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Auctions and Competition

Roth et al: 10 players submit bids (first price auction) on a prize worth $10 after a few rounds everyone is bidding $9.95 typical of games in a competitive environment “Cournot” example with seven firms…competition or Cournot?
Dominance and The Prisoner’s Dilemma Game

<table>
<thead>
<tr>
<th></th>
<th>cooperate</th>
<th>cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooperate</td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td>cheat</td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

- Has a unique dominant strategy equilibrium cheat-cheat
- This is Pareto dominated by cooperate-cooperate
- Role for altruism?
Public Goods Experiment

Players randomly matched in pairs
May donate or keep a token
The token has a fixed commonly known public value of 15
It has a randomly drawn private value uniform on 10-20
V=private gain/public gain
So if the private value is 20 and you donate you lose 5, the other player
gets 15; V= -1/3
If the private value is 10 and you donate you get 5 the other player gets
15; V=+1/3
Data from Levine/Palfrey, experiments conducted with caltech
undergraduates, based on Palfrey and Prisbey
## Coordination Results

<table>
<thead>
<tr>
<th>V</th>
<th>donating a token</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>100%</td>
</tr>
<tr>
<td>0.2</td>
<td>92%</td>
</tr>
<tr>
<td>0.1</td>
<td>100%</td>
</tr>
<tr>
<td>0</td>
<td>83%</td>
</tr>
<tr>
<td>-0.1</td>
<td>55%</td>
</tr>
<tr>
<td>-0.2</td>
<td>13%</td>
</tr>
<tr>
<td>-0.3</td>
<td>20%</td>
</tr>
</tbody>
</table>
Weak Dominance and the Second Price Auction

- bidding your value is weakly dominant
- BDM mechanism with random “second highest bid”
- The endowment effect
This ticket is worth $2.00 to you.
You can sell it.
Name your offer price.
A price will be posted shortly

The posted price was drawn randomly between:

[$0 and $6]

If your offer price is below the posted price then you sell your ticket at the posted price.

If your offer price is above the posted price then you do not sell your ticket but you do collect the $2.00 value of the ticket.

You can view the posted price after you have named your price.

Indicate the appropriate amount.

My offer price is below the posted price.
Pay me the posted price of $__________.
My offer price is above the posted price.
Pay me $2.00.
Coordination Games

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>2,2</td>
<td>0,0</td>
</tr>
<tr>
<td>D</td>
<td>0,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

three equilibria (U,L) (D,R) plus mixed

too many equilibria?? introspection possible?

the rush hour traffic game – introspection clearly impossible, yet we seem to observe Nash equilibrium

equilibrium through learning?

Coordinate on efficient equilibrium?
Coordination Experiments

Van Huyck, Battalio and Beil [1990]

Actions $A = \{1, 2, \ldots, 7\}$

Utility $u(a_i, a_{-i}) = b_0 \min(a_j) - b a_i$ where $b_0 > b > 0$

14-16 players

Everyone doing $a'$ the same thing is always a Nash equilibrium

$a' = \bar{e}$ is efficient, the bigger is $a'$ the more efficient, but the “riskier”

A model of “riskier” some probability of one player playing $a' = 1$

Story of the stag-hunt game
Coordination Results

treatments: \( A \ b_0 = 2b \), \( B \ b = 0 \)

- In final period treatment A:
  - 77 subjects playing \( a_i = 1 \)
  - 30 subjects playing something else
  - minimum was always 1

- In final period treatment B:
  - 87 subjects playing \( a_i = 7 \)
  - 0 playing something else
  - with two players \( a_i = 7 \) was more common
Approximate Equilibria and Near Equilibria

- exact: \( u_i(s_i | \sigma_{-i}) \geq u_i(s'_i | \sigma_{-i}) \)

  approximate: \( u_i(s_i | \sigma_{-i}) + \varepsilon \geq u_i(s'_i | \sigma_{-i}) \)

