Appendix

Appendix 1 (Equivalence of the market clearing rules): Let us denote the number of shareholders (sellers) and non-shareholders (buyers) who have submitted a bid higher than the market clearing price $p^*$ by $n_s$ and $n_b$. It follows then that

<table>
<thead>
<tr>
<th></th>
<th># of shareholders</th>
<th># of non-shareholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>higher bids than $p^*$</td>
<td>$n_s$</td>
<td>$n_b$</td>
</tr>
<tr>
<td>lower bids than $p^*$</td>
<td>$M - n_s$</td>
<td>$n - M - n_b$</td>
</tr>
<tr>
<td>total</td>
<td>$M$</td>
<td>$n - M$</td>
</tr>
</tbody>
</table>

When the clearinghouse follows the first approach, i.e., $p^* = \sup \{ p : D(p) = S(p) \}$, market excess demand at $p^*$ is equal to zero. That is, $n_b - (M - n_s) = 0$. When it follows the alternative approach, i.e., arranging all bids in a descending order, the market clearing price is equal to the $M^{th}$ highest bid. Since there should be $M$ bids higher than $p^*$, $n_s + n_b = M$. Both approaches result in the same equation for market clearing. Q.E.D.

Appendix 2 (Equivalence of the buyer's and seller's strategy under price taking): The optimal bid for buyer $i$ in round $\tau$ given his private signal and market information $Y^*$ is a solution to the following:

$$\max_{p} \mathbb{E}[(V - p) \cdot 1_{\{p \geq x\}} | X_i = x, Y^*]$$  \hspace{1cm} (A.1)
The maximization problem for seller $i$ is

$$\max_b E[V \cdot 1_{(b \geq p)} + p \cdot 1_{(b < p)} | X_i = x, Y^*]$$  \hspace{1cm} (A.2)$$

Since $V \cdot 1_{(p \leq b)} + p \cdot 1_{(p > b)} = V \cdot 1_{(b \geq p)} + p \cdot (1 - 1_{(b \geq p)}) = (V - p) \cdot 1_{(b \geq p)} + p$, it becomes

$$\max_b E[(V - p) \cdot 1_{(b \geq p)} | X_i = x, Y^*] + E[p | X_i = x, Y^*]$$  \hspace{1cm} (A.3)$$

Since $E[p | X_i = x, Y^*]$ is not affected by their bid, a maximization problem (A.3) for seller $i$ is equivalent to that for buyer $i$ given in (A.1). Hence, the optimal bid for trader $i$ is the same irrespective of the identity as a buyer or a seller. \hspace{1cm} Q.E.D.

Appendix 3 (Updated bids do not change the equilibrium price): Suppose that each trader, observing the market clearing price $p^*$, submits an updated bid in the next round. The problem faced by a trader $i$ who has signal $x$ and price information $p^*$ is to find a bid to solve the following:

$$\max_b E[U(V, X_i, b) | X_i = x, p^*]$$

Since they can infer $Y$ from price information, their updated bid is equal to $\varphi(x, y)$. Traders who submit a bid higher than the market clearing price are those whose signal $x$ is greater than $y$. Since the updated bid $\varphi(x, y)$ is still higher than the market clearing price $\varphi(y, y)$, this does not change a price determined in the first round. The same argument applies for traders whose signals are smaller than $y$. That is,
\[ \varphi(x, x) < \varphi(x, y) < p^* \quad \text{for} \quad x < y \]
\[ p^* < \varphi(x, y) < \varphi(x, x) \quad \text{for} \quad x > y \]
\[ \varphi(x, y) = \varphi(x, x) = p^* \quad \text{for} \quad x = y \]

A trader who tendered a bid higher (lower) than \( p^* \) will find it optimal to submit a new bid which is smaller (higher) than his initial bid but still higher (lower) than the market clearing price \( p^* \). Price information does not affect a trader whose initial bid is \( p^* \). Hence the updated bids do not change the market clearing price determined in the first round.

