W(p^{\mathrm{A}},\gamma) + [1-\phi(s(\gamma))]W(p^B,\gamma)] \tag{24}$$

$$V_j = \phi(s(\gamma))W_j(p^A,\gamma) + (1-\phi(s(\gamma)))W_j(p^B,\gamma) - (C^A(\gamma) + C^B(\gamma)) \tag{25}$$

$$s = \frac{1}{2} + b\theta(\gamma) + (1-\alpha)f[W(p^A,\gamma) - W(p^B,\gamma)] + \alpha h(C^A(\gamma) - C^B(\gamma)) \tag{26}$$

The shock can be a boom, bust, or no shock at all, and the intensity of the shock does matter. Let $\gamma > 0$ signify a boom, $\gamma < 0$ a recession, and $\gamma = 0$ normal times. We will assume that

[100] Again, this same holds for the micro-founded uninformed voters where $U^u(C) \perp \beta^u$. Then the utility of uninformed voters from contributions is also directly influenced, i.e. $U^u(C,\gamma) \equiv u^u(C^A,\gamma) - u^u(C^B,\gamma)$. On the other hand, it will indirectly change the preferences of uninformed voters by influencing confidence, which in turn influences the voter ideologies.

[101] This could imply that statistical independence between fixed and pliable policy/contributions preference is violated, however the underlying statistical independence is not violated. The reason is that the SIGs and politicians know how the utilities and fixed policy preferences change with the shock, and the shock itself does not change the independence between the two, but rather just shifts both. This implies that the statistical independence between the fixed preferences and pliable policy preferences for the informed, and between the fixed preferences and contribution levels for the uninformed, persists.

[102] The micro-founded uninformed voters vote for party A if $U^u(C,\gamma) \equiv u^u(C^A,\gamma) - u^u(C^B,\gamma) \geq \beta^u(\theta(\gamma))$, where $U^u(C,\gamma) \perp \beta^u(\theta(\gamma))$.

the welfare of informed voters and of the SIG decreases in a recession, and increases during a boom. Let us also assume that both the SIG and informed voters are loss averse, meaning that they lose more welfare with a recession than they gain with a boom. This implies that the welfare functions are strictly concave. Precisely, let $W(p, \gamma)$ and $W_j(p, \gamma)$ be continuous functions of γ with $\frac{\partial W(p,\gamma)}{\partial \gamma} > 0$, $\frac{\partial W_j(p,\gamma)}{\partial \gamma} > 0$, $\frac{\partial^2 W_j(p,\gamma)}{\partial \gamma^2} < 0$ and $\frac{\partial^2 W(p,\gamma)}{\partial \gamma^2} < 0 \; \forall p, \forall \gamma$, and let $W(p, 0) = W(p)$ and $W_j(p, 0) = W_j(p)$.

Furthermore, let us assume that the unexpected shock effects the welfare of the SIG more than it does the informed voters, meaning that in a boom the SIG gains more welfare than the informed voters, and loses more than them in a recession. This captures the idea that the SIGs are often large corporations and companies that are leading the economy, meaning that they will usually be the first to reap the benefits of a boom and get the largest piece of the cake, however they will also suffer the largest costs from recessions. Precisely this translates into the following condition: $\frac{\partial W_j(p,\gamma)}{\partial \gamma} > \frac{\partial W(p,\gamma)}{\partial \gamma} > 0$. Finally, let a boom contribute toward optimism and a recession contribute toward pessimism, that is let $\frac{\partial \theta(\gamma)}{\partial \gamma} > 0$.

4.3 SIG

In order to obtain the final result of the paper we need to introduce one final variable. It is reasonable to assume that the SIG will not have infinite resources for contributing. Let these total available resources be called *reserves*, $Z(\gamma)$, such that they decrease with recessions and increase with booms, $\frac{\partial Z(\gamma)}{\partial \gamma} > 0$. Let the reserves function be continuous and strictly concave. Lets assume that in normal times the SIG has larger reserves than total contributions necessary, that is $Z(0) > C^A(0) + C^B(0) \equiv C^T(0)$. The reason for this assumption is that most likely only the SIGs that do have these funds available will be able to form SIGs influential enough to pressure the government into adopting policy platforms that are not in the interest of informed voters. Let us further assume that the SIG has two methods of influencing the policy decisions; it can contribute for the influence motive or its members can strike. Let striking be more costly than contributions, therefore the SIG will contribute as long as the funds permit it to do so. When the point is reached that the amount of necessary contributions exceeds the amount of reserves the members of the SIG will strike, which means that due to insufficient reserves their only alternative is to strike. If the strike is unsuccessful, the SIG will fail. Precisely, as long as $C^A(\gamma) + C^B(\gamma) \equiv C^T(\gamma) \leq Z(\gamma)$ the SIG will contribute instead of strike. Finally, assume that the reserves increase quicker in booms than total contributions, and decrease quicker in recessions than total contributions. Precisely, $\frac{\partial Z(\gamma)}{\partial \gamma} > \frac{\partial C^T(\gamma)}{\partial \gamma}$. This implies that there exists a shock such that it leaves the SIG with with just enough reserves for the total contributions necessary. Precisely, $\exists! \tilde{\gamma} : Z(\tilde{\gamma}) = C^T(\tilde{\gamma})$. Then there exists a set of shocks such that if a shock takes place that is within this set strikes will occur, that is $\Gamma = \{\gamma < \tilde{\gamma} : Z(\gamma) < C^T(\gamma)\}$. More precisely let $\Gamma_N = \{\gamma < \tilde{\gamma} : Z(\gamma) < C^T(\gamma)\}$ be the set with a neutral confidence level (i.e. $\theta(\gamma) = \theta$), and $\Gamma_C = \{\gamma < \tilde{\gamma} : Z(\gamma) < C^T(\gamma)\}$ the set when confidence is allowed to vary with the shock.

4.4 Political Equilibrium

We want to find a subgame perfect Nash equilibrium of this three stage, noncooperative game. In the first stage the SIGs independently and simultaneously announce their contribution schedules, one for each party. In the second stage, the parties choose their policy platforms. In the third stage the voters decide whether to protest or not. We assume that promises are kept, hence after the policies are set, the contributions are paid and campaigns waged.

Definition 31. *An equilibrium consists of a pair of feasible policy vectors ($p^{Ao}(\gamma), p^{Bo}(\gamma)$) and a set of contribution schedules ($C_j^{Ao}(p^{Ao}, \gamma), C_j^{Bo}(p^{Bo}, \gamma)$), one for each SIG j, such that*

1. p^{Ao} maximizes s given p^{Bo}, $\gamma \notin \Gamma$, $\{C_j^{Ao}(p^{Ao}, \gamma)\}$, $\{C_j^{Bo}(p^{Bo}, \gamma)\}$ subject to constraints 23 and 24

2. p^{Bo} maximizes $(1 - s)$ given p^{Ao}, $\gamma \notin \Gamma$, $\{C_j^{Ao}(p^{Ao}, \gamma)\}$, $\{C_j^{Bo}(p^{Bo}, \gamma)\}$ subject to constraints 23 and 24

3. each $C_j^k(p^k, \gamma)$ is continuous and differentiable when positive, with $C_j^k(p^k, \gamma) \geq 0$ for all p^k and $\gamma \notin \Gamma$

4. for each lobby j, there does not a exist a profitable deviation to alternative feasible contribution schedules $\tilde{C}_j^A(p^A, \gamma)$ and $\tilde{C}_j^B(p^B, \gamma)$, such that

$$\begin{aligned}&\phi(\tilde{s}(\gamma))W_j(\tilde{p}^A, \gamma) + [1 - \phi(\tilde{s}(\gamma))]W_j(\tilde{p}^B, \gamma) - \tilde{C}_j^A(\tilde{p}^A, \gamma) - \tilde{C}_j^B(\tilde{p}^B, \gamma)\\ &> \phi(s(\gamma))W_j(p^{Ao}, \gamma) + [1 - \phi(s(\gamma))]W_j(p^{Bo}, \gamma) - C^A(p^{Ao}, \gamma) - C^B(p^{Bo}, \gamma)\end{aligned}$$

   where $\tilde{p}^A$ maximizes and $\tilde{p}^B$ minimizes

$$\frac{1-\alpha}{n_I}\int_{i \in I} F[u^i(p^A, \gamma) - u^i(p^B, \gamma)]di + \alpha H[\sum_{z \neq j} C_z^{Ao}(p^A, \gamma) + \tilde{C}_j^A(p^A, \gamma) - \sum_{z \neq j} C_z^{Bo}(p^B, \gamma) - \tilde{C}_j^B(p^B, \gamma)]$$

   and

$$\tilde{s}(\gamma) = \frac{1-\alpha}{n_I}\int_{i \in I} F[u^i(\tilde{p}^A, \gamma) - u^i(\tilde{p}^B, \gamma)]di + \alpha H[\sum_{z \neq j} C_z^{Ao}(\tilde{p}^A, \gamma) + \tilde{C}_j^A(\tilde{p}^A, \gamma) - \sum_{z \neq j} C_z^{Bo}(\tilde{p}^B, \gamma) - \tilde{C}_j^B(\tilde{p}^B,$$

The first two conditions specify the Nash equilibrium among the parties in the policy setting stage, while the last conditions ensures there are no profitable deviations for the SIGs. Note that the third stage has parties who want to avoid protests in the policy setting stage. This equilibrium definition assumes that the parties anticipate the contribution schedules of the SIGs. Such a strong assumption is an oversimplification, however with time it is reasonable to assume that parties learn what contributions different policies lead to. Finally, notice that this definition allows for several SIGs, however as this paper considers only the single SIG case, the definition collapses to it by setting $z = j = 1$.

4.5 Timing

We are interested in finding the effect of sentiments with and without a shock, as well as the effect of the shock itself. There is a natural timing to this setup, which will simplify the notation. We will assume that the first vote takes place without the exogenous shock, in normal times. Therefore, in the first period we will be looking only at the effect of emotional voters. Then the unexpected shock takes place which changes the environment and new equilibrium values have to be obtained, hence the second period will allow us to analyse the effect of a shock. More precisely, let $t \in T = \{1, 2\}$ signify time such that no unexpected shock takes place in $t = 1$ ($\gamma = 0$), and the unexpected shock occurs only in $t = 2$ ($\gamma \neq 0$). We will use t as a subscript (or its associated values) whenever it is beneficial to do so.

5 Equilibrium Analysis

For simplicity, the following analysis will be based on a single lobby. This makes the problem one of direct control, because the single SIG can implement any policy it prefers if it can provide enough contributions and/or there are enough uninformed voters. If there were no uninformed voters, or the parties did not care about contributions for any reason (e.g. contribution schedule not related to policies implemented), the contributions will be null, $C^A = C^B = 0$. Then the parties will only care about the informed voter and the policy platforms implemented will be the average informed voters optimal ones:

$$p_t^* = arg \max_{p \in \mathcal{P}} W_t(p, \gamma) \tag{27}$$

where $W_t(p, \gamma) = \frac{1}{n_I} \int_{i \in I} F[u_t^i(p^A, \gamma) - u_t^i(p^B, \gamma)] di$. Note further that both parties would choose the same policy, $p_t^A = p_t^B = p_t^*$.[103] Furthermore, if the SIG did not have any constraints it had to satisfy it would choose its unconstrained optimal policy, that is:

$$p_{j,t}^* = arg \max_{p \in \mathcal{P}} W_{j,t}(p, \gamma) \tag{28}$$

On the other hand, when the lobby does contribute it will choose their optimal value bearing in mind that it has to provide sufficient contributions to satisfy the participation constraint of the parties, and to avoid protests. The lobby will use backward induction to be able to solve this problem::

$$\begin{aligned} \max_{(p_t^A, p_t^B) \in \mathcal{P}^2} V_{j,t} =& \phi_t^A W_j(p_t^A, \gamma) + \phi_t^B W_j(p_t^B, \gamma) - (C_t^A + C_t^B) \\ \text{s.t. } C_t^k \geq& \delta \left[W(p_t^*, \gamma) - W(p_t^k, \gamma) \right] \\ \tau(\theta_t)(1-\alpha) W(p_t^*, \gamma) - \chi \leq& (1-\alpha)[\phi_t^A W(p_t^{\mathrm{A}}, \gamma) + \phi_t^B W(p_t^B, \gamma)] \end{aligned} \tag{29}$$

where $\phi_t^A = \phi(s_t)$,$\phi_t^B = (1 - \phi(s_t))$, $\delta = \frac{(1-\alpha)f}{\alpha h}$, and $k = A, B$. Let both $W_j(\cdot)$ and $W(\cdot)$ be twice differentiable functions with respect to p with second order conditions satisfied such that

[103] If informed voters are a representative sample of all voters (distribution of utility functions among informed and uninformed equal) then p_t^* maximizes the social welfare function (equivalently with homogenous individuals).

they achieve a unique maximum for each γ, particularly lets assume they are strictly concave functions. That is, $\frac{\partial^2 W_j(\cdot)}{\partial p^2} < 0$ and $\frac{\partial^2 W(\cdot)}{\partial p^2} < 0$.

If the lobby did not satisfy the PC, the party would simply implement p^*, as it gets a higher payoff from it. If the lobby did not satisfy the NPC, the party would also implement p^*; as we assume that protests are the worst cases for the parties, because they lose popularity and potentially office.[104]

Recall that $\tau(\theta) \in [1, \frac{1}{(1-\alpha)}]$. Note that, as the fraction of uninformed voters increase, the level of trust is allowed to take higher values. Furthermore, notice how important the cost of organising to protest is. Suppose $\chi = 0$, then the only way the NPC can bind is if the uninformed voters fully trust the government, i.e. $\tau = 1$ and the SIG does not contribute (the parties implement p^*), otherwise the NPC would be violated and voters would protest. As the cost of organising increases, the SIG can offer policies closer to its own ideal, and is therefore willing to contribute. As trusts increases ($\tau \downarrow$), it forces the SIG to improve its policy offer, and with a small enough fraction of informed voters, high enough trust can make contributions worthless, as the SIG cannot make the politicians propose any message other than p^*. In fact, the maximum value trust can take, such that the NPC is satisfied, is:

$$\tau(\theta_t) \leq \frac{\chi}{(1-\alpha)W(p_t^*, \gamma)} + \frac{\phi_t^A W(p_t^{\mathrm{A}}, \gamma) + \phi_t^B W(p_t^B, \gamma)}{W(p_t^*, \gamma)}$$

Note that the effect of the cost of protesting explodes as the fraction of uninformed voters comes close to 1. Since χ is interpreted as the informed group of voters cost of organising, it is intuitive that when there are very few informed voters around, the cost of organising the informed and uninformed voters to protest will be high. That is, if there are very few informed voters, the cost of convincing the uninformed voters to protest will be very high.

5.1 Influence motive

In this case the SIG contributes just enough to make the parties choose policies that the SIG finds constrained optimal. There is no electoral motive present, as the lobby does not contribute "extra" funds to get their preferred candidate into office. This implies that the PC (equation 23) binds for both parties. This implies that the combined PC and NPC become:

$$C_t^k \leq \bar{C}_t^k := \frac{\delta}{\tau(\theta_t)} \left[\left(\phi_t^k - \tau(\theta_t)\right) W(p_t^{\mathrm{k}}, \gamma) + \phi_t^l W(p_t^l, \gamma) + \frac{\chi}{(1-\alpha)} \right] \tag{30}$$

where $k \neq l$ for $k, l \in \{A, B\}$. We are interested in situations when the SIG and the informed voters prefer different policies. By shifting the contribution schedule such that the policies of the parties move away from p^*, it is decreasing the average informed voter welfare (i.e. $W(p, \gamma) \downarrow$). That implies that the RHS of the NPC is falling the further the policy is from the average informed voter ideal. For analytic simplicity we will assume that the difference in the preferred policies between the SIG and the informed voters is large enough that the NPC will always bind. That is,

[104] This assumption would need further specification. In a two party system the voters do not have many outside options, hence if one lost popularity, the other would gain it. But this complication can be avoided by assuming that if protests do occur, a new party would enter, win, and implement the average informed voters optimal policy. This party could be composed of the informed voters.

as the parties want to avoid protests at all costs the NPC will bind, hence the combined constraint (equation 30) also binds in both time periods. Therefore, $C_t^k = \max\{0, \bar{C}_t^k\}$. When $\bar{C}_t^k > 0$ for both parties, the preferred policy chosen by the SIG will be:

$$p_t^k = arg\max_{p \in \mathcal{P}} \left[\phi_t^k W_j(p_t^k, \gamma) + \delta W(p_t^k, \gamma) \left(1 - \frac{2\phi_t^k}{\tau(\theta)}\right)\right] \tag{31}$$

The lobby sets contributions such that it makes parties set policies that maximize a weighted sum of the lobbies and average informed voters welfare. In comparison to the benchmark model[105] the weight on the average informed voters welfare is decreased by, $-\frac{2\phi_t^k}{\tau(\theta)}$, due to the no protest constraint. This is somewhat surprising, as one would expect that when voters have the possibility of protesting they will also gain more power in the policy decision process, relative to the benchmark model, by being able to limit the extent to which the policies chosen are allowed to deviate from p^*. However, this is not the case due to the implicit assumption within the NPC. By specifying this form of protest constraint we are assuming that the SIG knows the exact calculation the informed voters do, as well as their valuations. The SIG is aware of what the informed voters expect the chosen policy will be, and hence the SIG can extract all possible surplus available from this information. Due to the fact that we assumed the NPC binds, the SIG extracts all of the information from the NPC and designs a contribution schedule such that the policy is in its own favour as much as possible, without causing protests.[106]

More specifically, the term $-\frac{2\phi_t^k}{\tau(\theta)}$ in the policy equation (equation 31) appears due to the fact that the informed voters welfare under the optimal policy depends on the welfare they expect to get from the SIGs policies. Since the NPC binds, the informed voters are aware their preferred policy will not be implemented, and the SIG learns about the NPC and surely avoids violating the NPC. From now on let the new term, $\mathcal{I} \equiv \frac{2\phi_t^k}{\tau(\theta)}$, be called the *information bias*. Note that all of the proofs are in the appendix.