- Approximate equilibrium can be very different from exact equilibrium

Radner's work on finite repeated PD

gang of four on reputation

upper and lower hemi-continuity

A small portion of the population playing "non-optimally" may significantly change the incentives for other players causing a large shift in equilibrium behavior.
Quantal Response Equilibrium
(McKelvey and Palfrey)

propensity to play a strategy

\[ p_i(s_i) = \exp(\lambda_i u_i(s_i, \sigma_i)) \]
\[ \sigma_i(s_i) = \frac{p_i(s_i)}{\sum_{s_i'} p_i(s_i')} \]

as \( \lambda_i \to \infty \) approaches best response

as \( \lambda_i \to 0 \) approaches uniform distribution
### Smoothed Best Response Correspondence Example

<table>
<thead>
<tr>
<th>( \sigma_1(U) = p )</th>
<th>( L(\sigma_2(L) = q) )</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{array}{c} 1,1 \ 0,0 \end{array} )</td>
<td>( \begin{array}{c} 1,1 \ 0,0 \end{array} )</td>
<td>( \begin{array}{c} 0,0 \ 1,1 \end{array} )</td>
</tr>
</tbody>
</table>

![Diagram showing best response correspondence](image-url)
Voting
Individual Behavior
Observations

- contains an unknown preference parameter $\lambda$
- $\lambda = 0$ play is completely random
- as $\lambda$ becomes large, the probability of playing the “best” response approaches one
- $\lambda$ kind of index of rationality.
- in the voting experiment we can estimate a common value of $\lambda$ for all players.
- corresponding equilibrium probabilities of play are given by the green curve
- does an excellent job of describing individual play
- it makes roughly the same predictions for aggregate play as Nash equilibrium
Limitations of QRE

- captures only the cost side of preferences
- recognizes – correctly – departures from standard “fully rational” selfish play are more likely if less costly in objective terms
- does not attempt to capture benefits of playing non-selfishly
- does not well capture, for example, the fact that under some circumstances players are altruistic, and in others spiteful.
Auctioning a Jar of Pennies

- surefire way to make some money
- put a bunch of pennies in a jar
- get together a group of friends
- auction off the jar of pennies
- with about thirty friends that you can sell a $3.00 jar of pennies for about $10.00
Winner’s Curse

- friends all stare at the jar and try to guess how many pennies there are.
- Some under guess – they may guess that there are only 100 or 200 pennies. They bid low.
- Others over guess – they may guess that there are 1,000 pennies or more. They bid high.
- Of course those who overestimate the number of pennies by the most bid the highest – so you make out like a bandit.
Nash Equilibrium?

- According to Nash equilibrium this shouldn’t happen.
- Everyone should rationally realize that they will only win if they guess high.
- They should bid less than their estimate of how many pennies there are in the jar.
- They should bid a lot less – every player can guarantee they lose nothing by bidding nothing.
- In equilibrium, they can’t on average lose anything, let alone $7.00.
QRE

- Recognize that there is small probability people aren’t so rational
- Very different prediction
- some most possible profit anyone can make by getting the most number of pennies at zero cost: call this amount of utility $U$
- some least possible profit by getting a jar with no pennies at the highest possible bid: call this amount of utility $u$
- QRE says ratio of probability between two bids that give utility $U, u$ is $\exp[\lambda(U - u)]$
- whatever is the difference in utility between two strategies it cannot be greater than that between $U$ and $u$
- probability of highest possible bid is at least $p > 0$
- depends on how many bids are possible, not on how many bidders or their strategies
QRE with Many Bidders

- each bidder has at least a $p$ probability of making the highest possible bid
- becomes a virtual certainty that one of the bidders will (unluckily for them) make this high bid

with enough bidders, QRE assures the seller a nice profit.
Mixed Strategies: How Do Athletes Do It?

