The Tripartite Auction Theorem

The Setting

- political contest between two groups providing or promising effort
- lobbying groups, political parties
- consider different mechanisms for resolving the contest
  - winner pays – first or second price auction: example – a politician to be bribed – common in the lobbying literature
  - everyone pays – all pay auction: example – an election, warfare

Are Large or Small Groups More Effective?

- Olson, Becker, Levine/Modica others argue that smaller groups are more effective at lobbying
- Levine/Mattozzi, others argue that larger groups are more effective at voting
- When groups of different sizes compete for the same prize when is the larger or smaller group more likely to be successful?
- Why should it be different for voting and lobbying?
- What factors determine the effectiveness of groups of different sizes?

Duties versus Chores

- effort provision a *duty*: we view voting as a civic duty so we receive a benefit for doing our duty that exceeds at least some of the cost of participating

  duty in the broad sense: a political demonstration or protest might be an enjoyable event - to be outdoors in good weather, meet new people, chant, march and sing

- effort provision a *chore*: a fixed cost of participation

cannot simply write a check for 32 cents to “anti-farm subsidies” must find the appropriate organization, learn about them, join up - and they have to vet me, process my application and so forth

considerable cost incurred even as I contributed absolutely nothing to the lobbying effort

- tend to think of voting as a duty and lobbying as a chore, but the cost structure is the fundamental distinction

The Political Contest Between Groups

two groups $k = S, L$ of size $N_L > N_S$ compete for a common prize worth V to the group and $v_k = V/N_k$ to each group member.

only difference between groups is their size

groups behave as single individuals

choose a social norm in the form of a per capita effort level $0 \leq \varphi_k \leq 1$

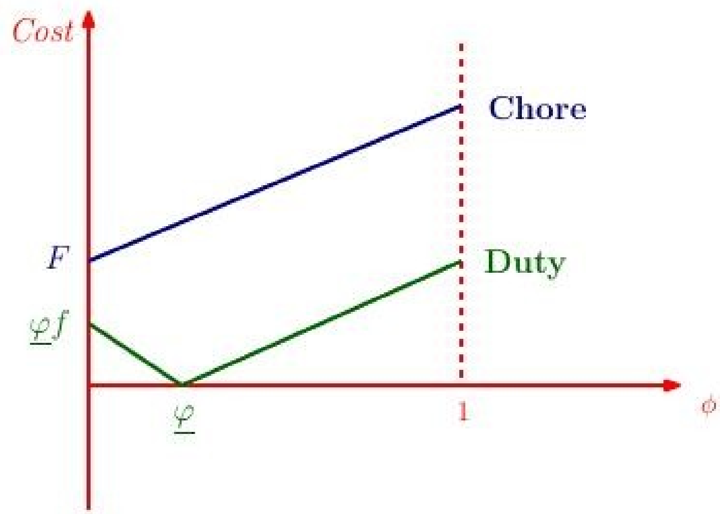
- marginal cost of per capita effort up to a threshold $\underline{\varphi} \geq 0 \text{ is } -f < 0$
- further effort requires a per capita fixed cost $F \geq 0$ plus a marginal cost of 1

group may “burn money” by choosing to pay the fixed cost without providing additional effort

Duties versus Chores

only allow two cases:

- effort *a duty:* $\underline{\varphi} > 0$ and $F = 0$
- effort is a *chore*: $\underline{\varphi} = 0$ and $F > 0$



Bids, Strategies and Payoffs

social norm φ_k in per capita terms results in total effort or *bid* $b_k = N_k \varphi_k$

pure strategy for group k is choice of accepting the fixed cost $q_k \in \{0, 1\}$ and a social norm φ_k satisfying the feasibility condition that $q_k = 1$ if $\varphi_k > \underline{\varphi}$

if group has probability p_k of winning the prize and follows pure strategy q_k, φ_k it receives per capita utility

$$p_k v_k - q_k F - \max\{0, \varphi_k - \underline{\varphi}\} + f \min\{\varphi_k, \underline{\varphi}\}$$

Willingness to Pay

willingness-to-pay is the greatest amount of effort group would be willing and able to provide to get the prize for certain.

$$W_k = \begin{cases} 0 & if & V < N_k F \\ N_k \underline{\varphi} + V - N_k F & if & V \in [N_k F, N_k((1-\underline{\varphi}) + F)] \\ N_k & if & V > N_k((1-\underline{\varphi}) + F) \end{cases}$$

benefit of duty f does not figure in because group can receive that benefit regardless of whether or not it wins the prize

if $V \leq N_k F$ for both groups we say that both groups are *disadvantaged*

otherwise a group with the highest willingness to pay is called *advantaged* and the other group *disadvantaged*

Size of the Prize

- prize is small if $V < FN_S$
- prize is medium if $N_S F < V < FN_L + N_S$
- prize is large if $V > FN_L + N_S$

Group Advantage

Theorem: *For a chore with a small prize both groups are disadvantaged. For a chore with a medium prize the small group is advantaged. For a large prize or a duty the large group is advantaged.*

Allocation Mechanisms

allocation mechanism determines the award of the prize and the contributions of the two groups based on their bids

1. Second-price auction. The highest bidder wins and provides an effort contribution equal to the bid of the lower bidder.

