## More on Dominance and the Second Price Auction

a single item is to be auctioned.
value to the seller is zero.
Many buyers $i=1, \ldots, N$
value $v_{i}>0$ to buyer $i$.
each buyer submits a bid $b_{i}$
the item is sold to the highest bidder at the second highest bid
suppose the bids are $b_{1}, \ldots b_{N}$
suppose that the second highest bid is $\hat{b}$ and that there are $M$ winning bidders
then a winning bidder gets $\frac{v^{i}-\hat{b}}{M}$ all other players get 0

## Application of Weak Dominance to Second Price Auction

the strategy of bidding $b_{i}=v_{i}$ weakly dominates all other strategies

Calculate utility. Let $\hat{b}$ be the highest bid by the other players.

| Other bid $\hat{b}$ | Your bid $b_{i}$ $v_{i}+x$ |  | $v_{i}-x$ |
| :---: | :---: | :---: | :---: |
| $\hat{b}<v_{i}-x$ | $v^{i}-\hat{b}>0$ | $v^{i}-\hat{b}>0$ | $\nu^{i}-\hat{b}>0$ |
| $\hat{b}=v_{i}-x$ | $v^{i}-\hat{b}>0$ | $v^{i}-\hat{b}>0$ | $\frac{v^{i}-\hat{b}}{M}>0$ |
| $v_{i}>\hat{b}>v_{i}-x$ | $v^{i}-\hat{b}>0$ | $v^{i}-\hat{b}>0$ | 0 |
| $\hat{b}=v_{i}$ | 0 | 0 | 0 |
| $v_{i}+x>\hat{b}>v_{i}$ | $v^{i}-\hat{b}<0$ | 0 | 0 |
| $\hat{b}=v_{i}+x$ | $\frac{v^{i}-\hat{b}}{M}<0$ | 0 | 0 |
| $v_{i}+x<\hat{b}$ | 0 | 0 | 0 |

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