Lemma 32. *A larger weight ($\tilde{\delta} > \delta$) on the preferences of the average informed voter ($W(\cdot)$) in the SIG's optimization problem (31) pushes both of the SIG's preferred policies (p_t^k) closer to the optimal policy of the informed voter (p_t^*), while a smaller weight pushes it closer to the SIG's preferred policy choice ($p_{j,t}^*$).*

[105] The policy chosen when only an influence motive exists in GH is $p^k = arg\max_{p \in \mathcal{P}} \left[\phi^k W_j(p^k) + \delta W(p^k)\right]$

[106] To clarify this, lets say that the informed voters know p^A will be chosen, $\phi^A = 1$. In the benchmark case this does not affect the participation constraint and the contributions remain equivalent, $C^k \geq \delta\left[W(p^*) - W(p^k)\right]$. The SIG has to give the politicians enough contributions to compensate for the loss in votes of the informed voters. When the NPC is added, the politicians get an extra piece of information (so does the SIG). The informed voters welfare under the optimal policy is bounded by the combination of their expected welfare from the SIGs preferred policies and their cost of protesting, $W(p_t^*, \gamma) \leq \frac{1}{\tau(\theta)}[\phi^A W(p_t^A, \gamma) + \phi^B W(p_t^B, \gamma)] + \frac{\chi}{\tau(\theta)(1-\alpha)}$. Therefore, through the NPC the politicians gain information about what $W(p_t^*, \gamma)$ really is, and the SIG can use this information to ask for policies that are more in their own favour than they could have in the benchmark GH model. In fact, by assuming the NPC binds, the model assumes the SIG extracts all the information and pushes the policy in its own favour as much as possible without causing protests.

In this new environment, if the informed voters know p^A will be chosen, $\phi^A = 1$, the NPC would lead to $W(p_t^*, \gamma) \leq \frac{W(p_t^A, \gamma)}{\tau(\theta)} + \frac{\chi}{\tau(\theta)(1-\alpha)}$ that leads to $C_t^k \geq \delta\left[\frac{1}{\tau(\theta)}\left(W(p_t^A, \gamma) + \frac{\chi}{(1-\alpha)}\right) - W(p_t^k, \gamma)\right]$. Suppose that the uninformed agents fully trust the government, then the contributions, when only the influence motive is present, will be $C_t^A \geq \frac{\chi}{(1-\alpha)}$ and $C_t^B \geq W(p_t^A, \gamma) - W(p_t^B, \gamma) + \frac{\chi}{(1-\alpha)}$. As we can see the NPC, the cost of protesting, and the extra information it provides the SIG, greatly changes the environment.

The intuition behind Lemma 32 is that the more the parties care about informed voters, the more they would like to endorse policies that favour them, and therefore the SIG is forced to pick policies that are more favourable for the average informed voter. Many results will be based on this simple logic.

5.1.1 The effects of the NPC

Introducing the NPC and trust has non trivial consequences on the policies. As we saw, it pushes the SIG's chosen policy closer to the SIG's optimum than it is under the benchmark GH model. Before looking at the results, suppose none of the constraints bind, then the NPC and PC place upper bounds on the average informed voters welfare from their own optimal policy:

$$W(p_t^*, \gamma) \leq \min \left\{ \underbrace{\frac{1}{\tau(\theta_t)} \left(\phi_t^A W(p_t^A, \gamma) + \phi_t^B W(p_t^B, \gamma) + \frac{\chi}{1-\alpha} \right)}_{\text{NPC}}, \underbrace{\frac{C_t^k}{\delta} + W(p_t^k, \gamma)}_{\text{PC}} \right\}$$

The NPC constraint depends on trust, confidence, and the cost of protesting. The higher the cost of protesting (or the more trust in the government) the higher can $W(p_t^*, \gamma)$ be. This constrains the whole problem significantly, since $W(p_t^*, \gamma) \geq W(p_t, \gamma)$ for all p_t. The PC depends only on the contributions and the combined parameter δ. Note that from the PC the SIG can select policies that improve its own welfare by increasing contributions, that is by increasing contributions the SIG can shift the policy away from the average informed voters optimal policy. From the NPC this is not possible. The convex combination of SIG policies has to be close enough the the average informed voters ideal policy, unless the cost of protesting is high, or the fraction of uninformed voters is high. The fact that the NPC is such a strong constraint drives the following results. Note that now we continue to assume that both of the PC's and the NPC bind. When all the constraints bind, the set of possible policies shrinks, and it does so benefiting the SIG, which has direct control over the policy choice.

Proposition 33. *If voters can protest, the SIG can select a policy (p_t^k) that is closer to its optimal policy ($p_{j,t}^*$), than under the benchmark. Therefore, the level of contributions is higher.*

Note that for this result to hold the payoffs have to be common knowledge, only then can the SIG infer the NPC, and use it to improve its own welfare. As previously described, the NPC provides the SIG with information as to how much they can shift policies in their own favour, whilst avoiding protests. Thereby, the possibility of protesting, and the cost of protesting, allow the SIG to shift the policy in its own favour by exploiting the information the NPC offers.

From the GH model we know that the more popular party will set policies that favour the SIG more than the informed voter, and reversely for the less popular one. The larger popularity leads to a larger probability of implementing its policies, which puts more weight on the welfare of the more popular party in the SIG's decision problem. This can be seen from the GH policy choice $p^k = arg\max_{p \in \mathcal{P}} \left[\phi^k W_j(p^k) + \delta W(p^k) \right]$ and applying Lemma 32. From the above proposition we know that the information bias leads both parties to put less emphasis on the welfare of the informed voters, $W(\cdot)$, and therefore equilibrium policies are shifted towards the SIG's.

Furthermore, the policy bias of the more popular party towards the SIG's welfare, $W_j(\cdot)$, is exacerbated as a result of the information bias because popularity (or legislative power)[107], ϕ_t^k, is not only attached to the welfare of the SIG, but also decreases the focus on the informed voters welfare. This means that the NPC makes both parties policies less favourable to the informed voters, which also also makes the bias of the more popular party, in favour of the SIGs, even worse. Note that this bias can be very strong. If $\phi_t^k = 1$ and $\tau(\theta_t) = 1$ the welfare of the informed voter will not be considered at all. Equivalently, with few informed voters and extreme distrust in the government ($\tau \to \infty$), we recover the benchmark equilibrium policies.

Proposition 34. *Wlog suppose $b\theta_t(\gamma) > 0$ (party A is ex-ante more popular). If voters can protest, the bias caused by popularity (being the more popular party) is stronger than in the benchmark.*

Proposition 34 tells us is that the popularity of the party, ϕ_t^k, will affect the policy the SIG will want to implement such that it will make the more popular party implement policies that are more in line with the SIG. This is true, because when both the PC's bind the share of votes party A receives simplifies to $s = \frac{1}{2} + b\theta(\gamma)$. Next it is important to look at the effect of ex-ante popularity, $b\theta$, on the welfares and contributions offered. Without loss of generality (wlog) suppose that party A is more ex-ante popular, $b\theta(\gamma) > 0$, then, in the influence motive case, party A will also be more popular, $\phi_t^A > \frac{1}{2} > \phi_t^B$. The next proposition describes the implications for welfare and contributions.

Proposition 35. *Wlog suppose $b\theta_t(\gamma) > 0$, then $\phi_t^A > \frac{1}{2} > \phi_t^B$. If voters can protest, the SIG bias keeps the same qualitative features (i.e. $C_t^A > C_t^B$, $W(p_t^B, \gamma) > W(p_t^A, \gamma)$, $W_j(p_t^A, \gamma) > W_j(p_t^B, \gamma)$), however it is stronger towards the more popular party than in the benchmark, i.e. $C_t^A > C^A > C^B$, $W(p^B) > W(p^A) > W(p_t^A, \gamma)$, $W_j(p_t^A, \gamma) > W_j(p^A) > W_j(p^B)$.*

From Proposition 35 we know that the SIG will induce the more popular party to endorse policies that are more beneficial to the SIG than to the informed voters, by offering them higher contributions. This drives the informed voters welfare from party A's platform below those of party B's platform, and the SIG welfare from party A's above that from party B. Combining this with Proposition 34 we know that the party popularity bias (incumbency advantage) is more pronounced in this model then in the benchmark GH model.

5.1.2 The effects of trust

When the uninformed voters have complete trust, $\tau(\theta_t) = 1$, the information bias is $2\phi_t^k$, and only the probability of implementing its policy, $\phi_t^k \in [0, 1]$, matters for the policy choice. Full trust therefore puts an upper bound on the information bias. On the other hand, if the uninformed voters are most distrustful, $\tau(\theta_t) = \frac{1}{1-\alpha}$, then the information bias becomes $(1 - \alpha)\phi_t^k$. Full distrust puts a lower bound on the information bias, determined by the fraction of uninformed voters in the economy. Specifically, the more uninformed voters (with maximum distrust) the smaller the information bias. This leads to a policy that is more favourable to the informed voters. This means that the more trust there is, the less will the policy favour the informed voters. As uninformed voters become more distrustful, the policy chosen will become more favourable to the

[107]Precisely, this is legislative power, the probability of succesfully implementing party k's policy. However, due to this being a function of the share of votes, this is also a proxy for popularity.

informed voters. Precisely, as $\tau(\theta_t)$ increases, the focus in policy decisions will slowly shift away from the welfare of the SIG towards the welfare of the informed voters, that is the "information bias" is decreasing in τ, $\frac{\partial \mathcal{I}}{\partial \tau} < 0$.

Proposition 36. *Higher distrust of the government (higher τ) shifts the equilibrium towards p_t^*. Higher trust (lower τ) shifts the equilibrium policy towards $p_{j,t}^*$.*

Note that $\tau(\theta) \in [1, \frac{1}{1-\alpha}]$, places boundaries on the information bias, $\frac{2\phi_t^k}{\tau(\theta)} \in [2\phi_t^k, \frac{2\phi_t^k}{1-\alpha}]$. From Proposition 36 we see that trust will have non-trivial effects on the equilibrium policy, such that a more distrustful society will have policies that favour the informed voters more than a trustful society. We are also interested in the effect of these changes in trust on welfare and contributions.

Proposition 37. *Higher trust (lower τ) increases SIG contributions, increases SIG welfare, and decreases voter welfare. Formally, let $\tilde{\tau} > \tau$, then $W_{j,t}^k(\cdot)\big|_{\tau=\tau} < W_{j,t}^k(\cdot)\big|_{\tau=\tilde{\tau}}$, $W_t^k(\cdot)\big|_{\tau=\tau} > W_t^k(\cdot)\big|_{\tau=\tilde{\tau}}$, and $C_t^k\big|_{\tau=\tau} < C_t^k\big|_{\tau=\tilde{\tau}}$ for both $k \in K = \{A, B\}$. With decreased trust the effect is reversed.*

Proposition 37 describes the effects of changes in trust on welfare and contributions. Trust has aggregate changes and therefore influences both parties' equilibrium policies, which changes welfare. From the discussion so far it is clear that confidence enters the model in two ways. It effects the model directly through the incumbency bias, $b\theta_t(\gamma)$, and indirectly through trust, $\tau(\theta_t(\gamma))$. The direct effect of changes in confidence through the incumbency bias, $b\theta_t(\gamma)$, only changes the relative popularity of the two parties. Therefore the aggregate direct effect on policies, welfares and contributions will be zero sum.

5.1.3 The effects of confidence

The direct effect operates through the effect on party popularity. Suppose wlog party A is the incumbent (i.e. is ex ante more popular), $b > 0$, and times are normal in period 1, $\theta_1 = 1$. Pessimism will unequivocally decrease the popularity of party A, and increase party B's popularity. Proposition 35 implies that this makes the new equilibrium p^A less favourable to the SIG (who also contribute less), and more favourable to the informed voter. The effect is opposite for party B.[108] Furthermore, the direct effect of pessimism can be divided into two categories; with weak pessimism party A loses popularity but remains ex-ante more popular, $b > b\theta_2 > 0$, with strong pessimism party A becomes ex-ante less popular, $b > 0 > b\theta_2$. Proposition 34 implies that weak pessimism makes the equilibrium policies converge, while strong pessimism makes them diverge,[109] and it can switch the incumbency bias.

On the other hand, optimism will make party A even more popular, $b\theta_2 > b > 0$. Proposition 34 implies that the two parties equilibrium policies diverge further.[110] Proposition 35 implies that A's policy will increase the SIGs welfare, decrease the informed voters welfare and garner more

[108] More formally: $W_j(p_1^A, \gamma) > W_j(p_2^A, \gamma)$ and $W_j(p_1^B, \gamma) < W_j(p_2^B, \gamma)$; $W(p_1^A, \gamma) < W(p_2^A, \gamma)$ and $W(p_1^B, \gamma) > W_j(p_2^B, \gamma)$; $C_1^A > C_2^A$ and $C_1^B < C_2^B$.

[109] With weak pessimism the norm between the policies is decreasing with pessimism, $\frac{\partial \|p_2^A - p_2^B\|}{\partial \theta} > 0$, while with strong pessimism it is increasing with pessimism, $\frac{\partial \|p_2^A - p_2^B\|}{\partial \theta} < 0$.

[110] With optimism the norm between the parties policies is increasing in optimism, $\frac{\partial \|p_2^A - p_2^B\|}{\partial \theta} > 0$.

contributions. Basically, the inequalities of Proposition 35 will intensify. There are actually five different cases to consider. Suppose wlog that party A is the incumbent, $b > 0$, and that a fall in confidence takes place from neutrality to pessimism, therefore let $\theta_1 = 1$.

1. When pessimism is not too strong, $\theta_2 \in (0, 1)$, party A remains the more popular party, $b > b\theta_2 > 0$, and the inequalities of Proposition 35 continue to hold, however the ex-ante party popularity changes. The difference in popularity of the two parties decreases, where party A loses popularity and party B gains it.[111] From Proposition 34 we know that this makes party A's policy, p_2^A, more favourable to the informed voters than it was, while party B's policy becomes more favourable to the SIG, therefore the policies become more similar, $\|p_1^A - p_1^B\| > \|p_2^A - p_2^B\|$. Proposition 35 furthermore implies that $W_j(p_1^A, \gamma) > W_j(p_2^A, \gamma)$ and $W_j(p_1^B, \gamma) < W_j(p_2^B, \gamma)$; $W(p_1^A, \gamma) < W(p_2^A, \gamma)$ and $W(p_1^B, \gamma) > W_j(p_2^B, \gamma)$; $C_1^A > C_2^A$ and $C_1^B < C_2^B$.

2. When pessimism makes the parties equally popular, $\theta_2 = 0$, that is, $b > b\theta_2 = 0$. Then both parties use the same equilibrium policies, $p_2^A = p_2^B \neq p^*$, which leads to the same welfare for the lobby, informed voters, and equivalent contributions.[112]

3. When pessimism reverses the popularity of the parties, $\theta_2 \in (-\infty, 0)$, then party B becomes the more popular party, $b > 0 > b\theta_2$. This leads to a reversal of the inequalities of Proposition 35, where the policy of party B will now become the more favourable to the SIG (garnering more contributions), and less favourable to the average informed voter, relative to party A.

4. In the single case of optimism, $\theta_2 \in (1, \infty)$, the more popular party further increases its popularity, $b\theta_2 > b > 0$. Proposition 34 and 35 imply that the equilibrium policies diverge, where p_2^A increases the SIGs welfare, decreases informed voters welfare, and garners more contributions from the SIG. The opposite holds for party B's policy.[113]

The final case is normal times, $\theta_2 = 1$, the popularity remains equal, $b = b\theta_2 > 0$, and nothing changes. We summarise the general direct effect of changes in confidence in the following proposition. The proofs simply follow Lemma 32, Proposition 34 and Proposition 35, therefore they are omitted.

