## Answers to Problem Set 4: Dynamic Game Theory

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## 1. Bayes Law

Let E be the evidence and let H be the event that the husband did it.
$\operatorname{pr}(\mathrm{H})=.8 ; \operatorname{pr}(\mathrm{E} \mid \mathrm{H})=.8 ; \operatorname{pr}(\mathrm{E} \mid \sim \mathrm{H})=.15$
apply Bayes law

$$
\begin{aligned}
\operatorname{pr}(H \mid E) & =\frac{p r(E \mid H) p r(H)}{\operatorname{pr}(E)}=\frac{p r(E \mid H) p r(H)}{\operatorname{pr}(E \mid H) \operatorname{pr}(H)+\operatorname{pr}(E \mid \sim H) \operatorname{pr}(\sim H)} \\
& =\frac{.8 \times .8}{.8 \times .8+.15 \times .20}=.96
\end{aligned}
$$

so a $96 \%$ chance the husband did it. In the second case

$$
\operatorname{pr}(H \mid E)=\frac{.8 \times .8}{.8 \times .8+.05 \times .20}=.98
$$

## 2. Mixed Strategy Equilibrium

a) D and R are strictly dominant strategies, so this is the only Nash equilibrium.
b)

|  | L | R |
| :--- | :--- | :--- |
| U | $3^{*}, 2^{*}$ | 0,0 |
| D | 0,0 | $2^{*}, 3^{*}$ |

Two pure equilibria as marked. To the symmetric mixed equilibrium let $p$ be the probability L. Then for player 1 to be indifferent, player 2 must mix according to $3 p=2(1-p)$ giving $p=2 / 5$ chance of Land a $3 / 5$ chance of R . For player 2 to be indifferent let $q$ be the chance of D ; we find that $q=2 / 5$ as well.
c)

|  | L | R |
| :--- | :--- | :--- |
| U | $4^{*}, 2$ | $3,5^{*}$ |
| D | $2,4^{*}$ | $4^{*}, 2$ |

1. No pure equilibrium. To find the mixed equilibrium, again, let $p$ be probability of L and $q$ be the probability of $D$. Then $4 p+3(1-p)=2 p+4(1-p)$ and $4 q+2(1-q)=2 q+5(1-q)$ so $p=1 / 3$ and $q=3 / 5$
