Mock Exam Game Theory

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Exercise 1: Where to go on holidays

Filipe (P1) and Goncalo (P2) are debating where to go on holidays. They have three options: either Rome, Spain or Portugal. Their payoff matrix is depicted below. Find all NE of the game.

	R	**S**	**P**
R	0, 0	2, -2	-2, 3
S	-2, 2	0, 0	2, -1
P	3, -1	-1, 2	1, 1

Exercise 2: A traffic light

Marta (P1) and Tuna (P2) are driving through Florence and they meet at an intersection. Unfortunately, the traffic light is broken, and they need to choose what to do: either they go ahead and cross the intersection, or they wait and let the other one go. Find all the symmetric NE. If there was a traffic light, what would be the (correlated) equilibrium? Here the traffic light sends fair "signals/messages" upon which players act.

	Go	**Wait**
Go	-10, -10	1, 0
Wait	0, 1	-1, -1

Exercise 3: Share the cake

Stav, Michael and Richard get as a present for their work as REPS a schiacciata of size 1. They want to share it and they come up with the following mechanism: Stav will divide the schiacciata in three pieces. Michael will be the first to pick one and Richard will go second, so that Stav gets the last piece. They all want as much schiacciata as possible. Can you come up with an equilibrium of the game using backward induction?

Exercise 4: Self-confirming equilibrium

Consider a three person centipede game in which player 1 can drop or pass, player 2 can drop or pass, and player 3 can drop or pass. If player 1 drops, the payoffs are (0,0,0); if player 2 drops the payoffs are (-1,0,0), if player 3 drops the payoffs are (0,-1,0) and if player 3 passes the payoffs are (2,2,2).

Sketch the extensive form and write down the normal form. Find all pure strategy Nash equilibria. Find all pure strategy subgame perfect equilibria. Find all pure strategy self-confirming equilibria.

Exercise 5: Long-run and short run players

Two players must decide whether to be hunters or gathers. If both are hunters, both receive 0; if both are gatherers both receive 1. If one is a hunter and one a gatherer, the hunter receives 3 and the gatherer 2.

Write the normal form of this game. Find all Nash equilibria of this game. Are there any dominated strategies? Find the minmax for both players. Find the pure and mixed Stackelberg equilibrium in which the long-run player, in this case player 1, moves first. What is the best dynamic equilibrium and for what discount factor is it attainable?

Exercise 6: Bar fight

Nature draws Player 1 as a strong type or a weak type. Player 1 is strong with probability 1/3. Player 1 then chooses whether to drink beer or eat quiche. Player 2 observes Player 1's food choice and chooses whether to fight her or not. Player 2's goal is to fight a weak (+1) and avoid a fight with a strong type (-1). Player 1's main goal is to avoid a fight (+2). But the types differ in their food preferences: strong types prefer beer, whereas weaks prefer quiche. They get a payoff of +1 from getting their favourite meal.

Draw the game tree and find all sequential equilibria.

Exercise 7: Unemployment benefits

Consider an economy where a continuum of agents face shocks which (temporarily) prevents them from work. In particular assume that their productivity can take two values: 0 and 1. Also assume, for simplicity, that these two events are i.i.d. with equal probabilities. Agents have utility functions given by $log(c_t)$. Production does not use capital, in particular total production is equal to total hours. The only good in this economy is perishable so all production has to be consumed during the *same period.*

Derive the Pareto optimal allocation (FB). What would happen if agents can claim that they are temporarily unable to work (their productivity is 0) and their claim is not verifiable? Prove it. Interpret the FB above as a (private) unemployment insurance market. Provide some argument why these market arrangements do not exist in the real world.