Proposition 38. *Wlog suppose $\theta_1 = 1$ and $b > 0$. The voters can become pessimistic, optimistic or remain neutral.*

- *Optimism, $\theta_2 \in (1, \infty)$: increased the popularity of the more popular party, which causes the equilibrium policies to diverge, with corresponding welfare effects ($W_j(p_1^A, \gamma) < W_j(p_2^A, \gamma)$ and $W_j(p_1^B, \gamma) > W_j(p_2^B, \gamma)$; $W(p_1^A, \gamma) > W(p_2^A, \gamma)$ and $W(p_1^B, \gamma) < W_j(p_2^B, \gamma)$; $C_1^A < C_2^A$* and $C_1^B > C_2^B$)

[111]More formally; $|\phi_1^A - \phi_1^B| > |\phi_2^A - \phi_2^B|$ such that $\phi_1^A > \phi_2^A$ and $\phi_1^B < \phi_2^B$

[112]More formally: $W_j(p_2^A, \gamma) = W_j(p_2^B, \gamma)$, $W(p_2^A, \gamma) = W(p_2^B, \gamma)$ and $C_2^A = C_2^B$

[113]More formally: $W_j(p_1^A, \gamma) < W_j(p_2^A, \gamma)$ and $W_j(p_1^B, \gamma) > W_j(p_2^B, \gamma)$; $W(p_1^A, \gamma) > W(p_2^A, \gamma)$ and $W(p_1^B, \gamma) < W_j(p_2^B, \gamma)$; $C_1^A < C_2^A$ and $C_1^B > C_2^B$

- *Weak pessimism, $\theta_2 \in (0,1)$: decreased popularity of the more popular party, which causes the equilibrium policies to be convergent, with corresponding welfare effects ($W_j(p_1^A,\gamma) > W_j(p_2^A,\gamma)$ and $W_j(p_1^B,\gamma) < W_j(p_2^B,\gamma)$; $W(p_1^A,\gamma) < W(p_2^A,\gamma)$ and $W(p_1^B,\gamma) > W_j(p_2^B,\gamma)$; $C_1^A > C_2^A$* and *$C_1^B < C_2^B$)*

- *Strong pessimism, $\theta_2 \in (-\infty,0)$: reversed popularity of parties, making party B more popular, with divergent equilibrium policies as pessimism increases. The welfare effects are the reverse of the case where party A is more popular.*

- *Goldilocks pessimism, $\theta_2 = 0$: the parties become equally popular, have the same equilibrium policy, welfare and contributions (where the SIG also gets the same welfare from either party).*

- *Neutrality, $\theta_2 = 1$: nothing changes.*

Proposition 38 identifies the direct effect of a change in confidence. Note that changes in party popularities are by definition zero sum, $\phi_t^A + \phi_t^B = 1$ and $\Delta\phi_t^A + \Delta\phi_t^B = 0$. The change in the optimal policy (and corresponding changes in optimal welfares and contributions), precipitated by a change in popularity, depends on the functional forms of the welfares and parameter specifications.

The indirect effect of confidence operates through trust, $\tau(\theta_t)$, such that optimism ($\theta_t > 0$) increases trust and pessimism ($\theta_t < 0$) decreases it. Recall that we assumed that higher optimism leads to more trusting uninformed voters (of the government), and that the effect is small, $-1 < \frac{\partial\tau(\theta)}{\partial\theta} < 0$. As the indirect operates through trust, it suffices to know how trust effects the equilibrium, which is described in Propositions 36 and 37, where note that the effect of confidence on trust is less than one for one.

5.1.4 The effects of unexpected shocks

One of the main aims of the paper is to analyse how the state of the economy feeds back into the voting decision, and how this is affected by emotions. As we have previously defined, the shock, γ, effects the model by directly influencing the welfare of the SIG, the welfare of informed voters and confidence. These changes will endogenously effect trust, contributions, and the equilibrium policies. We have previously seen the reasons for the following assumptions, I repeat the assumptions for convenience: $\frac{\partial W_j(p,\gamma)}{\partial\gamma} > \frac{\partial W(p,\gamma)}{\partial\gamma} > 0$, $\frac{\partial^2 W_j(p,\gamma)}{\partial\gamma^2} < 0$, $\frac{\partial^2 W(p,\gamma)}{\partial\gamma^2} < 0$, and $\frac{\partial\theta(\gamma)}{\partial\gamma} > 0$.

In the following we will show that when a shock occurs, the informed voters optimal policy, and the SIGs preferred policy, both change. Furthermore, we will see that total contributions decrease in a recession and increase in a boom. Finally, we will show that if the recession is bad enough the members of the SIG will strike, and that pessimism will make the recession that leads to strikes more severe. This means that due to pessimism the members of the SIG will require a stronger recession for them to go on a strike.

Remember that the game in the first period, $t = 1$, occurs in normal times, $\gamma = 0$, with a neutral confidence level, $\theta_1(0) = 1$. The election takes place, the policy platforms are chosen, the welfares realised, and finally the policies of one party implemented. In the second period, $t = 2$, an unexpected shock, $\gamma \neq 0$, takes place. This shock is assumed exogenous, therefore the agents cannot anticipate it. In both periods the shock is treated as a parameter rather than as a function or a random variable, because the shock is always first observed, taken into consideration, and only then the policy optimisation is done by the informed voters and the SIG. This allows us to greatly simplify the problem, where instead of having to use multivariate analysis in each optimisation or probabilistic reasoning (with appropriate expectations), we can use the envelope theorem and treat the problem as a single variable case per realisation of the shock. A further simplification is that we assume that the policy choices do not affect the shock parameter.

The exogenous shock may, or may not, change the informed voters and SIGs optimal policy ($p_t^*, p_{j,t}^*$), which depends on the functional forms of the informed voter and SIG welfare functions ($W(\cdot), W_j(\cdot)$). Without further assumptions on each of the welfare functions[114] we cannot know whether the unconstrained optimal policies change. Nevertheless, as we assumed the welfare functions as strictly concave, we know that the unconstrained problem will always have a unique solution.

Lemma 39. *There exists a unique optimal policy (p_2^*) and a unique constrained optimal policy (p_2^k) for each possible shock for both parties.*

Due to the fact that there is always a unique unconstrained welfare maximising policy we know that for all realisations of the shock there exists an optimal policy, p^* (and p_j^*) which defines an envelope connecting the optimised welfare, $W(p^*, \gamma)$ (and one for $W_j(p_j^*, \gamma)$), for all γ. Since there exists a pair of optimal policy, p^*, and optimised informed voters welfare, $W(p^*, \gamma)$, for all realisations of the shock, there exists a line connecting all of these points. Let this line be called the envelope of maximised informed voters welfares, $N^*(\gamma)$ (and equivalently one for the SIG $N_j^*(\gamma)$). Furthermore, since there exists a constrained optimal policy for each party for each realisation of the shock, there also exists an envelope connecting the pairs of constrained optimal policies of party k, p^k, and the informed voters welfare from this policy, $W(p^k, \gamma)$ for all realisations of the shock (and equivalently for the SIG welfare). Let these lines be called the envelopes of the informed voters welfare under the equilibrium constrained optimal policies, $N^A(\gamma)$ and $N^B(\gamma)$ for the informed voters welfare, and $N_j^A(\gamma)$ and $N_j^B(\gamma)$ for the SIG welfare.

A final technical assumption that is made is that the direct effect of the shock on the informed voters welfare function is such that a boom increases it in a parallel way, and a bust decreases it in a parallel way. This means that at any two fixed policy choice set, $\check{p}$ and $\grave{p}$ where $\check{p} \neq \grave{p}$, the shock changes the informed voters welfare equivalently, that is $W(\check{p}, 0) - W(\check{p}, \gamma) = W(\grave{p}, 0) - W(\grave{p}, \gamma)$.

Proposition 40. *Suppose confidence is unaffected by shocks, $\frac{\partial \theta}{\partial \gamma} = 0$. In booms the equilibrium policy shifts in the SIGs favour (i.e. $||p_1^k - p_{j,1}^*|| > ||p_2^k - p_{j,2}^*|| \; \forall k$), while in recessions it shifts in the informed voters favour (i.e. $||p_1^k - p_1^*|| > ||p_2^k - p_2^*|| \; \forall k$). This increases the contributions to each party in booms and decrease them in recessions ($\frac{\partial C^T(\gamma)}{\partial \gamma} > 0$).*

Proposition 40 identifies the effect of a shock on the policies and contributions, when we are disregarding the effect of changes in confidence. The intuitive reason behind this result is that when a shock takes place the optimal and constrained optimal policies will change due to the shock itself changing the welfares, however the equilibrium policies (the constrained optimal ones, p_t^k) will change for an extra reason. More precisely, due to the assumption that in booms the SIG gets more benefits than the informed voters, while it experiences more losses in recessions,

[114] For example, parallel shift of welfare due to unexpected shocks, which would keep the new unconstrained optima at the same location as without the shock.

$\frac{\partial W_j(p,\gamma)}{\partial \gamma} > \frac{\partial W(p,\gamma)}{\partial \gamma} > 0$, the unexpected shock will change the weighting in the SIGs optimisation problem (equation 31). In booms the equilibrium policies will shift to the benefit of the SIG (closer to p_j^*), so the informed voters welfare from the constrained optimal policies will increase by less than their welfare from the optimal policies (p^*). From the binding participation constraints (equations 23) we know that this will increase total contributions. Furthermore, from the combined constraint (equation 30) we can see that as long as the welfares of the informed voters increase in booms, so will the contributions. On the other hand, in recessions the SIGs preferred policies will both shift in favour of the informed voters (closer to p^*), therefore the informed voters welfare from the constrained optimal policies will fall by less than from the optimal policies, therefore total contributions will decrease. This effect is caused by the fact that there will not only be the direct effect of the shock but additionally the indirect effect of the altered equilibrium policies given that $\gamma \neq 0$. The following proposition describes the effect of shocks that operate through confidence, on top of the effect identified by Proposition 40.

Proposition 41. *The indirect effect of a shock exacerbates the results from Proposition 40. A boom shifts the equilibrium policies further in the SIGs favour (by increasing trust), while a recession shifts it more in the informed voters favour. Therefore, booms have higher contributions and recessions lower. Additionally, a boom increases the incumbency advantage, while a recession decreases it.*

Proposition 41 tells us that sentiments have a non trivial consequence on the effect of an exogenous shock on the equilibrium policies and contributions. In fact, it describes the extra effect confidence has through trust, due to an unexpected shock. For example, a boom has three effects: i) it directly influences the welfares of the informed voters and SIG, increasing the SIGs welfare by a larger amount, ii) it increases the popularity of the more popular party, and iii) it increases trust. Effects i) and iii) directly decrease the weight that is placed on the informed voters welfare in the equilibrium policy calculations, whereby the equilibrium policies are shifted in favour of the SIG. Effect ii) increases the popularity of the more popular party, giving them a stronger incumbency advantage. The effect of a recession is reversed.

We know that the SIG only has a finite amount of reserves which it can use for contributing, and that its members will strike once the contributions necessary to implement its preferred policy exceed the reserves available, that is $C^A(\gamma) + C^B(\gamma) = C^T(\gamma) > Z(\gamma)$. Furthermore, we assumed that with an increasing shock the reserves increase quicker than total contributions, that is $\frac{\partial Z(\gamma)}{\partial \gamma} > \frac{\partial C^T(\gamma)}{\partial \gamma}$.

Theorem 42. *There exist recessions severe enough ($\gamma \in \Gamma$) such that the members of the SIG will strike. With sentiments (i.e. pessimism), a more severe recession is required for SIG members to strike. That is pessimism decreases the set of shock parameters for which strikes occur ($\Gamma_C \subset \Gamma_N$).*

The main result of the paper tells us that strikes occur in recessions and that strong enough recessions will make the SIG members strike. Furthermore, the recession induced pessimism will make a stronger recession necessary for strikes to occur, that is pessimism makes strikes less likely. This is a consequence of the fact that pessimism shifts the equilibrium policy in favour of the informed voter, which decreases contributions, thereby making it harder for the SIG to deplete its reserves.[115] More precisely, a recession causes pessimism ($\theta \uparrow$), which decreases trust in the government ($\tau \downarrow$), which shifts the policy in favour of informed voters (Proposition 36)

[115]Note that this result holds for interior equilibria, that is, as long as the contributions are positive. When the

consequently decreasing contributions (Proposition 37 and the participation constraint) making it harder to deplete reserves.[116] This result is puzzling as one would expect the fall in confidence to lead to more strikes, rather than less strikes. Therefore, in the appendix a case is treated where strikes take place only in recessions and pessimism leads to a quicker approach to the point where strikes occur.

From Proposition 41 we know that optimism will increase the total contributions in booms. If optimism is strong enough such that the rate of increase of total contributions is higher than that of reserves in booms ($\frac{\partial Z(\gamma)}{\partial \gamma} < \frac{\partial C^T(\gamma)}{\partial \gamma}$), strikes could also occur in booms. Since we would need further assumptions for this, we will not formally treat this possibility. Before proceeding to the empirical section, note that the appendix contains an analysis of the effect of the parameter δ, as the effects are equivalent to the benchmark GH model. Furthermore, a basic analysis of the electoral motive is also treated in the appendix.

6 Empirical Analysis

We will be testing Theorem 8: Does the amount of strikes change when there is a recession, and does this further change with optimism/pessimism. The empirical aspect is not the main part of this paper, therefore this analysis will only be suggestive rather than comprehensive. We will first look at data construction, and then the empirical methodology and results.

6.1 Data construction

We will look at eight countries from 1980 onwards, where the series finishes in 2008 for some and 2010 for others, therefore the panel is unbalanced. A single source with all the data does not exist, and for simplicity we limit our attention to only a few independent variables. The dependent variable is "days lost due to strikes and lockouts as a fraction of total hours worked in the economy", called *strike fraction*. This should make the data between different countries comparable, as it gets rid of the potential level bias.

First I obtained the total days lost due to strikes and lockouts [117] from ILOSTAT.[118] A final point to make is that for some years there was data available for both ISIC2 and ISIC3 of "International Standard Industrial Classification of All Economic Activities". Since ISIC3 is the more recent revision, I chose that value.[119] The countries for the days of strikes was chosen with the other independent variables in mind, such that there can be the most overlap possible over time, in order to maximize the number of observations in the sample.

SIG is unwilling to contribute, continuity is lost and one needs to consider several cases. For example, one might be in a situation where the contributions fall to zero, however the SIG still has positive reserves. Then the recession can still increase in strength without effecting the contributions, but only decreasing the reserves. Only when the reserves turn negative would a strike occur. This could easily be incorporated into the above result, but a bit more bookkeeping would be necessary.

[116]Note that this effect is small, due to the assumption that confidence weakly feeds into trust: $-1 < \frac{\partial \tau(\theta)}{\partial \theta} < 0$.

[117]One can find the definitions from ILOSTAT: http://www.ilo.org/ilostat/faces/help_home/conceptsdefinitions?_adf.ctrl-state=77nl95hgj_38&clean=true&_afrLoop=640619222811385

[118]http://www.ilo.org/ilostat/faces/home/statisticaldata/bulk-download?_adf.ctrl-state=15lfadi220_446&clean=true&_afrLoop=7333

[119]E.g. look at http://unstats.un.org/unsd/cr/registry/regcst.asp?Cl=2

All of the other data was obtained from the OECD statistical database.[120] First, it was necessary to obtain a measure of total hours worked in the economy. We construct this by multiplying the average yearly hours worked per worker with the total labour force, finally dividing it by 24 to make it in days as with strikes.[121] This allows us to construct the fraction that is the dependent variable.

As the focus of this analysis is whether confidence and the recession influence the amount of strikes, we will use only GDP (PPP, US$ current) or its growth rate as the only independent variable that will essentially serve as a control. We could have added bargaining power and looked further into the analysis of strikes, however as mentioned, this is only a first analysis of the problem at hand. In order to proxy for optimism/pessimism we will use the consumer confidence index that is OECD standardised and amplitude adjusted, so as to have a comparable measure amongst the countries. This is measured monthly, therefore I have averaged it for each year. This loses valuable variance, however there was no data for strikes that is more frequent than yearly.

Finally, I have created a dummy variables for the recession. I use the textbook definition of a recession, i.e. three consecutive quarters of negative growth, and constructed the growth level from quarterly GDP (PPP), as it had a longer range into the past. *Recession1* is defined as: equal 1 in both year t and $t+1$ if the three negative quarters overlap those years, or if there is no overlap then only that year equals 1, and 0 otherwise.

As many countries do not have the data, at least not readily available, we look at: Australia, Canada, Denmark, France, Italy, the Netherlands, the UK and the US. As there are only 238 realisations of the dependent variable and more realisations of the independent variables, this will be an unbalanced panel with 31 realisations of recession1. Unfortunately there are 10 recession observations not incorporated due to the most recent crisis 2008 financial crisis and the European debt crisis, as data on strikes was not sufficiently recent.

In order to make this empirical exercise more than suggestive we would need to include a variable that captures political events, to rule out strikes and lockouts for political reasons, and obtain a clearer result for recessions and confidence. A potential proxy for this could be "changes in the leading party". This is not done in this exercise, however it should be done in the future.