2. First-price auction. The highest bidder wins and provides an effort contribution equal to their own bid.

3. All-pay auction. The highest bidder wins and both bidders provide an effort contribution equal to their own bid.

- for chores if neither group chooses to incur fixed cost the prize is canceled and both groups receive zero
- for auctions if there is a tie the winner is determined endogenously.

Equilibrium

Nash equilibrium of the game between groups (two-player game) with the following refinements:

1. Second-price auction: weakly undominated strategies

2. First-price auction: the “honest bidding” refinement from menu auctions – a bid that loses with probability one must be equal to the willingness-to-pay.

3. All-pay auction: none

Tripartite Auction Theorem

d the disadvantaged group

if $W_d \geq N_{-d}\underline{\varphi}$ it costs the advantaged group $W_d - N_{-d}\underline{\varphi} + N_{-d}F$ to match the bid of the disadvantaged group

if $W_d < N_{-d}\underline{\varphi}$ it costs nothing to overmatch the bid of the disadvantaged group

surplus is the difference between the value of the prize and cost of matching the bid of the disadvantaged group if this is positive, zero otherwise.

Theorem: *In the second-price, first-price and all-pay auction a disadvantaged group gets 0 and an advantaged group gets the surplus.*

Observations

small group gets a positive surplus when there is a medium prize and a chore: fungibility (Levine/Modica) and resource constraints

rent dissipation: if the value of the prize is medium and groups are of similar size then value of prize dissipated

Proof

this is NOT the revenue equivalence theorem

here the size of the prize is common knowledge

the result for first and second price auction is obvious

the result for the all-pay auction is not

prove the special case where $F = \underline{\varphi} = 0$, large prize $V > \eta_S$ so the large group is advantaged

No Ties and No Pure Strategy Equilibrium

if there was a positive probability of a tie the large party would raise its bid slightly for small cost and increase the probability it wins by ½

with pure strategies and no tie one party must lose with probability 1 and so must be bidding 0 . But if one party bids 0 the other party should bid the smallest number bigger than zero and there is no such number.

Key Idea

one party must get 0 and both parties must bid arbitrarily close to W_S.

second fact will implies that it is the disadvantaged party that gets 0.

why does this mean that the large party gets $V - W_S$?

One Party Must Get Zero

let $\underline{b}$ be the lowest bid by either party.

cannot be that bidding $\underline{b}$ leads to a tie with positive probability

so one party k must face an opponent who has zero probability of bidding $\underline{b}$ or less

means that k must be almost certain to lose if it bids near $\underline{b}$

so if it is bidding near $\underline{b}$ it must be getting 0 in equilibrium

if k is not bidding near $\underline{b}$ then the other party $-k$ must be and these are losing for sure so $-k$ must be getting 0 in equilibrium.

What is a Mixed Strategy

and why you need to know math to be a theorist

a probability distribution represented by a cumulative distribution function over bids, that is, a G_k is a non-decreasing function on $(-\infty, \infty)$ with $G_k(b) = 0$ for $b < 0$ and $G_k(1) = 1$. It is right continuous and if it fails to be left continuous at a bid b the height of the jump at b is the probability with which b is bid – it is an *atom* in the probability distribution. At points of continuity of G_k the probability of the bid is zero.

Both Parties Bid Close to W_S

if the highest bid is less than W_S the party getting an expected payoff of zero should bid a shade higher because this would turn a profit.

moreover, one party cannot have a higher highest bid than the other, since the party with the higher highest bid could lower its bids, saving cost and still winning with probability 1. Hence both parties must bid near W_S.

Equilibrium Strategies

show there are no gaps and no atoms...

use the indifference condition to find that the small group has an atom at the bottom, the large group has an atom at the top, and that both use a uniform distribution in between

easy enough to compute exactly

Why not Split a Large Group?

with a positive fixed cost why doesn't the larger group “act like a smaller group” by appointing a smaller subgroup to act on its behalf?

a subgroup of size $M_k < N_k$ will only receive a share of the prize: $(M_k/N_k)V$

so raw willingness of the subgroup to pay is

$$M_k\underline{\varphi} + \frac{(M_k/N_k)V - M_kF}{c} = \frac{M_k}{N_k}\left(N_k\underline{\varphi} + \frac{V-N_kF}{c}\right) = \frac{M_k}{N_k}r_k$$

a fraction M_k/N_k of the raw willingness of the entire group to pay.

problem involves “renegotiation” subgroup will collude not to do it