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## **Cournot and Bertrand**

#### The Cournot Model

- a market with *n* identical firms facing constant marginal cost *c*
- demand given by p = a bx

so that the competitive solution is (a-c)/b units of output and the monopoly solution is (a-c)/2b units of output

let  $\bar{x}$  be output of representative firm profits of a representative firm

$$\pi_i = [a - b(x_i + (n-1)\overline{x})]x_i - cx_i$$

### **Reaction Function**

$$\frac{d\pi_i}{dx_i} = \left[a - b(2x_i + (n-1)\overline{x})\right] - c = 0$$

in a symmetric equilibrium  $x_i = \overline{x}$ , so

$$a-b(n+1)\overline{x}=c$$
 giving the result

$$\overline{x} = \frac{a-c}{b(n+1)}$$
 per firm

$$\bar{x} = \frac{a-c}{b(n+1)}$$
 per firm

industry output 
$$\frac{n}{(n+1)} \frac{a-c}{b}$$

when n = 1 this gives the usual monopoly solution as  $n \to \infty$  this approaches the competitive solution

# **Bertrand Competition**

Firms choose prices rather than quantities

- two facing constant marginal cost c
- demand given by p = a bx

so that the competitive solution is (a-c)/b units of output and the monopoly solution is (a-c)/2b units of output

consumers buy from the lowest price firm: demand for firm *i* 

$$x_{i} = \begin{cases} 0 & p_{i} > p_{-i} \\ \frac{a - p_{i}}{2b} & p_{i} = p_{-i} \\ \frac{a - p_{i}}{b} & p_{i} < p_{-i} \end{cases}$$

Suppose in equilibrium  $p_{-i} > c$  profits are

$$\pi_{i} = \begin{cases} 0 & p_{i} > p_{-i} \\ (p_{i} - c) \frac{a - p_{i}}{2b} & p_{i} = p_{-i} \\ (p_{i} - c) \frac{a - p_{i}}{b} & p_{i} < p_{-i} \end{cases}$$

this problem does not have a solution

as  $p_i \bigwedge p_{-i}$  profits approach

$$(p_i - c) \frac{a - p_i}{b}$$

always undercut by a little bit

if  $p_i = p_{-i} = c$  then we have a Nash equilibrium

Bertrand competition between two firms is competitive

### Bertrand vs. Cournot

- choosing output is a commitment not to produce more
- in Bertrand competition firms will provide whatever amount the market wants

# Bertrand Competition in the Hotelling Model

$$\frac{\text{firm 1}}{0} \frac{\text{firm 2}}{1}$$

- consumers are located on the line between 0 and 1
- firms are located on each edge
- a consumer gets b units of satisfaction from buying 1 unit, minus x where x is the distance traveled to purchase the good, minus the price
- both firms have constant marginal cost c

indifference of a consumer between stores

$$b-x*-p_1 = b-(1-x*)-p_2$$

solving for x gives demand for firm 1 firm 2 demand is 1-x

$$x^* = \frac{1 - p_1 + p_2}{2}$$

#### **Reaction Function**

profit 
$$\pi_1 = (p_1 - c) \frac{1 - p_1 + p_2}{2}$$

differentiate

$$\frac{d\pi_1}{dp_1} = \frac{1 - p_1 + p_2 - p_1 + c}{2} = 0$$

in symmetric equilibrium  $p_1 = p_2$ 

$$p_1 = c + 1$$

this is valid provided

$$b-1/2-c-1 \ge 0$$