6.2 Data analysis

A good way to do a preliminary overview of the data is by looking at the data. Below are the graphs for gdp growth and average confidence, and for the movement of the strike fraction over time for all of the countries:

[120] http://stats.oecd.org/

[121] This division does not effect the underlying variance

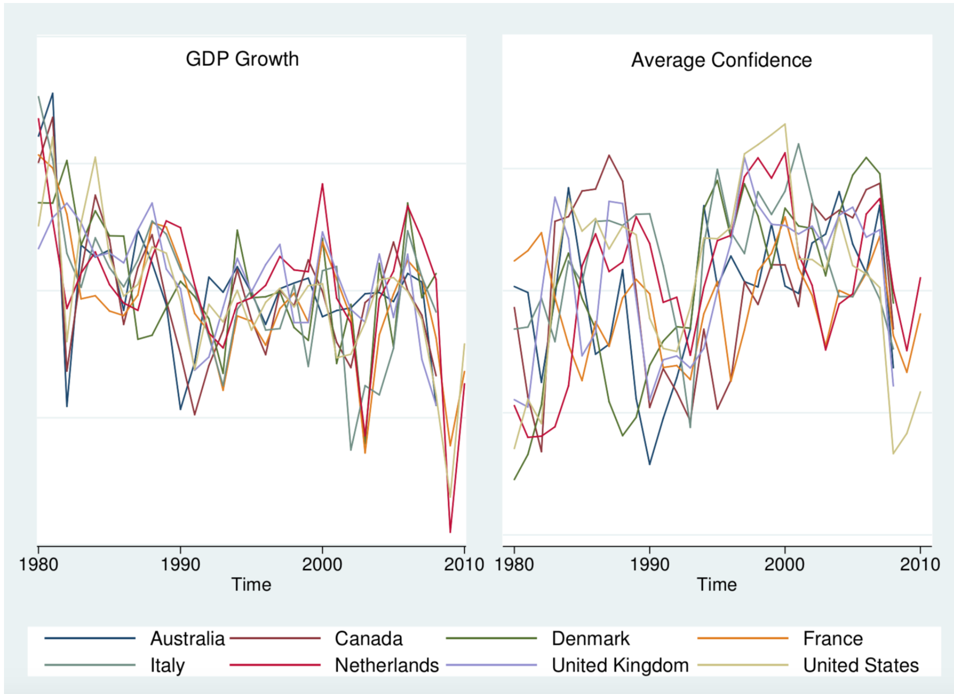
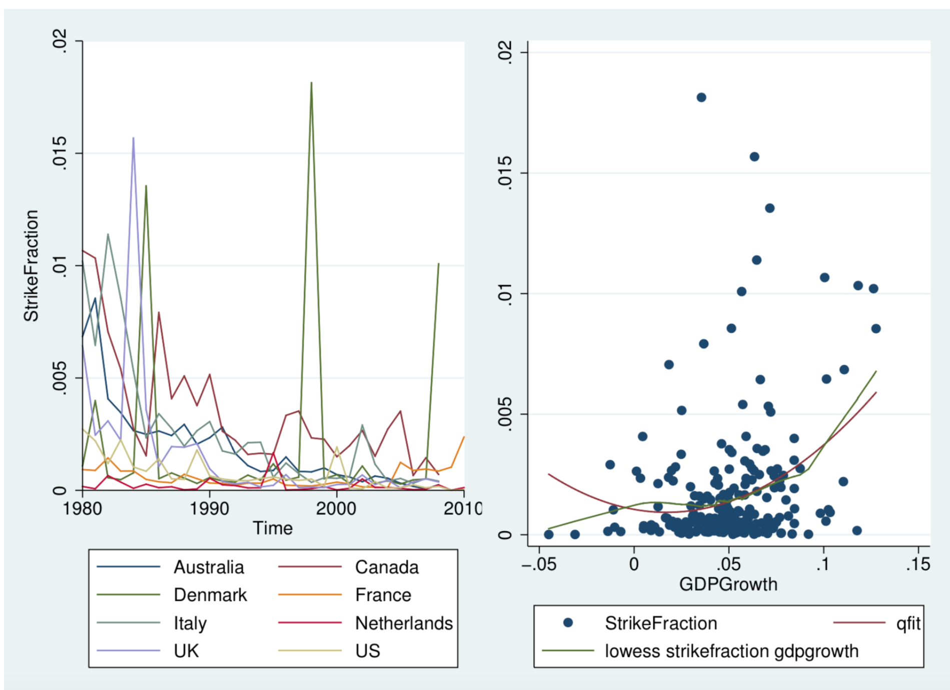
Figure 3: 

Figure 4: 

From the first graphs we can see that the gdp growth moves quite similarly for all of the countries, so there should be relatively little between variation. On the other hand, the average confidence also moves relatively similarly for the countries, however, there is more variance and the series are less closely related, which should result in some between variation. Note that there is a strong correlation between gdp growth and average confidence, which was found in the regressions too, but will not be presented. On the other hand, from the second graphs we can see that the fraction of strikes actually differ quite a bit between the countries. This *Strike-Fraction* is the dependent variable. Furthermore, as a first look I fit quadratically and by lowess the relationship between the strike fraction and gdp growth. Somewhat counter intuitively the higher the growth the larger the strike fraction. One could expect that the worse the situation the more strikes, and not the opposite. However, one could also expect that higher GDP results in more freedom, which then increases the amount of strikes. This relationship is found in all the regressions done. This would suggest that the theoretical model should allow for the possibility of strikes in booms. A possible way of theoretically including strikes in booms was suggested in the text. The quadratic relationship would suggest that any non-normal activity (at the tails of the growth level) could result in increased striking, however this is misleading as the lowess one shows this relationship most likely holds only at the top end of growth. One important thing to notice is that the fraction of strikes has been trending down over the past 30 years in these countries. This will complicate the analysis, however we will somewhat ignore this issue, at most first differencing the data. Some basic summary statistics are (mean, sd, min, max): *strikefraction* (.0016359, .0026047, 9.24e-06, .0181384), *gdpgrowth* (.0500023, .025652, -.0450082, .1276128), *averageconfidence* (100.098, 1.263795, 96.90773, 102.7285) and *changeconfidence* (.0000952, .0100279, -.0279288, .0372899). Finally, it is interesting to note that there is approximately a 15% chance of going from no recession to recession and 40% from recession to recession in our sample. This is only a preliminary view of the data, therefore it is very brief.

Now we look at regressions, however, in order to do this, first, it was important to know whether there was serial correlation in the variables or error terms of our regression, or unit roots. Strikefraction is autocorrelated with its first lag at 0.43, so quite strongly. When doing a basic pooled OLS (strikefraction on gdpgrowth time recession1 averageconfidence) with clustered standard errors by country, we find that there is some autocorrelation in the error (1st lag: 0.321, 2nd: 0.235, 3rd: 0.186, 4th: 0.219, 5th: 0.177, 6th:0.190) that is decreasing, however remains around 0.2 for 6 periods. Only after 10 lags does it fall below 0.1, however, at 13 lags increases again. This is most likely due to the sudden jumps in striking activity seen on the graphs, which is part of the unexplained. Unfortunately this does not change significantly when looking at first differenced data, where the first lag is more significant, then it decreases, but sporadically increases greatly. Looking at how a single lag is correlated across the years we find that in the original specification it is highly correlated in many years. Same story with the differenced dependent variable. This however did not exploit the panel structure.

Serial correlation would produce less efficient standard errors (smaller) and a higher R squared, so testing would be problematic. Performing the Wooldridge test for serial correlation finds no serial correlation in the original model specified above at $F(1,7) = 1.78$ ($p = 0.2239$), i.e. the probability of observing a value as high as observed under the assumption that the null (of no serial correlation) is true is high, so we fail to reject the null. It is unlikely that there is serial correlation.

Before discussing this further we will look at the whether there is a unit root in the dependent variable. The Levin-Lin-Chiu, Harris-Tzavalis, and Breitung test require the correlation coefficient to be the same across correlation structure amongst the countries, as well as a strongly balanced panel. Our panel has only 6 observations that make it unbalanced, therefore we do not loose much data from balancing it (only 2009 and 2010 for France, the Netherlands and the US).

All three tests reject the null of a unit root in the dependent variable, or the independent variables, except the Breitung test for strikes at $p = 0.1354$. The Im-Pesaran-Shin and Fisher-type tests allow for unbalanced panels, as well as the correlation structure to vary across individuals. Both of these reject the null of a unit root in all of the involved variables. Therefore it seems unlikely that there is a unit root.

Vars	OLS_pool	OLS_pa_rob	BE	FE	FE_rob	RE	RE_rob
gdpgro	0.0119	0.0074	-0.2051*	0.0141*	0.0141*	0.0124	0.0124*
	0.0069	0.0050	0.0755	0.0073	0.0065	0.0077	0.0067
time	-0.0001**	-0.0001***	-0.0024**	-0.0001***	-0.0001**	-0.0001***	-0.0001***
	0.0000	0.0000	0.0004	0.0000	0.0000	0.0000	0.0000
bust	0.0011*	0.0009*	0.0015	0.0010**	0.0010*	0.0011**	0.0011**
	0.0005	0.0005	0.0049	0.0005	0.0004	0.0005	0.0005
avconf	0.0001	0.0001	0.0004	0.0001	0.0001	0.0001	0.0001
	0.0001	0.0001	0.0012	0.0001	0.0001	0.0001	0.0001
_cons	0.1919**	0.2033**	4.6919**	0.1753***	0.1753**	0.1883***	0.1883***
	0.0689	0.0789	0.8441	0.0394	0.0677	0.0418	0.0687
N	238	238	238	238	238	238	238
r2	0.1616		0.9268	0.1736	0.1736		
r2_o			0.1182	0.1605	0.1605	0.1615	0.1615
r2_b			0.9268	0.3046	0.3046	0.3175	0.3175
r2_w			0.1243	0.1736	0.1736	0.1729	0.1729
sigmau				0.0011	0.0011	0.0002	0.0002
sigmae				0.0022	0.0022	0.0022	0.0022
rho				0.1865	0.1865	0.0093	0.0093

legend: b/se; * p<.1; ** p<.05; *** p<.01

Table 2:

The pooled OLS makes the strongest assumptions amongst these, that the independent variables are exogenous, and that we cannot decompose the error term into the idiosyncratic part and the fixed part. In a fixed effects model (FE) we assume that the dependent variable may be correlated with the error term (as we take out *sigmau* - the fixed part of the error), since the regressor may be correlated with some unobservable the unobservable time-invariant individual effect. This is likely, as we are analyzing macro data which is describing connected societies. Unfortunately, the time frame is not very long, which makes prediction more difficult with fixed effects, since the fixed effect (sigmau) is more difficult to estimate. With random effects (RE) we are assuming that there is no fixed effect,[122] and we need to use feasible generalized least squares due to otherwise wrong standard errors (and therefore tests). It is important not to have any correlation between the independent variables and the error term. The advantage is, however, that even time invariant coefficients can be estimated (unlike in FE), however they will be inconsistent if there is serial correlation or endogeneity. Finally, note that RE restricts serial correlation to be the same at all lags. As I have demonstrated above, this is unlikely. In our dataset all of the countries involved are highly developed therefore they may share some similar characteristics, which would point to the usefulness of FE, however, at the same time it is highly likely that there

[122]That is, the group means are random variables and we do not have to de-mean the model (its all pure randomness).

exist some endogeneity between the error term and the independent variable, which points to the usefulness of RE. It is unclear whether one is strictly preferred.

From the table above we can see that the time dummy is always significant, which is expected as we saw in the data that there is a trend. Furthermore, gdp growth does not matter under simple OLS and population average OLS, however it does start having a significant relationship in the more panel specialized regressions, i.e. the between estimator, both fixed effects and only the random effects with robust standard errors. The relationship is as expected from the initial graph, the larger the gdp growth, the more strikes there are. This result is somewhat at odds with Theorem 42, where strikes occur only in recessions, however the model could be expanded to include this possibility too. Potentially, it could work in the following way: in booms the contributions are increasing, if we assumed that a ceiling existed, or that the necessary contributions grew faster than the reserves the SIG has, we would end up with a result stating that strikes occur in booms. Note that in the population average OLS we assumed an AR1 error correlation structure.

The particularly important result for the theory of the paper is that a recession influences the amount of strikes. This is seems to be empirically confirmed. The unfortunate result is that we cannot clearly disentangle whether this effect comes from the change in perception or the pure structural change that the recession brings with it. Average confidence was supposed to capture this, however, as we can see it is insignificant in all regressions. This is misleading though, because this is done in levels, and not changes. Below we have the same table, but it presents only the results for recession (*bust*) and change of confidence (*chgcnf*).

Vars	OLS_pool	OLS_pa_rob	BE	FE	FE_rob	RE	RE_rob
bust	0.0008	0.0007	0.0013	0.0007	0.0007	0.0008	0.0008*
chgcnf	-0.0334	-0.0260	0.4597	-0.0329**	-0.0329	-0.0334**	-0.0334

legend: * p<.1; ** p<.05; *** p<.01

Table 3:

As we can see, the change in confidence is significant in two out of the six regressions, and almost every time in the same direction, i.e. the more optimism there is the less strikes there will be. This is inconsistent with Theorem 42, which predicts that pessimism will lead the SIG to consumer its reserves more slowly, whereby a stronger recession is necessary for strikes to occur. This suggests that pessimism should decrease the incidence of recession, while the data predicts that pessimism increases the incidence of strikes. This could be as pessimism may lead the SIG to consume all of its resources quicker (rather than slower, as the theory suggested), hence strikes and protests would occur quicker and are more likely. The story is opposite with optimism. The only regression with the opposite direction is the between estimator, which implies that the more optimism the more strikes there will be, which is counterintuitive, but in line with the theory, however it is insignificant.

From Table 2 one may conclude that recessions have a significantly positive effect on the amount of strikes, however Table 3 changes this slightly. When we include the change in confidence in Table 3, rather than average confidence (as in Table 2), most of the coefficients on the dummy for the recession become insignificant. Therefore, when not including the change in confidence, but average confidence, the recession dummy is incorporating some of this drop in confidence, and when we control for this by including the change in confidence, the recession dummy becomes much less significant (the standard error increases). An important point to make is that even with including the change in confidence, instead of the average confidence, the direction of the effect of recessions remains the same (recessions are correlated with increased striking activity). This is suggestive that the effect really works in this direction, which is in line with 42, recessions increase the incidence of strikes.

From this basic empirical analysis we were able to obtain a first look at the data, which weakly confirmed the expected effect of a recession, as well as the possible effect that optimism/pessimism have. It would be interesting to include trust, and how it would effect strikes and the other relations involved, however, that is left for a future exercise.

7 Conclusion

The main purpose of this paper was to analyse how the state of the economy affects the special interest groups behavior, given that the economy is composed of emotional voters who can protest, and special interest group members can strike. The paper first described the benchmark model, where each of the emotional aspects was individually introduced and its effect, in relation to the benchmark model, was analyzed. The unexpected shock was introduced into this modified framework, where the entire structure was analyzed in unison. More precisely, the main purpose of this paper was to analyse what happens to a GH type framework when we include trust and confidence, and then to see what happens to those equilibrium values when an unexpected shock takes place. We were particularly interested in finding out whether a recession will increase the amount of strikes and how pessimism influences this effect. In order to do this we augmented the GH model by adding sentiments, allowing voters to protest, adding an unexpected shock, and giving SIG members the possibility to strike (if the SIG depletes their reserves for contributions).

We saw that the more distrustful the uninformed voters are in the government, the more the policy the SIG chooses will be in favour of the informed voters. This will increase the informed voters welfare. This seems reasonable, because being distrustful of politicians implies uninformed voters will trust informed voters more. Furthermore, we found that confidence will have a direct and indirect effect. The direct effect will be that pessimism will decrease the more popular parties incumbency advantage, potentially reversing it, while optimism makes the incumbent more popular and the challenger less popular. The indirect effect of confidence operates through trust, such that pessimism will slightly decrease trust and hence make the policies chosen more favourable to the informed voters, thereby decreasing the total contributions the SIG gives. The opposite result occurs with optimism. That is, an increase in optimism increases trust in the government, whereby the policy shifts away from the informed voters optimal policy, increasing the total contributions the SIG gives. These results seem reasonable. Finally, we also showed that if the reserves of the SIG are decreasing faster than total contributions, there will exist a recession that is strong enough so the SIG members will strike. With pessimism the recession will have to be stronger in order to induce strikes. Even though the logic[123] is reasonable, it seems counterintuitive that pessimism leads to less strikes. The model may be analysing second order effects, whilst ignoring some more important first order effect. This is puzzling, therefore in the

[123] Pessimism decreasing trust in the government, leading to a higher focus on the informed voters welfare, which decreases total contributions whereby a stronger recession is necessary for SIGs to deplete their reserves and strike

appendix a case is treated which makes pessimism lead to strikes faster, rather than slower. When testing these results empirically, we find that recessions most likely do lead to more strikes, and confidence does influence it; that is, higher pessimism further increases the amount of strikes. Unfortunately, the empirical exercise was only a first look at the problem, and the results are very fragile due to the limited amount of data and the potential complications with it.

The theoretical model offers itself to many extensions, such as, it would be interesting to see whether these results hold under the electoral motive. It would also be interesting to see what the effect would be of making the exogenous shock a random variable that could be somewhat anticipated. Furthermore, it would be interesting to see how the equilibrium changes if we allowed the policy choices to influence the shock. A natural extension is to look at the effect of multiple SIGs competing for influence. A final example to which this model lends itself to analysing is the question of lying. Politicians will endorse some policies in normal times and the policies they will endorse will change with the shock. While all these questions, and many more, would be interesting to analyse within this framework, they are beyond the scope of this paper.