- Holmes, Moriarity, Canterbury and Dover
- once in Japan catchers equipped with mechanical randomization devices to call the pitch
- later ruled unsporting and banned from play
- good tennis players in important matches do it right
- professional soccer players do it right
- submarine captains and the RAND corporation
Goeree and Holt: Matching Pennies

Symmetric

<table>
<thead>
<tr>
<th></th>
<th>50% (48%)</th>
<th>50% (52%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% (48%)</td>
<td>80,40</td>
<td>40,80</td>
</tr>
<tr>
<td>50% (52%)</td>
<td>40,80</td>
<td>80,40</td>
</tr>
<tr>
<td></td>
<td>12.5% (16%)</td>
<td>87.5% (84%)</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>50% (96%)</td>
<td>320,40</td>
<td>40,80</td>
</tr>
<tr>
<td>50% (4%)</td>
<td>40,80</td>
<td>80,40</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>(80%)</th>
<th>(20%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% (8%)</td>
<td>44,40</td>
<td>40,80</td>
</tr>
<tr>
<td>50% (92%)</td>
<td>40,80</td>
<td>80,40</td>
</tr>
</tbody>
</table>
Subgame Perfection and Best Shot
Prasnikar and Roth

\[
W(\max(x_1, x_2) - C(x_1), W(\max(x_1, x_2) - C(x_2))
\]
<table>
<thead>
<tr>
<th>$x$</th>
<th>$W(x)$</th>
<th>$C(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>1</td>
<td>$1.00</td>
<td>$0.82</td>
</tr>
<tr>
<td>2</td>
<td>$1.95</td>
<td>$1.64</td>
</tr>
<tr>
<td>3</td>
<td>$2.85</td>
<td>$2.46</td>
</tr>
<tr>
<td>4</td>
<td>$3.70</td>
<td>$3.28</td>
</tr>
<tr>
<td>5</td>
<td>$4.50</td>
<td>$4.10</td>
</tr>
<tr>
<td>6</td>
<td>$5.25</td>
<td>$4.92</td>
</tr>
<tr>
<td>7</td>
<td>$5.95</td>
<td>$5.74</td>
</tr>
<tr>
<td>8</td>
<td>$6.60</td>
<td>$6.50</td>
</tr>
</tbody>
</table>
Discussion of Best Shot

if the other player makes any contribution at all, it is optimal to contribute nothing

unique subgame perfect equilibrium player 1 contributes nothing

another Nash equilibrium player 2 to contributes nothing regardless of player 1’s play
Best-Shot Results

Hirshleifer-Harrison partial information, but alternating roles

Prasnikar-Roth fixed roles, both partial and full information

In the full information case and partial information heterogeneous case
player 2 occasionally contributes less than 4 when player 1 has
contributed nothing; Note that the player who contributes nothing gets
$3.70 against $0.42 for the opponent who contributes 4

- full information case: player 1 never contributed anything
- partial information case: sometimes roles reverse
Subgame Perfection and Ultimatum Bargaining

player 1 proposes how to divide $10 in nickles
player 2 may accept or reject

Nash: any proposal by player 1 with all poorer proposals rejected and equal or better proposals accepted

Subgame Perfect: First player gets at least $9.95
### US Data for Ultimatum

<table>
<thead>
<tr>
<th>$x$</th>
<th>Offers</th>
<th>Rejection Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.60$</td>
<td>3</td>
<td>33%</td>
</tr>
<tr>
<td>$4.25$</td>
<td>13</td>
<td>18%</td>
</tr>
<tr>
<td>$5.00$</td>
<td>13</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td></td>
</tr>
</tbody>
</table>

US $10.00$ stake games, round 10
Centipede Game: Palfrey and McKelvey

This game has a unique Nash equilibrium path; in it player 1 with probability 1 plays $T_1$. 

Numbers in square brackets correspond to the observed conditional probabilities of play corresponding to rounds 6-10, stakes 1x below.