**Appendix 4 \((\phi(Y) = \varphi(Y, Y)\) is a unique function satisfying (3.8)):**

When \( \phi(Y) = \varphi(Y, Y) \), \( E[V|X_i = Y, p = \phi(Y)] \) is equal to \( \phi(Y) \) since \( E[V|X_i = Y, p = \phi(Y)] = \varphi(Y, \phi^{-1}(p)) = \varphi(Y, Y) \). Since \( E[V|X_i = x, p = \phi(Y)] \) is increasing in \( x \), \( E[V|X_i = x, p = \phi(Y)] > (<) \phi(Y) \) for \( x > (<) Y \).

Next, let us prove that the function satisfying (3.8) is unique. Suppose that it is not and there is another function \( q(Y) \). Then, there should exist at least one point of \( Y = y' \) such that \( q(y') \neq \varphi(y', y') \). Since \( q(Y) \) satisfies (3.8), \( E[V|X_i = y', p = q(y')] \) should be equal to \( \phi(y') \). It contradicts \( q(y') \neq \varphi(y', y') \) since \( E[V|X_i = y', p = q(y')] = \varphi(y', y') \). \( Q.E.D. \)

**Appendix 5 \((Naive traders' bidding strategy after circuit breakers have been triggered)):** By the same reasoning used in a proof of Theorem 2, the optimal bidding price of naive trader \( i \) as a solution to (3.15) is given as \( \delta_N = \varphi(x', x') \). As far as \( \varphi(x', x') \) is an admissible bidding price, trader \( i \) will submit it as his own bid. On the other hand, when \( \varphi(x', x') \) is greater (smaller) than the limit price, his optimal bid becomes the maximum (minimum) bid allowed by the exchange. Hence, (3.16) is
optimal for trader $i$. Since the market clearing price $\bar{p}$ is the $M^{th}$ highest bid,
$\bar{p} = \varphi(y', y')$ where $y' = y + (1 - \gamma) \cdot E[Y | X_i = y, Y \geq c]$. Notice that
$\varphi(y', y') > \varphi(y, y)$ since $y' > y$. Hence, the market clearing price determined in a
market with circuit breakers is greater than the one determined in a market without
circuit breakers. \textit{Q.E.D.}

Appendix 6 (A proof of lemma 2): Sophisticated trader $i$'s maximization problem is
given in (3.20). Suppose that the price functional $\pi(Y)$ is equal to $\varphi(Y, Y)$. Then, the
problem for trader $i$ degenerates into the one shown in the benchmark model without
circuit breakers. The optimal strategy $\bar{b}_i$ is equal to $\varphi(x, x)$. Next, suppose that
$\pi(Y) > \varphi(Y, Y)$. Then, the optimal bid is a solution to the following:

$$\max_{\bar{b} \in \mathbb{R}_+} \int_c^{\pi^{-1}(\bar{b})} \{\varphi(x, \omega) - \pi(\omega)\} h(\omega / x, \omega \geq c) \, d\omega$$

Notice that $\varphi(x, Y) - \pi(Y)$ is positive at a sufficiently small value of $Y$ and negative at
any value of $Y$ greater than $x$. Since $\varphi(x, Y) - \pi(Y)$ is a monotonically decreasing
function in $Y$, there exists a unique value of $Y$ denoted by $y'$ such that
$\varphi(x, y') = \pi(y')$. Since $\varphi(x, Y) - \pi(Y)$ is negative when $Y = x$, $y'$ is smaller than $x$.
Regardless of the conditional density of $Y$, the maximum is achieved by integrating
over $Y$ such that $\{Y | \varphi(x, Y) - \pi(Y) \geq 0\}$. Hence, $\pi^{-1}(\bar{b}) = y'$ \textit{i.e.}, $\bar{b}_i = \pi(y')$. Since
$\bar{b}_i = \pi(y')$ and $\pi(y') = \varphi(x, y') < \varphi(x, x)$, the optimal bid $\bar{b}_i$ is smaller than $\varphi(x, x)$.
In other case when $\pi(Y) < \varphi(Y, Y)$, we can be prove using similar arguments. \textit{Q.E.D.}
References


Camerer, C. and Weigelt, K., Informational Mirages in Experimental Asset Markets,


Korea Stock Exchange, Korea Stock Exchange, 1992


Lee, In Ho, Market Crashes and Informational Cascades, 1992, Mimeo.