The balance between self motivated interest groups, politicians and voters is an interesting one, and it allows for quite a complex environment, where small changes in the assumptions can have large effects. This paper tried to expose some of the many interconnections between these parties, and their effects in a simple electoral game.

H Proofs

Lemma 32. *A larger weight ($\tilde{\delta} > \delta$) on the preferences of the average informed voter ($W(\cdot)$) in the SIG's optimization problem (31) pushes both of the SIG's preferred policies (p_t^k) closer to the optimal policy of the informed voter (p_t^*), while a smaller weight pushes it closer to the SIG's preferred policy choice ($p_{j,t}^*$).*

Proof. Let $\tilde{\delta}_t > \delta_t$, then $p_t^k = arg\max_{p \in \mathcal{P}} \left[\phi_t^k W_j(p_t^k, \kappa) + \tilde{\delta}_t W(p_t^k, \kappa) \right] \to p_t^*$ as $\tilde{\delta}_t \to \infty$ because in the limit only the welfare of the average informed voter is considered. We know from 27 and from the assumptions of well behaved functions with a unique maximum, that no other policy is optimal. Therefore, as long as $W_j(p_t, \kappa) \neq W(p_t, \kappa)$[124] we know that any weight on the SIG will make the policy less optimal for the informed voter. This is so because the welfare of informed voters is decreasing the further away the policy implemented is from their optimal policy, precisely stated $W(p_t^k, \kappa)$ is monotonically decreasing as $\|p_t^* - p_t^k\|$ is increasing. The opposite is true when the weight on the informed voters welfare is decreasing. In the limit, as $\tilde{\delta}_t \to 0$, the SIG will only consider its own welfare when deciding on the policy it will endorse for each of the two parties, corrected by the probability of each of the parties' policies being implemented, ϕ_t^k. □

Proposition 33. *If voters can protest, the SIG can select a policy (p_t^k) that is closer to its optimal policy ($p_{j,t}^*$), than under the benchmark. Therefore, the level of contributions is higher.*

Proof. Follows directly from Lemma 2, because the no protest constraint decreases the weighting on the preferences of the average informed voter by the information bias. □

[124]If the welfare of informed voters and the interest groups was equivalent for each policy choice, $W_j(p_t, \kappa) = W(p_t, \kappa)$, then there would be no point to form SIGs in the first place.

Proposition 34. *Wlog suppose $b\theta_t(\gamma) > 0$ (party A is ex-ante more popular). If voters can protest, the bias caused by popularity (being the more popular party) is stronger than in the benchmark.*

Proof. Without loss of generality (wlog) suppose party A is more popular, $\phi^A > \phi^B$. This will make the weight on the SIGs welfare larger for party A , $\phi_t^k W_j(p_t^k, \gamma)$, than for party B, $\phi_t^k W_j(p_t^k, \gamma)$, as in the GH model. The information bias is the innovation in this model, an additional term that makes the party A put less weight on the informed voters welfare than party B. The proof for the effect of the innovation follows directly from Lemma 2, while the proof of the first part is equivalent to the benchmark model so will not be presented. □

Proposition 35. *Wlog suppose $b\theta_t(\gamma) > 0$, then $\phi_t^A > \frac{1}{2} > \phi_t^B$. If voters can protest, the SIG bias keeps the same qualitative features (i.e. $C_t^A > C_t^B$, $W(p_t^B, \gamma) > W(p_t^A, \gamma)$, $W_j(p_t^A, \gamma) > W_j(p_t^B, \gamma)$), however it is stronger towards the more popular party than in the benchmark, i.e. $C_t^A > C^A > C^B$, $W(p^B) > W(p^A) > W(p_t^A, \gamma)$, $W_j(p_t^A, \gamma) > W_j(p^A) > W_j(p^B)$.*

Proof. Again let $\lambda_t^k = \delta\left(1 - \frac{\phi_t^k}{\tau(\theta)}\right)$.

1) From the definition of the SIGs optimization problem (31), we know that it is optimal for party k to set policy p_t^k instead of p_t^l where $k \neq l$, $\phi_t^k W_j(p_t^k, \gamma) + \lambda_t^k W(p_t^k, \gamma) > \phi_t^k W_j(p_t^l, \gamma) + \lambda_t^k W(p_t^l, \gamma)$. Subtracting party B's result from party A's, we obtain that $(\phi_t^A - \phi_t^B) W_j(p_t^A, \gamma) > (\phi_t^A - \phi_t^B) W_j(p_t^B, \gamma)$, and therefore the lobbies welfare is higher from the more popular party, $W_j(p_t^A, \gamma) > W_j(p_t^B, \gamma)$.

2) Again, using the result that for party B it is optimal to implement p_t^B, from part 1, we know that $\phi_t^B W_j(p_t^B, \gamma) + \lambda_t^B W(p_t^B, \gamma) > \phi_t^B W_j(p_t^A, \gamma) + \lambda_t^B W(p_t^A, \gamma)$, it is also true that $\phi_t^B \left[W_j(p_t^B, \gamma) - W_j(p_t^A, \gamma)\right] > \lambda_t^B \left[W(p_t^A, \gamma) - W(p_t^B, \gamma)\right]$. We know that $\phi_t^B \geq 0$ and $\lambda_t^B \geq 0$, as well as part 1 tells us that $W_j(p_t^A, \gamma) > W_j(p_t^B, \gamma)$, therefore we can see that party A's policy leads to a lower welfare for the informed voters than party B's policy, $W(p_t^B, \gamma) > W(p_t^A, \gamma)$.

3) Finally we can show that the more popular party gets higher contributions. Let the expected welfare from the SIGs policy for the informed voters be $E(p_t^A, p_t^B) = \frac{1}{\tau(\theta)}\left(\phi^A W(p_t^A, \gamma) + \phi^B W(p_t^B, \gamma) + \frac{\chi}{(1-\alpha)}\right)$. Then the combined constraint (30), which binds in the influence motive case, becomes $C_t^k = \delta\left[E(p_t^A, p_t^B) - W(p_t^k, \gamma)\right]$. Subtracting the contributions to the less popular party, C_t^B, from the more popular parties, C_t^A, we obtain $C_t^A - C_t^B = \delta[W(p^B, \gamma) - W(p^A, \gamma)]$ where $\delta \geq 0$. Since from part 2 we know that $W(p_t^B, \gamma) > W(p_t^A, \gamma)$, therefore the SIG will contribute more to party A than to party B, $C_t^A > C_t^B$. □

Proposition 36. *Higher distrust of the government (higher τ) shifts the equilibrium towards p_t^*. Higher trust (lower τ) shifts the equilibrium policy towards $p_{j,t}^*$.*

Proof. Let $\lambda_t^k = \delta\left(1 - \frac{2\phi_t^k}{\tau(\theta)}\right)$. From Lemma 2 we know that a larger weight on the informed voters welfare will make the policy chosen be in their favour. In this proposition the weight parameter is λ instead of δ, and we can use it in this proof because changes in trust do not change party popularity, ϕ^k, therefore it is not endogenous.[125] Note that λ is increasing with distrust, $\tau(\theta)$, precisely $\frac{\partial \lambda}{\partial \tau} > 0$. Therefore with increasing distrust the policy becomes more in favour of the informed voters, that is $\|p_t^* - p_t^k\|$ is decreasing as $\tau(\theta)$ is increasing. The reverse is true for when

[125] When looking at the effect of confidence (changes in θ) ϕ^k becomes endogenous, therefore we will have to alter the argument.

uninformed voters are becoming more trustful, $\tau(\theta)$ decreasing. Note that for any chosen party popularity, ϕ_t^k, the range where trust is defined, $\tau(\theta) \in [1, \frac{1}{1-\alpha}]$, will place boundaries on the information bias, $\frac{\phi_t^k}{\tau(\theta)} \in [\phi_t^k, \frac{\phi_t^k}{1-\alpha}]$. $\square$

Proposition 37. *Higher trust (lower τ) increases SIG contributions, increases SIG welfare, and decreases voter welfare. Formally, let $\tilde{\tau} > \tau$, then $W_{j,t}^k(\cdot)\big|_{\tau=\tau} < W_{j,t}^k(\cdot)\big|_{\tau=\tilde{\tau}}$, $W_t^k(\cdot)\big|_{\tau=\tau} > W_t^k(\cdot)\big|_{\tau=\tilde{\tau}}$, and $C_t^k\big|_{\tau=\tau} < C_t^k\big|_{\tau=\tilde{\tau}}$ for both $k \in K = \{A, B\}$. With decreased trust the effect is reversed.*

Proof. From Proposition 36 we know that with increased trust the policy shifts in favour of the SIG, and against the favour of the informed voters, for both parties. Let $\tilde{\tau} > \hat{\tau}$. This leads to a higher welfare for the SIG under both parties policies, $W_{j,t}^k(\cdot)\big|_{\tau=\hat{\tau}} < W_{j,t}^k(\cdot)\big|_{\tau=\tilde{\tau}}$, and a lower welfare for the informed voters under both parties policies, $W_t^k(\cdot)\big|_{\tau=\hat{\tau}} > W_t^k(\cdot)\big|_{\tau=\tilde{\tau}}$.

Since the NPC binds we can use the combined constraint or the PC itself. From the PC, $C^k \geq \delta[W(p^*, \gamma) - W(p^k, \gamma)]$, and the above result, we can see that the contributions to either party has to decrease, if the optimal policy does not shift due to changes in trust. The proof is reversed for decreases in trust. $\square$

Lemma 39. *There exists a unique optimal policy (p_2^*) and a unique constrained optimal policy (p_2^k) for each possible shock for both parties.*

Proof. Since we assumed that the exogenous shock changes the welfare of the informed voter, from the maximization of the informed voters welfare (equation 28) we know that under the same policy the welfare would not be the same with and without a shock, $W(p_1^*, 0) \neq W(p_1^*, \gamma)$. The optimal policy would change for all functional forms $W(\cdot)$, but certain special cases. Equivalently, since the shock changes the SIG's welfare we know that the welfare from the constrained optimal policy changes (decision problem 29), $W_j(p_1^k, 0) \neq W_j(p_1^k, \gamma)$. The constrained optimal policy would also change for all functional forms $W_j(\cdot)$, but certain special cases.

We assumed that both $W(\cdot)$ and $W_j(\cdot)$ are strictly concave functions, which means that there exists a unique unconstrained optimal policy that maximizes $W(\cdot)$ for every possible realization of the shock, and a unique constrained optimal policy solving the maximization problem 29 for every possible shock realization. $\square$

Proposition 40. *Suppose confidence is unaffected by shocks, $\frac{\partial \theta}{\partial \gamma} = 0$. In booms the equilibrium policy shifts in the SIGs favour (i.e. $||p_1^k - p_{j,1}^*|| > ||p_2^k - p_{j,2}^*|| \; \forall k$), while in recessions it shifts in the informed voters favour (i.e. $||p_1^k - p_1^*|| > ||p_2^k - p_2^*|| \; \forall k$). This increases the contributions to each party in booms and decrease them in recessions ($\frac{\partial C^T(\gamma)}{\partial \gamma} > 0$).*

Proof. Recall the assumptions made: $\frac{\partial W_j(p,\gamma)}{\partial \gamma} > \frac{\partial W(p,\gamma)}{\partial \gamma} > 0$, $\frac{\partial^2 W_j(p,\gamma)}{\partial \gamma^2} < 0$, $\frac{\partial^2 W(p,\gamma)}{\partial \gamma^2} < 0$, $\frac{\partial^2 W_j(p,\gamma)}{\partial p^2} < 0$, and $\frac{\partial^2 W(p,\gamma)}{\partial p^2} < 0$. The assumptions $\frac{\partial W(p,\gamma)}{\partial \gamma} > 0$ and $\frac{\partial^2 W(p,\gamma)}{\partial \gamma^2} < 0$, both $\forall \gamma$ and for every p, mean that the envelopes $N^*(\gamma)$ is increasing and strictly concave (and equivalently for $N_j^*(\gamma)$). Since we assumed the SIG gets more benefits than the informed voters in booms, while it experiences more losses in recessions, we can use Lemma 32 to directly obtain the result. In other words, when $\gamma > 0$, the welfare of the SIG increases by more than the informed voters welfare. This relative difference can be treated as if the weight on the SIGs welfare increased, where we can then use Lemma 32 that implies the equilibrium policy shifts in favour of the SIG for both parties, $||p_1^k - p_1^*|| < ||p_2^k - p_2^*|| \; \forall k$. In a recession, $\gamma < 0$, the welfare of the SIG falls by more than that of the informed voter, which implies the weight on the SIG decreased, and hence the policy will shift in the informed voters favour for both parties, $||p_1^k - p_1^*|| > ||p_2^k - p_2^*|| \; \forall k$.

When both PCs bind we know the total contributions are: $C_t^T := C_t^A + C_t^B = \delta \left[2W(p_t^*, \gamma) - W(p_t^A, \gamma) - W(\right.$ Therefore, $C_2^T < C_1^T \iff 2\left[W(p_2^*, \gamma) - W(p_1^*, \gamma)\right] > W(p_2^A, \gamma) - W(p_1^A, \gamma) + W(p_2^B, \gamma) - W(p_1^B, \gamma)$. Note that the policy is function of the recession, therefore $\frac{\partial W(p^k(\gamma), \gamma)}{\partial \gamma} = \frac{\partial W(p^k(\gamma), \gamma)}{\partial p^k} \frac{p^k(\gamma)}{\partial \gamma} + \frac{\partial W(p^k(\gamma), \gamma)}{\partial \gamma}$, where we saw that $\frac{\partial W(p^k(\gamma), \gamma)}{\partial p^k} \frac{p^k(\gamma)}{\partial \gamma} < 0$ since boom pushes the policy away from the informed voters preferred policy. Furthermore, from the envelope theorem we know that at the optimum $\frac{\partial W(p^*(\gamma), \gamma)}{\partial \gamma} = \frac{\partial W(p^*(\gamma), \gamma)}{\partial \gamma}$. Using the assumption that γ shifts voter welfare in a parallel fashion,[126] implies that the the envelope of optimal policies, $N^*(\gamma)$, is increasing quicker than either of the envelopes of the SIGs preferred policies, $N^k(\gamma)$, because the SIGs preferred policy changes will diminish the rate of change of $N^k(\gamma)$. That is, $\frac{\partial N^*(\gamma)}{\partial \gamma} > \frac{\partial N^k(\gamma)}{\partial \gamma}$. Therefore, each parties contributions increase in booms and fall in recessions, the SIGs welfare increases more than the informed voters in booms (and falls more in recessions). □

Proposition 41. *The indirect effect of a shock exacerbates the results from Proposition 40. A boom shifts the equilibrium policies further in the SIGs favour (by increasing trust), while a recession shifts it more in the informed voters favour. Therefore, booms have higher contributions and recessions lower. Additionally, a boom increases the incumbency advantage, while a recession decreases it.*

Proof. In a boom, $\gamma > 0$, the welfare of the SIG will increase by more than the welfare of the informed voter. This relative difference can be treated as if the weight on the SIGs welfare increased, which we know from Lemma 2 means that the SIGs preferred policy will shift in favour of the SIG for both parties, $||p_1^k - p_1^*|| < ||p_2^k - p_2^*|| \; \forall k$. The optimism induced by the boom, $\theta(\gamma) > 1$, has a direct effect (Proposition 38) changes the parties popularities. The effect of the change in popularity is described by Proposition 35, that is, the popularity bias identified in the original model is exacerbated. The indirect effect of a change in confidence operates through changes in trust, that is, a boom will increase trust (which decreases $\tau \to 1$). Notice that a decrease in τ decreases the weight on the informed voters welfare, shifting the policy even further in the SIGs interest. Precisely, with $\lambda_t^k = \delta \left(1 - \frac{2\phi_t^k}{\tau(\theta)}\right)$ we can see that $\frac{\partial \lambda}{\partial \tau} > 0$, so an increase in trust decreases λ.

A boom has three effects: i) it directly influences the welfares of the informed voters and SIG, increasing the SIGs welfare by a larger amount, ii) it increases the popularity of the more popular party, and iii) it increases trust. Effects i) and iii) directly decrease the weight that is placed on the informed voters welfare in the equilibrium policy calculations, whereby the equilibrium policies are shifter in favour of the SIG. Effect ii) increases the popularity of the more popular party, giving them an extra benefit. The effect of a recession is reversed. □

Theorem 42. *There exist recessions severe enough ($\gamma \in \Gamma$) such that the members of the SIG will strike. With sentiments (i.e. pessimism), a more severe recession is required for SIG members to strike. That is pessimism decreases the set of shock parameters for which strikes occur ($\Gamma_C \subset \Gamma_N$).*

[126] The direct effect of the shock on the informed voters welfare function is such that a boom increases it in a parallel way, and a bust decreases it in a parallel way. Precisely, at any two fixed policy choices, p_1, p_2, the shock changes the informed voters welfare equivalently, that is $W(p_1, 0) - W(p_1, \gamma) = W(p_2, 0) - W(p_2, \gamma)$.

Proof. From Proposition 40 we know that $\frac{\partial C^T(\gamma)}{\partial \gamma} > 0$, therefore the amount of reserves also unambiguously increases in booms and decreases in recessions, $\frac{\partial Z(\gamma)}{\partial \gamma} > \frac{\partial C^T(\gamma)}{\partial \gamma} > 0$.[127] Since the recession leads to decreasing total contributions, and faster decreasing reserves, there exists a point where total contributions equal total reserves, let it be $\tilde{\gamma}$. When the recession is strong enough (i.e. $\gamma < \tilde{\gamma}$) and there are no confidence considerations, $\gamma \in \Gamma_N$, and the members of the SIG will strike.

From Proposition 41 we know that pessimism changes the popularity of the two parties, however it also decreases trust (higher τ), whereby it increases the weight on the informed voters welfare. That is, a recession shifts the equilibrium policies in favour of the informed voters and decreases contributions. Given our assumption, in recessions the required contributions to ensure equilibrium policies (different from p^*)[128] are decreasing at a faster rate than the reserves, however sentiment concerns are shifting policies towards the informed voters optimum, which implies that sentiments increase the rate at which contributions are shrinking (i.e. the total contributions decrease by more in recessions with sentiments).

Therefore, the recession necessary to reach the point when reserves equal total contributions has to be stronger than without confidence considerations. Let $\tilde{\gamma}_N < 0$ be such that $Z(\tilde{\gamma}_N) = C_N^T(\tilde{\gamma}_N)$ (note that if $\gamma > \tilde{\gamma}_N$ then $Z(\gamma) > C_N^T(\gamma)$) when there are no confidence considerations, where $C_N^T(\cdot)$ is the contribution function without confidence considerations, and $C_C^T(\cdot)$ with confidence considerations. With pessimism we know that $C_N^T(\tilde{\gamma}_N) > C_C^T(\tilde{\gamma}_N)$, therefore $Z(\tilde{\gamma}_N) > C_C^T(\tilde{\gamma}_N)$. This implies that there exists a stronger recession, $\tilde{\gamma}_C < \tilde{\gamma}_N < 0$, such that it leads to reserves equalling total contributions, that is $Z(\tilde{\gamma}_C) = C_C^T(\tilde{\gamma}_C)$. Finally, this implies that the set of recessions for which the SIG members will strike shrinks due to pessimism, that is $\{\gamma < \tilde{\gamma_C} < 0 : Z(\gamma) < C^T(\gamma)\} = \Gamma_C \subset \Gamma_N = \{\gamma < \tilde{\gamma}_N < 0 : Z(\gamma) < C^T(\gamma)\}$. □

I Alternative reason for strikes and protests occurring during recessions

As the welfare of the SIG decreases 'quickly' with the recession, we will assume that there exists a minimum amount of acceptable welfare, $\tilde{W}_j(\tilde{p}_2, \tilde{\gamma})$, such that if the welfare falls below this point it will cause the SIG to fall apart causing a discontinuous negative jump in its welfare. Lets call this the "failure welfare". This is supposed to capture the idea that since many of these SIGs are likely to be companies, at some point there exists a recession strong enough, $\tilde{\gamma} < 0$, such that after this point the companies will go bankrupt. The policy the SIG will choose at this point is $\tilde{p}_2^k = arg\max_{p \in \mathcal{P}} \left[\phi_2^k W_j(p_2^k, \tilde{\gamma}) + \delta W(p_2^k, \tilde{\gamma})\left(1 - \frac{\phi_2^k}{\tau(\theta)}\right)\right]$, and the necessary contributions are $\tilde{C}_2^A + \tilde{C}_2^B = \tilde{C}_2$.[129] Since the SIG will want to avoid such an outcome at all costs, we will suppose that when their welfare reaches this failure level they will use extra resources to make it seem as if they are doing better than they are in reality. They will try to artificially keep the policy more in their favour than it really should be. Let $R(\gamma) = G\left(W_j(p_2^A, \gamma) + W_j(p_2^B, \gamma)\right)$ be these

[127]Note that the assumption that $\frac{\partial Z(\gamma)}{\partial \gamma} > \frac{\partial C^T(\gamma)}{\partial \gamma} > 0$ is problematic, since $C^T(\gamma)$ is determined in equilibrium. Nevertheless, it simplifies the analysis, and can be relaxed in a future version.

[128]We are looking at interior solutions. If the policies were to reach p^*, then the SIG would not contribute anything.

[129]More precisely, when $\gamma < \tilde{\gamma}$ then $W_j < \tilde{W}_j$ such that $W_j + y = \tilde{W}_j$, where y is a positive nontrivial number, hence the fall is discontinuous.

extra resources that the SIG will spend in the case when the recession becomes strong enough to reach the failure level, where $R(\gamma)$ is increasing with the strength of a recession, $\frac{\partial R(\gamma)}{\partial \gamma} < 0$, and where $G(\cdot)$ is a function of the recession which ensures that the SIG does not spend more than it needs to. We know that in a recession the contribution level is decreasing, and if the SIG wants to leave the impression that it will not fail, it needs to keep policies at $\tilde{p}_2^k$, therefore the extra resources spent will have to compensate for the difference in the decrease in the welfare of the SIG relative to the decrease in the welfare of informed voters. At strong enough recessions, $\gamma < \tilde{\gamma}$, the welfare of the SIG from contributing will therefore changes to:

$$\max_{(p^A,p^B)\in\mathcal{P}^2} V_{j,2} = \phi^A W_j(p_2^A,\gamma) + \phi^B W_j(p_2^B,\gamma) - (C_2^A + C_2^B) + \mathbb{1}_{\{W_j \leq \tilde{W}_j\}} G\left(W_j(p_2^A,\gamma) + W_j(p_2^B,\gamma)\right) \tag{32}$$

$s.t.$PC (23) and NPC (24)

$$\text{leading to } \tilde{p}_2^k = arg\max_{p\in\mathcal{P}}\left[\phi_2^k W_j(p_2^k,\gamma) + \delta W(p_2^k,\gamma)\left(1 - \frac{\phi_2^k}{\tau(\theta)}\right) + G\left(W_j(p_2^A,\gamma) + W_j(p_2^B,\gamma)\right)\right] \tag{33}$$

We can see that $G(\cdot)$ is the function that determines the extra resources the SIG needs to spend in order to keep the policy at $\tilde{p}_2^k$ when $\gamma < \tilde{\gamma}$.[130] This leads to the last part of the effect of the shock on contributions. At recessions that are strong enough, $\gamma < \tilde{\gamma}$, the contributions are not composed anymore of only the direct resources spent on political parties, but also of the indirect cost the SIG needs to spend on making the policy be more in its favour than it should realistically be, that is $C_2^A + C_2^B + R(\gamma)$. Since the SIGs preferred policy does not change, from the total contributions we can see that with a strengthening recession the actual contributions will depend on the change of the informed voters welfare under the optimal policy and under the SIGs artificial policy, that is on $\frac{\partial N^*(\gamma)}{\partial \gamma}$ and $\frac{\partial N^k(\gamma)}{\partial \gamma}$. Since the policy does not change lets assume that the envelopes have the same slope and rate of change. This suggests that the total contributions do not change with a strengthening recession, however we know that as the recession strengthens the extra resources spent will increase, $\frac{\partial R(\gamma)}{\partial \gamma} < 0$. This means that at very strong levels of recession, $\gamma < \tilde{\gamma}$, the total resources spent by the SIG will start increasing again.

Finally, it is reasonable to assume that the SIG will not have infinite resources for spending for the influence motive nor for keeping the policy artificially fixed, and that these resources will diminish with a recession and increase with a boom. Let these total available resources be called $Z(\gamma)$, such that $\frac{\partial Z(\gamma)}{\partial \gamma} > 0$, and assume that the SIGs have enough funds in booms, and weak recessions, $\frac{\partial Z(\gamma)}{\partial \gamma} > \frac{\partial\left(C_2^A + C_2^B\right)}{\partial \gamma}$ $\forall \gamma > \tilde{\gamma}$. The reason behind this last assumption is that most likely only the SIGs that do have these funds available will be able to form SIGs influential enough to pressure the government into adopting policy platforms that are not in the full interest of informed voters. However, there do exists recessions strong enough that might lead to the SIG not having sufficient funds to avoid the failure welfare, call this recession $\hat{\gamma}$. This will most likely happen when the recession is very strong. At this point the members of the SIG will use every resource available to them to avoid bankruptcy, particularly if they do not have sufficient funds

[130]Note that there is a plus because the SIG is pretending it has a higher welfare than it really does. Furthermore the politician and voter from this setup know it, only the uninformed voter does not know the strength of the recession but neither does he know the policy.

their only alternative is to strike. Therefore, under some reasonable assumptions a recession will lead to strikes. When we include pessimism into this calculation, from Lemma 2, Proposition 12, and Proposition 13, from the SIGs preferred policy in weak recessions (equation 33) the weight on the informed voters welfare increases, which implies the SIG's total contributions decrease, however as there recession becomes very strong (to reach the failure rate) from equation 33 we see that the SIG starts placing less weight on the informed voters welfare and more on its own welfare, which shifts the equilibrium policy in the SIG's favour. This implies that total contributions increase with strong enough recessions, making the SIG reach the total available resources $Z(\gamma)$ sooner than without confidence. That is, when the recession is very strong (threatening bankruptcy for the SIGs) pessimism stops being the driving factor for strikes (as we assumed it had a small effect), and the failure welfare and threat of bankruptcy become the leading cause for the SIGs depleting their reserves and thereby leading to strikes. Thereby, with pessimism in strong recessions the reserves the SIG has to spend to avoid the failure welfare increases, thereby causing the SIG to deplete their resources quicker. Thereby, with strong recessions pessimism increases strikes, that is, pessimism implies that a weaker (strong) recession is necessary for strikes to occur.

Finally, if we assume that immediately after the SIG fails the politicians cannot, or do not want to, change the policy to the optimal policy of the informed voter, the no protest constraint will be violated and the voters will want to protest. This happens because with a SIG failing its contributions will immediately fall to zero violating the participation constraint and no protest constraint immediately. Confidence again plays only the role that the pessimism will lead to a quicker downfall of the SIG. We will summarise the previous discussion into the alternative main theorem of the paper.

Theorem 43. *When Propositions 40 and 41, and Lemma 39, hold, the following holds. If we allow for recessions to be strong enough ($\gamma < \tilde{\gamma}$) that a failure point of the SIG ($\tilde{W}_j(\tilde{p}_2, \tilde{\gamma})$) exists, the SIG will expend extra resources ($R(\gamma)$) in order to avoid this failure point at any cost. If there exists a ceiling to how much resources the SIG has at its disposal, a strong enough recession ($\hat{\gamma} < \tilde{\gamma}$) will reach it, and the members of the SIG will strike. Pessimism causes this point to be reached sooner for strong enough recessions ($\gamma < \tilde{\gamma}$). If the politicians cannot immediately change the policies after the disintegration of the SIG the no protest constraint will stop binding and voters will protest.*

J Combined Parameter δ

Proposition 44. *We know that $\delta = \frac{(1-\alpha)f}{\alpha h}$ using equation 31. Then when constraint 30 binds for both parties, an increase will increase $W(p^k, \gamma)$ and decrease $W_j(p^k, \gamma)$ for all $k \in K$.*

Proof. Follows directly from Lemma 2. □

From this proposition we can clearly see what factors lead to a policy that is more in line with the preferences of the average informed voter, whatever increases δ. f is the sensitivity of informed voters' votes to differences in the policy payoffs (the relative preference of A over B), and $\frac{\partial \delta}{\delta f} > 0$, i.e. the more sensitive the votes the larger the benefit the voters get. An alternative interpretation of f is that it is a parameter measuring the diversity of ex ante views about the parties (i.e. β^i, where small β^i means less diversity of preference over ideologies). Remember that $u^i(p^A,\gamma) - u^i(p^B,\gamma) \geq \beta^i$ where β^i is the preference for B's fixed position. With range of β^i small even informed voters who are almost indifferent between the pliable policies prefer A (i.e. $\beta^i \to 0 \Rightarrow u^i(p^A,\gamma) \approx u^i(p^B,\gamma)$ vote for A). Then f large means the informed voters are more sensitive to the differences in induced pliable policy preferences, i.e. informed voters close to indifferent between p^A and p^B vote for A (if $f \uparrow \to F[u^i(p^A) - u^i(p^B)] \uparrow \to$a larger mass of people vote for A, equivalenty for small β). Therefore, it is reasonable that $f \uparrow \to \delta \uparrow \to p^k : W(p^k) \uparrow$ (i.e. the welfare of the average informed voter increases, due to policies geared more towards them). More intuitively, the more almost indifferent informed voters there are makes it more costly for parties to neglect the public interest. Further, h is the sensitivity of uninformed voters' votes to the difference in campaign spending (susceptibility of uninformed voters to campaign spending) with $\frac{\partial \delta}{\delta h} > 0$. The more votes the parties can obtain from campaigning, the less important will the informed voters be. α is the proportion of uninformed voters, with $\frac{\partial \delta}{\delta \alpha} < 0$ and the obvious interpretation.

The parameters that are not in GH tell the following. The parameter $\frac{\phi^k}{\tau}$ comes from the no protest constraint. ϕ^k is the probability that party k gets the majority, and $\frac{\partial \delta}{\delta \phi} > 0$, and τ measures trust with $\frac{\partial \delta}{\delta \tau} < 0$. We have previous seen why more herd behaviour decreases the weight on informed voters, due to the decreased set of implementable policies and bargaining positions. The story is similar with ϕ^k, as the larger the chance party k wins, the more weight it will have in deciding the policy that will determine whether voters protest or not. More precisely, the more popular party will have more weight in deciding the benefit to not protesting for the voters, and the lobbyists are aware of that. An interesting implication of this is that the more popular party gets less welfare as it has to tend more to the voters.

K Electoral Motive

Now we will look into when there is reasons to give "extra" contributions and to whom.

$$\max_{(C_t^A, C_t^B)} V_j = \phi^A W_j(p^A,\gamma) + \phi^B W_j(p^B,\gamma) - (C^A + C^B) \Rightarrow \phi'(s)\alpha h[W_j(p^k) - W_j(p^l)] = 1 - \lambda^k$$

Where $k, l \in K$, $k \neq l$. Note that if the constraint binds for k, $\lambda^k = 0 \Rightarrow W_j(p^k) > W_j(p^l)$, which implies that $\lambda^l > 0$.

Proposition 45. *The SIG will contribute "extra" to at most one party for electoral reasons.*

Proof. Suppose $C_t^k \geq \delta \left[\frac{1}{\tau(\theta_t)}\left(\phi_t^A W(p_t^{\mathrm{A}},\gamma) + \phi_t^B W(p_t^B,\gamma) + \frac{\chi}{(1-\alpha)}\right) - W(p_t^k,\gamma)\right]$ for both k. Then SIG can decrease both contributions equally, until one binds, keep the relative popularity amongst uninformed voters equal, but increase its own utility (from a lower cost). Hence a profitable deviation exists. Contradiction. $\square$

Notice that now $s_t(\Lambda_t)$, unlike in the influence motive case where $\Lambda_t = 0$, therefore we need to modify the probabilities ϕ^k. Since b is ex ante unknown, i.e. it is a realisation of a random variable $\tilde{b}$ with cumulative distribution function (CDF) $F_{b\gamma,t}(\cdot)$ which is defined through $s \geq \frac{1}{2} \iff b\theta(\gamma) \geq -\Lambda_t$. For the next result we need to make an assumption: the realisation of $\tilde{b}$ is such that $s = \frac{1}{2}$, however the probability that A wins the majority of seats when $\phi^A \equiv 1 - F_{b\gamma}(-\Lambda_t) > \frac{1}{2} \Rightarrow F_{b\gamma}(-\Lambda_t) < \frac{1}{2}$. Note that in $t = 2$ we can treat γ, and therefore $\theta(\gamma)$, as an exogenous parameter or a realisation of a random variable.[131] Now we can show which party will be the beneficiary of electoral contributions.

Proposition 46. *Let A be the more popular party ($b\theta_t(\gamma) > 0$). Then a) the participation constraint for B binds; and b) if it slacks for A then $W_j(p^A, \gamma) > W_j(p^B, \gamma)$, $W(p^A, \gamma) < W(p^B, \gamma)$ and $\bar{C}^A > C^A > C^B$, where $\bar{C}^A = C^{AE} + C^{AI}$ is the electoral and interest motive and $C^{AI} \equiv C^A$.*

Proof. I first show that the more popular party gets the extra contributions, which will then imply result b).

a) Suppose

$$\hat{C}_t^A = \delta \left[\frac{1}{\tau(\theta_t)} \left(\phi_t^A W(p_t^A, \gamma) + \phi_t^B W(p_t^B, \gamma) + \frac{\chi}{(1-\alpha)} \right) - W(p_t^A, \gamma) \right]$$
$$\hat{C}_t^B > \delta \left[\frac{1}{\tau(\theta_t)} \left(\phi_t^A W(p_t^A, \gamma) + \phi_t^B W(p_t^B, \gamma) + \frac{\chi}{(1-\alpha)} \right) - W(p_t^B, \gamma) \right]$$

Therefore SIG prefers p^B in expectation ($W_j(p^B, \gamma) > W_j(p^A, \gamma)$). Suppose it switches its contributions to C^A, C^B which induce p^B, p^A respectively, i.e. they switch the contributions from one party to the other ($\hat{C^A} = C^B$ and $\hat{C^B} = C^A$). The individual constraints are still satisfied as well as the total one$C^A + C^B = \hat{C^A} + \hat{C^B}$. The SIG prefers contribution combination (C^A, C^B) because it obtains a higher payoff from it as $1 - F_{b\gamma}(-\Lambda_t) > \frac{1}{2}$. To see this clearly, notice $[1 - F_{b\gamma}(-\Lambda_t)]W_j(p_t^A, \gamma) + F_{b\gamma}(-\Lambda_t)W_j(p_t^B, \gamma) - (C^A + C^B) \equiv V_j \geq \hat{V}_j$ which is strict if $\hat{C^A} \neq \hat{C^B}$. This implies that there exists a profitable deviation, hence $(\hat{C^A}, \hat{C^B})$ could not have been an equilibrium. Contradiction.

b) This follows from Proposition 11 and 12, with the only difference that now $\bar{C}^A > C^A > C^B$ due to the contributions for electoral motives. $\square$

Notice that allowing the SIGs to offer "extra" contributions will worsen the initial bias towards the more popular party, because not only will it receive more contributions due to its popularity, but it will also receive extra contributions. These extra contributions will lead to a higher probability of winning a majority, i.e. $\bar{C}^A > C^A \Rightarrow \bar{s} > s \Rightarrow \bar{\phi}^A > \phi^A$. Finally, rewriting the optimisation problem where A is the more popular party using $C^A = C^{AE} + C^{AI}$, we obtain $\Lambda_t = \alpha h C_t^{AE} \Rightarrow s_t = \frac{1}{2} + b\theta_t(\gamma) + \alpha h C_t^{AE}$ and first order conditions (FOC) with respect to C^{AE}

$$F'_{b\gamma}(-\Lambda_t)\alpha h[W_j(p_t^A, \gamma) - W_j(p_t^B, \gamma)] = 1$$

If $\alpha h \uparrow$ the marginal benefit from extra contributions increases for a fixed cost, therefore $C^{AE} \uparrow$. The lobby will start giving extra contributions to the more popular party, if and only if by adding a single unit from zero has a higher marginal benefit to the marginal cost, i.e. $F'_{b\gamma}(0)\alpha h[W_j(\hat{p}_t^A, \gamma) - W_j(\hat{p}_t^B, \gamma)] > 1$ where $\hat{p}_t^k$ satisfies 31 with $\phi^A = 1 - F_{b\gamma}(0)$ and $\phi^B = F_{b\gamma}(0)$. Furthermore, GH have shown that even if $C^{AE} > 0$, both contribution levels will still be positive ($C^k > 0,\ \forall k$).

[131] If γ is a realization of the random variable $\tilde{\gamma}$ with CDF F_γ, and suppose $\tilde{\gamma} \perp \tilde{b}$, then their joint CDF is $F_{b\gamma}(\cdot) = F_b(\cdot)F_\gamma(\cdot)$.

Part III

Is a divided society more prone to populism?

Is a divided society more prone to populism?

by Filip Lazarić and Mathijs Janssen[0]

Abstract

This paper introduces a simple theoretical electoral competition model. It looks at the effect of carving out policies in the center of the distribution of preferences. It finds that the force that lead to the median voter theorem remains present in the simplest case, however further restricting the model leads to drastically different results. In fact, if the candidates are on opposite sides of the median there exists a strong force pushing their policies towards the extreme. Furthermore, if the politicians can influence the division in society, and the amount by which they can influence division is limited, we identify the condition when the unique equilibrium has two politicians running with the most extreme positions. However, if politicians have enough power so that they can arbitrarily influence division in society, pure strategy equilibria fail to exist. The main result is that when there is division in society grows, so does the amount of different policy positions that can exist, whereby popular policies also become sustainable. Furthermore, in certain situations division in society leads to extremism being the only possible equilibrium.

[0]We would like to thank Mustafa Kaba for helpful discussions and comments.

1 Introduction

This paper looks at how divisions in society effect the politicians equilibrium positions. We find that (under certain conditions) if the division in society is greater, there exists a multiplicity of equilibria. This implies that policies further from the optimal can be supported in equilibria, including more populist ones. Therefore, with division the politicians may also take populist positions. When we allow politicians to influence the division in society by a small amount, we find that only the highest division in society with most extreme messages can exist in equilibrium. Finally, by allowing politicians to fully influence the division in society, we find that pure strategy equilibria disappear.

Populism seems more likely to be present in situations where there is more perceived division (such as income, ethnic, national..., i.e. cleavages in any of those categories), and more dissatisfaction with the status quo. The paper finds situations where this occurs, that is higher division in society may lead to situations where politicians are free to use populist policies.

The paper is structured as follows. Section 2 looks at the most basic model, where only gaps are introduced. Section 3 uses the politicians identity to restrict the message space so that each party is willing to run on its own side of the median only (e.g. leftist use leftist messages). Section 4 allows the candidates to choose the size of the gap size around the median, however the amount they can change the gap size is bounded, capturing the intuition that it is hard for politicians to influence division in society. Section 5 allows politicians the freedom to choose any gap size and any position. Section 7 concludes.

2 Location model with gaps and messages

The underlying model is an extension of the standard spatial competition model. [Hotelling, 1929] wrote the seminal paper on spatial competition, where two sellers need to position themselves on a beach so as to attract the most customers. [Downs, 1957] later applied this location model to an electoral competition environment. An important paper that formalized a lot of the results of spatial economic competition is [Eaton and Lipsey, 1975], while important work from the spatial electoral competition side was done by [Cox, 1987], who identifies what possible positions could be taken by politicians, and [Cox, 1990] which surveys his own work. A nice survey of early spatial competition literature can be found in [Shepsle, 1991].

In order to keep the notation and interpretation constant throughout the paper, we can think of the Downsian two party competition model, and we will refer to the individuals who have to position themselves within some space as candidates, or parties, interchangeably. Suppose there are two candidates x_1 and x_2, and a continuum of voters with single peaked preferences, where the voter preferences are uniformly defined on the interval $V = [0, 1]$. Furthermore, suppose the voters vote sincerely[132]. Suppose candidates can costlessly run with any policy platform in the support of voter preferences, i.e. V.[133] The only equilibrium has both candidates running with

[132]Each votes for the candidate with a statement closest to his own preference.

[133]This brings about an implicit assumption: lying is costless. Given that each party may have some identity not equal to the message they state, lying is important, however we ignore it in this paper. An example of an environment where candidates endogenously decide whether to enter, and what message to state, given their identity (endogenously giving rise to lying costs) is treated in the first part of this thesis, the Politician-Candidate model.

the median platform, i.e. $x_1 = x_2 = 0.5$. This follows from the fact that at any other policy pair $(x_1, x_2) \neq (0.5, 0.5)$, the losing candidate[134] has a profitable deviation to ensure victory by moving slightly closer to the center.

We relax the assumption about the voter preference distribution being continuous. Suppose the distribution is still uniform, however now there is a gap around the center $G = (g_l, g_u) = (0.5 - k, 0.5 + k)$ where $0 \leq k \leq 0.5$, which has the property that if a candidate runs with a message within that interval, no one will vote for him, and the voters outside the gap are still uniformly distributed.[135] This exogenous variation in the size of the gap around the median is supposed to capture the intuition that when the gap increases, the population is getting more divided, as the preferences of the individuals on one side of the gap are increasing in distance from the individuals on the other side of the gap. There is also a more subtle interpretation: if we suppose that centrist policies are the status quo, then as the gap increases the support for center policies, even though they still exist as possible statements, is disappearing. Using the logic of the median voter theorem we obtain the following corollary describing the only possible pure strategy equilibria.

Corollary 47. *There are three possible pure strategy equilibria:*

- *Pooling at* $0.5 - k$
- *Pooling at* $0.5 + k$
- *Separating at* $\{0.5 - k, 0.5 + k\}$

Proof. For any message pair other than at the very edge of the gap (i.e. $x_1, x_2 \in \{0.5-k, 0.5+k\}$), the losing candidate has a unilateral profitable deviation towards the center (getting closer to the median, or as close to it). The details rest on the same logic as the median voter theorem in the Hotelling-Downsian model, so the proof is omitted. □

Notice that at each of these equilibria, each candidate is indifferent between stating any of the two messages, but if he states any more extreme message he will lose, as his opponent will get more votes. Furthermore, this holds for all k, i.e. the size of the gap does not change that these are the only pure strategy equilibria. More intuitively, even when there is a huge divide in society, the equilibrium strategy of politicians remains the same, get as close as possible to the median. More precisely, the only change is that an expanding gap shifts the closest point to the median (the edges of the gap) towards the extremes. This implies that the changes in messages happen mechanically, and the amount of votes (and hence the probability of winning) each candidate receives is unchanged.

This result is a mechanical application of the median voter theorem to a marginally adjusted environment. Therefore, the force pushing the candidates towards the center in the Hotelling-Downs continues to drive candidate positions towards the median. The candidates desire to win pushes them to propose messages to the edges of the gap, i.e. as close as they can get to the median.

[134]When the candidates tie away from the median, then both have a profitable deviation.

[135]This is a very strong assumption, since it makes the voter preference distribution put zero mass on the gap, and shifts that to the remaining sections outside the gap. This assumption can be weakened by assuming that there are voters within the gap will vote sincerely, however any of the voters outside of the gap will not vote for any message inside the gap. This has the benefit of reducing the reliance on this very restricted distribution of preferences that the gap imposes in this model. This will be included in a future version of this paper.

3 Restricting the message space of the candidates through identity

Here we introduce candidate identity. Let x_i^I for $i \in \{1, 2\}$ be the candidate identity, and let there be a cost to using a message further from ones true identity. Precisely, the cost of lying is: $|x_i^I - x_i|$. Finally, suppose also that there exist an exogenous benefit to winning the election: $W > 0$. We can obtain the following result, which significantly restricts the possible equilibria:

Proposition 48. *Each candidate will use messages on own side of the median.*

Proof. Wlog suppose $x_1^I \leq 0.5$, and $k > 0$ so the gap is non-empty. Then the expected benefit in equilibrium for candidate i is: $\frac{W}{2} - |x_i^I - x_i|$. Suppose the candidates are pooling at $0.5 + k$, then candidate x_1^I has a profitable deviation to the separating equilibrium: $\frac{W}{2} - (0.5 + k - x_1^I) < \frac{W}{2} - |x_1^I - (0.5 - k)| \iff |x_1^I - (0.5 - k)| < 0.5 + k - x_1^I$. This can be split into two cases: (i) if $x_1^I \geq 0.5 - k \implies x > 0.5$ which contradicts $x_1^I \leq 0.5$, and (ii) $x_1^I < 0.5 - k \implies 0 > 2k$ which contradicts $k > 0$. The proof is equivalent for the separation case where at least one candidate is using his further message. □

The intuition behind this result is straightforward: when a leftist candidate (left of median) plays against a rightist candidate, each candidate can get as many votes as his opponent by positioning himself at exactly the same distance from the median, but on his own side of the median. This is driven by the fact that lying is costly, and the cost is increasing in the size of the lie. Proposition 48 implies that if $(x_1^I, x_2^I) \in [0, 0.5] \times [0.5, 1]$, then we know that $(x_1, x_2) \in [0, 0.5] \times [0.5, 1]$. This means that each candidate will run with a policy proposal on his own half of the ideological spectrum, and neither will run with a message on the other side of ideology. In other words, a left candidate will never run with a right message, and vice versa. In the next result, we assume that each candidate occupies a different side of the median.

Theorem 49. *Suppose $(x_1^I, x_2^I) \in [0, 0.5] \times [0.5, 1]$, then equilibrium behavior depends on gap size:*

1. *If $G = \emptyset$, the median voter theorem continues to hold.*
2. *If $G \neq \emptyset$, the size of the gap determines equilibrium policies. Define the set of possible policy configurations:*

$$\{(x_1, x_2) \,|x_1 = 0.5 - d_1,\ x_2 = 0.5 + d_2,\ \textit{where } d_1, d_2 \in [k, \min\{3k, 0.5\}],\ 0 \leq k \leq 0.5\}$$

   - *As the gap increases, the acceptable distance between candidate statements is increasing*
   - *When $k = \frac{1}{6}$ we know that any possible message pair may occur in equilibrium*

Proof. The first result follows from the first theorem, the logic of the median voter theorem, and the fact we assumed that the idea spaces of both candidates intersect at the median.

For the second result, suppose the gap is very small, i.e. $k = \epsilon$ which implies that $G = (0.5 - \epsilon, 0.5 + \epsilon)$. If $x_2 = 0.5 + \epsilon$, then x_1 can win at most with probability equal to the probability that x_2 wins, i.e.

$$P\left(x_1 \text{ win} | x_1 \in [0, 0.5 - \epsilon] \text{ and } x_2 = 0.5 + \epsilon\right) \leq P\left(x_2 \text{ wins} | x_1 \in [0, 0.5 - \epsilon] \text{ and } x_2 = 0.5 + \epsilon\right)$$

To identify what messages make this inequality bind, we need to check what messages x_2 can state and receive the same number of votes as x_1. We know that if x_1 positions himself symmetrically around the median he will get the same number of votes as his opponent, however due to the gap x_1 can also make more extreme statements within a well defined range. Precisely, x_1 can at most position himself at $x_1 = 0.5 - 3\epsilon$. By increasing the gap (distance between edges) by 2ϵ and keeping one candidate fixed at his edge, the other candidate can move away from the edge by the size of the gap (when the halfway point between the candidates reaches the deviators internal edge of the gap). At that point, his opponent (i.e. x_2) captures all of the votes within the gap, and since there are no votes within it this does not effect the equilibrium outcome. This implies that the maximum distance between their statements, when one of the candidates takes the edge position, is 4ϵ. Therefore, when $k = \epsilon$ and $x_1 = 0.5 - \epsilon$, any message pair $(x_1, x_2 | x_2 - x_1 \leq 4\epsilon$ and $x_2 \in [0.5 + \epsilon, 0.5 + 2\epsilon])$ can occur in equilibrium.

The maximum distance x_2 can position himself against $x_1 = 0.5 - \epsilon$ is 4ϵ, however this is not the maximum possible distance between the candidates. If we let x_1 also vary, for any symmetric messages we know they will both win with the same probability. However, if the two messages are very far from one another, there will exist a profitable deviation towards the median, since by doing so one can start capturing votes from his opponents voter base (i.e. on the other side of the gap, internal to opponents voter base).

More concretely, when $k = \epsilon$ (gap is of distance 2ϵ) then the furthest symmetric messages without any profitable deviations available to either players is at a distance of 3ϵ from the median, i.e. $G = (0.5 - \epsilon, 0.5 + \epsilon)$ where $x_1 = 0.5 - 3\epsilon$ and $x_2 = 0.5 + 3\epsilon$ then the candidates split the votes exactly at the median. If one deviates towards the median, he can do so by moving at most 2ϵ. Suppose the x_2 deviates to $x_2 = 0.5 + 2\epsilon$ (i.e. movement of ϵ towards the median), now the candidates still have equal probability of winning, however they now split the votes at $0.5 - \frac{\epsilon}{2}$. That is, for any unilateral movement of ϵ towards the median, the location where the candidates split votes will move towards the opponent by $\frac{\epsilon}{2}$. This is natural, since the location where they split votes is the midpoint between them, i.e. a simple average of their positions. So the problem of identifying the maximum distance between two symmetric candidates, becomes a problem of identifying how far the midpoint of (x_1, x_2) can move and still remain within the gap, which turns out to be the size of the gap itself.

From the previous analysis we know that when $k = \epsilon$, the distance between the candidates is somewhere inside $[2\epsilon, 6\epsilon]$, it is 2ϵ when both candidates position themselves at the edge of the gap, while it is 6ϵ when both position themselves at the most extreme equilibrium messages. We can define all the possible equilibrium message pairs as follows:

$$\{(x_1, x_2)\,|x_1 = 0.5 - \epsilon - d_1,\ x_2 = 0.5 + \epsilon + d_2,\ \text{where } d_1, d_2 \in [0, 2\epsilon]\}$$

This defines the message pairs such that they are outside of the gap, and that the distance between them is never too large. In other words, when division in society is very small, the candidates messages will be close the the edges, but each candidate can freely state anything within 2ϵ distance from his edge. Notice that if we generalize the above definition for k, we also need to limit k. More precisely, for any k,

$$\{(x_1, x_2)\,|x_1 = 0.5 - d_1,\ x_2 = 0.5 + d_2,\ \text{where } d_1, d_2 \in [k, \min\{3k, 0.5\}] \text{ and } 0 \leq k \leq 0.5\}$$

□

When there is no gap the result doesn't change, we observe the median position as the only equilibrium. This shows that the force pushing the equilibrium behavior towards the center is still present when politicians are split into left and right politicians.

When a gap is present the set of possible message pairs that can arise in equilibrium is defined by the size of the gap. In particular, the set is expanding around the edges of the gap, and each candidate can state something more extreme than the edge position by the size of the gap. This implies that when $k > 0$ the maximum possible distance between the two candidates is $\min\{6k, 1\}$. Any message pair that is not inside the gap, but is within $2k$ of the edge of the gap, can be an equilibrium policy configuration.

To better understand how the policy space changes with k, suppose $x_2 = 0.5 + k$. When $k = \frac{1}{6}$ we know that the gap is of size $\frac{1}{3}$, so each candidate can say anything within a third of his edge, but since the gap is positioned at $G = \left(\frac{1}{3}, \frac{2}{3}\right)$ this means they can state any message. To further confirm this suppose $(x_1, x_2) = (0, 1)$. Each of the candidates can at most shift their message by a third closer to the median (i.e. to ones own edge of the gap), which will shift the average of their position by $\frac{1}{3}\frac{1}{2} = \frac{1}{6}$, i.e. the location where they split the votes will be internal to the gap exactly at the non-deviator's edge. Therefore, the deviation will not be profitable.

As the gap increases the maximum supported distance between candidates increases, which implies that more extreme policy configurations can be supported in equilibrium. Note that this does not imply that the most distant message pair will occur in equilibrium, rather it means that if the most distant message pair is observed, it will be an equilibrium. Intuitively, as the division in society grows, with two parties that have opposite ideological leanings, the set of possible equilibrium policy configurations is growing (by allowing more extreme policy configurations). This implies that division in society allows for populist policies to exist in equilibrium. In particular, note that if both of the candidates are at the edge of the gap, each can propose a more extreme (by $2k$) position, and still have an equilibrium policy configuration. Therefore, in a divided society nothing is stopping politicians from stating more extreme (than the message closest to the median) messages. Finally, note that there is only a single possible message configuration where both candidates have policies at the opposite gap edges, however there is an infinite amount of more extreme policy configurations. Therefore, when society is divided, non-centrist policy configurations will occur with a higher probability.

The final bullet point specifies the threshold at which the gap becomes large enough such that no feasible message pairs midpoint can move past the gap and steal votes from the other candidate. Precisely, when $k = \frac{1}{6}$ the gap will be of size $\frac{1}{3}$, so the most either candidate can move is within his voter base (which is of size $\frac{1}{3}$), and there exists no message pair which would allow him to encroach on his opponents voter base. More intuitively, as the division in society grows, at some point it reaches a situation where anything feasible (i.e. $x_1, x_2 \in V \backslash G$) can arise in equilibrium, even two candidates running with the most extreme messages (i.e. $(0, 1)$). The intuitive implication is related to populism in the following way: only when there is enough division in society can we observe equilibria where candidates are using extreme policy platforms.

4 Allowing candidates to change the gap size by a small amount

This is supposed to capture the fact that politicians usually spend a lot of resources on advertising, and often actually smear their opponents. This may have a twofold effect, it makes their own voter base more opposed to the competitor, it may make the undecided switch towards his voter base, and the effect on the opponents voter base is not clear. This potential effect can be boiled down to a more basic effect, by spending resources the politician may increase the divide in society. Naturally, the candidate may also spend resources on decreasing the divide in society, however we will see that, under certain restrictions, the force pushing towards extremism is more powerful.

In order to most simply capture this, we will assume that each politician can effect the size of the gap without incurring any cost. As in the previous case, this is a simultaneous game where each of the two candidates actions are choosing a message x, but now they are also choosing the size of the gap k. Finally, suppose that at any one time the candidates can change the gap only by an amount a. We will suppose that $0 < a < \frac{1}{6}$. This makes intuitive sense, as it seems unreasonable to assume that within a small time period a candidate can wildly change the division in society.[136] The following theorem shows that when politicians can endogenously change the gap size (marginally), the effect on equilibria is catastrophic:

Theorem 50. *When candidates can marginally change the gap size (i.e. k), then the only equilibrium in pure strategies is the maximal gap with the most extreme statements, i.e. $G = (0, 1)$ with $(x_1, x_2) = (0, 1)$.*

Proof. First, suppose there is no gap and $x_1 = x_2 = 0.5$. Then each candidate has a profitable deviation by increasing the gap size and message, which makes the opponent's message remain within the gap (making the opponent lose certainly), and ensuring victory for the deviator. More precisely, if $x_2 = 0.5 + k$ where $G = (0.5 - k, 0.5 + k)$ and $k = a > 0$, then $x_1 = 0.5 \in G$ means the deviator wins certainly. This holds true for any symmetric positions at the edges of the gap. Suppose $x_1 = 0.5 - k$ and $x_2 = 0.5 + 3k$ where $k \leq \frac{1}{6}$, then x_1 can decrease the gap to $\hat{k} = k - a$ and change his policy to $x_1 = 0.5 - \hat{k}$ ensuring victory. Furthermore, $x_2 = 0.5 + 3k$ can also increase the gap size by $\tilde{k} = k + a$, leaving x_1 in the gap.

Finally, we also need to check for situations where the policies are neither at the edge of the gap, nor at the most highest possible distance from the edge of the gap. Suppose $0 \leq x_1 < 0.5 - k$ and $0.5 + k < x_2 \leq 1$. First, suppose that $k < \frac{1}{6}$. Then either candidate can decrease the gap by epsilon, move to his gap edge, and ensure victory. Suppose that $k \geq \frac{1}{6}$, then any policy configuration outside the gap is possible when gaps cannot be changed. The furthest distance of x_i from the gap edge is $2k$, when $k = \frac{1}{6}$. Since we assumed that $a < \frac{1}{6}$, we know that at $k = \frac{1}{2}$ and $(x_1, x_2) = (0, 1)$, no profitable deviation is possible. For any lower k, one can decrease the gap and position himself at the edge, whereby he steals votes from his opponents voter base. □

The main intuition driving this result is that for messages close to the median, one can increase the gap size unilaterally which makes the opponent's message be part of the gap making him certainly lose. This holds true for all of the messages closest to the median (at the gap edge). When either of the candidates occupies the most extreme allowed position, his opponent can decrease the gap and move closer to the median, ensuring victory. Finally, when the policies are neither at the edge, nor the extreme, for all policy/gap configurations (except $(x_1, x_2, k) = (0, 1, 0.5)$), one of the candidates can either decrease or increase the gap, and move their policy, in such a way to ensure victory. The fact that $(x_1, x_2, k) = (0, 1, 0.5)$ remains as the unique equilibrium is an

[136] In the next section we relax this assumption, and the result is that then there are no pure strategy equilibria.

effect of the assumption that gap size cannot be changed significantly, and $a < \frac{1}{6}$ was picked so as to decrease the multiplicity of equilibria the most.[137] The unique pure strategy that remains has both candidates running with the most extreme messages, where they have made the population as divided as possible.

When candidates can effect the division within society, under certain restrictions, it is in the candidates best interest to divide society as much as possible, and state policies as extreme as possible. This seems pretty in line with what happened in the Trump-Clinton/Sanders election, and seems like a common occurrence in situations where emotions take over the electoral debate, rather than policies. Populism seems to be a situation where emotions very often take over the debate, where rhetoric is used to smear the opponent, as well as the status quo, and generally divide the population. Now that we have obtained an intuition for the model, the last section will extend it.

5 Allowing candidate to freely change gap size

Suppose that each of the two politicians can select a gap parameter, and the gap size will be determined as an average of the two politicians choices. That is, each of the politicians $x_i \in [0, 1]$ picks a "gap"-size parameter $k_i \in [0, \frac{1}{2})$. The gap choice induces a gap $(\frac{1}{2} - k, \frac{1}{2} + k)$, where $k = \frac{k_1+k_2}{2}$. Voters vote for the closest politician that is not in the gap. Voter mass is uniform on the intervals outside the gap. Therefore, the probability that candidate 1 wins is:

$$\Pi_1 = \begin{cases} 1 & \text{if } k < |x_1 - 0.5| < |x_2 - 0.5| \text{ or if } |x_2 - 0.5| < k < |x_1 - 0.5| \\ \frac{1}{2} & |x_1 - 0.5| < k \wedge |x_2 - 0.5| < k \text{ or if } |x_1 - 0.5| = |x_2 - 0.5| > k \\ 0 & \text{otherwise} \end{cases}$$

Proposition 51. *There can be no equilibrium in which either x_i or k_i is chosen as a pure strategy.*

Proof. First, note that the equilibrium identified in the previous section fails, since each extremist can decrease the gap enough. For example, let $(x_1, x_2, k_1, k_2) = (0.25 + \epsilon, 1, 0, 0.5 - \epsilon)$, which implies that $k = 0.25 - \frac{\epsilon}{2}$. Then x_1 certainly wins. Therefore, no configuration exists where each candidate chooses their policy and gap parameter (x_i, k_i) with probability 1 (i.e. purely).

Suppose that k_1 is chosen by politician 1, who may still mix for his choice of x_1. For a fixed value of k, there is a clear optimal choice of x, which is $x \in \{\frac{1}{2} - k, \frac{1}{2} + k\}$. With this optimal choice, the politician wins with certainty, unless his opponent also makes the optimal choice on x with probability 1. Given k_1, candidate 2 can pick a value for k_2 such that $Pr(x_1 \in \{\frac{1}{2} - k, \frac{1}{2} + k\}) < 1$ and pick $x_2 \in \{\frac{1}{2} - k, \frac{1}{2} + k\}$, winning with a probability greater than $\frac{1}{2}$ (given that candidate 1 places some probability on using non-optimal policies). This disproves this can be an equilibrium.

Suppose next that x_1 is chosen by politician 1, who may still mix on k_1. If $|x_1 - 0.5| < \frac{1}{4}$, then the strategy $x_2 = 1, k_2 = 2|x_1 - 0.5| + \kappa < \frac{1}{2}$, where $\kappa > 0$, guarantees victory, as politician 1 will always be inside the gap and politician 2 never. If $|x_1 - 0.5| > \frac{1}{4}$, then the strategy $x_2 = \frac{1}{4}$

[137] For example, if we assumed that $a < \epsilon$, then for $k > \frac{1}{6}$, any policy configuration outside the gap can be supported. As we allow for a large change in k, the multiplicity of equilibrium configurations is shrinking, up to $a < \frac{1}{6}$. If we allow for $a > \frac{1}{6}$, then no pure strategy equilibria exist.

or $x_2 = \frac{3}{4}$ and $k_2 = 0$ guarantees victory, as politician 2 is closer to the median and never in the gap. This leaves $|x_1 - 0.5| = \frac{1}{4}$. Suppose $Pr(k_1 \leq \kappa) < \frac{1}{2}$, for some $0 < \kappa < \frac{1}{4}$ then the strategy $x_2 = 1, k_2 = \frac{1}{2} - \kappa$ wins with probability $Pr(2 \text{ wins}) = Pr(|x_1 - 0.5| < k) = Pr(\frac{1}{2} < k_1 + \frac{1}{2} - \kappa) = 1 - Pr(k_1 \leq \kappa) > \frac{1}{2}$. Suppose instead that $Pr(k_1 \geq \frac{1}{2} - \eta) < \frac{1}{2}$ for some $0 < \eta < \frac{1}{4}$, then the strategy $x_2 = \frac{1}{4} + \eta$ or $x_2 = \frac{3}{4} - \eta$ and $k_2 = 0$ wins with probability $Pr(2 \text{ wins}) = Pr(|x_2 - 0.5| > k) = Pr(\frac{1}{4} - \eta > \frac{k_1}{2}) = 1 - Pr(\frac{1}{2} - \eta \leq k_1) > \frac{1}{2}$. Since it cannot be that both $Pr(k_1 > \frac{1}{2}) > \frac{1}{2}$ and $Pr(k_1 < \frac{1}{2}) > \frac{1}{2}$, this exhausts all possibilities. $\square$

Proposition 51 shows that any equilibrium must be fully mixing, that is neither the value of x_i nor of k_i is known ex ante with certainty in equilibrium. In other words, both players mix over x_i and over k_i. To give a general equilibrium characterization, we define the events of a particular player winning and of drawing, given a choice of strategy for that player:

$$W(x_1, k_1) = W_I(x_1, k_1) \cup W_{II}(x_1, k_1)$$
$$W_I(x_1, k_1) = \{(x_2, k_2) \in [0,1] \times [0, \tfrac{1}{2}) : \tfrac{k_1 + k_2}{2} < |x_1 - 0.5|, |x_2 - 0.5| > |x_1 - 0.5|\}$$
$$W_{II}(x_1, k_1) = \{(x_2, k_2) \in [0,1] \times [0, \tfrac{1}{2}) : \tfrac{k_1 + k_2}{2} < |x_1 - 0.5|, \tfrac{k_1 + k_2}{2} > |x_2 - 0.5|\}$$
$$D(x_1, k_1) = \{(x_2, k_2) \in [0,1] \times [0, \tfrac{1}{2}) : \tfrac{k_1 + k_2}{2} > |x_1 - 0.5|, \tfrac{k_1 + k_2}{2} > |x_2 - 0.5|\}$$

A symmetric equilibrium is now a probability density function $f(x, k)$ with support $S \subset \Omega = [0,1] \times [0, \dfrac{1}{2})$ such that:[138]

$$\iint_{W(x,k)} f(\xi, \kappa) d\xi d\kappa + \iint_{D(x,k)} \frac{1}{2} f(\xi, \kappa) d\xi d\kappa = \frac{1}{2} \quad \forall (x, k) \in S$$
$$\iint_{W(x,k)} f(\xi, \kappa) d\xi d\kappa + \iint_{D(x,k)} \frac{1}{2} f(\xi, \kappa) d\xi d\kappa \leq \frac{1}{2} \quad \forall (x, k) \in \Omega \backslash S$$

This only specifies the structure the mixed strategy equilibrium will take. The structure imposes a full weight being attached to situations where politician 1 wins, and half the weight for ties. The mixed strategies have to be such that both players have the same probability of winning, that is, both mixed strategies need to give each player half a probability of winning. In order to find the mixed strategy equilibria we can use the intuition that strictly dominated actions cannot be played. This will simplify the analysis, however we have yet to specify the mixed strategy equilibrium for this game.

6 Empirical Analysis

The main result from the theoretical section demonstrated that when division in society grows, a larger set of policy proposals can be supported as equilibrium messages. Furthermore, if politicians can influence the division in society, there exists a strong force pushing the policy proposals parties use towards extremism. These results imply that when division in society grows we should observe an increase in the variance of the candidates policy proposals, and furthermore a tendency towards extremism. The final theoretical section finds that pure strategy equilibria are

[138] Assuming that there are no atoms and that in particular the probability of a tied vote is 0.

not possible when politicians have full flexibility about how divided society will be. This result seems least empirically relevant, therefore this will not be investigated.

The purpose of this section is to analyze the main theoretical results. Is it true that when division in society increases the variance in policy proposals increases? Do the policy proposals become more extreme? In order to answer these research questions the paper would first approach the problem using descriptive statistics, looking at policy proposals of a handful of countries with a long enough dataset, which can be coupled with data that could be used to back out some proxy for division in society. After a descriptive analysis of the problems, inferential methods would be used to test the research questions. Finally, if the problem requires tools from computer science (such as deep learning and other machine learning techniques) we would use these tools to test our models hypothesis.

For example, division in society is a statistic that can be proxied in many ways. One way would be to use survey data through which a distribution of preferences about some issue are backed out, such as inequality and taxes. Then we could look at how the politicians policy proposals are related to this division. Alternatively, one could use deep learning to train a model for predicting the most likely policy outcomes, which could be compared to actual policies chosen. Furthermore, this trained model could be compared to a different time, where the division of society changed, and test the predicted policies in comparison to actual policies.

We believe that this paper is well suited for looking at the problem through data. Even though the model is very simple, the predictions are clear and testable, therefore the empirical exercise would form a crucial part of the paper.

7 Conclusion

In this short paper we look at how divisions in society affect the equilibrium policies of the two parties involved in the spatial electoral competition game. We find that division have a non trivial effect, where depending on the exact specifications, the results change. If gaps are introduced in the voter distribution of preferences mechanically, then the result is similar to the median voter theorem, in that the only possible equilibrium messages are the ones closest to the median. If the candidates have an identity on opposite sides of the median, we get a multiplicity of equilibria, where the same equilibria hold as in the previous case, however more extreme policy configurations are also possible.

We then alter the approach. First, we let politicians choose the gap size, where we limit the size of the gap they can impose. We find that, under specific conditions, both politicians want to increase the gap size as much as possible. In fact, for specific values of how much politicians can influence the division in society, the unique equilibrium is composed of two parties using the most extreme messages. When we relax the restriction limiting how much politicians can influence division, and let both politician choose any gap size, this result fails. In fact, we then find that no pure strategy equilibria exist. Therefore, the fact that most extreme messages are a unique equilibrium is not a robust result, however the force pushing politicians to use more extreme messages is robust for a large set of parameters (defining how much politicians can influence division).

This paper gives basic support of the common claims that cleavages and divisions in society may lead to situations where populism and extremism occurs. It does this through several theoretical specifications, yielding testable results. It would be interesting to check whether these results could be found in the data.

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