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## Duopoly and the Prisoner's Dilemma

BMG, the giant media corporation, has a rival CBA. Each has produced a movie "The Bird in the Hood" and "The Sparrow in the Hat" to be released on DVD. From the consumer point of view the two movies are the same.

Both media giants can produce and distribute their DVDs at a unit cost of $c \$$

Let $x_{1}$ denote the output of BMG and $x_{2}$ the output of CBA. Inverse demand for software depends upon the output of the two firms

$$
p=f\left(x_{1}+x_{2}\right)
$$

How many DVDs should each firm produce?

## Simplified Duopoly

$p=a-b x$
$a-c=16, b=1$
so that the competitive solution is 16 units of output and the monopoly solution is 8 units of output
restrict the firms to produce either 4 units of output, or 5 units of output
profits $\pi_{i}=\left[a-b\left(x_{1}+x_{2}\right)\right] x_{i}-c x_{i}=\left[16-\left(x_{1}+x_{2}\right)\right] x_{i}$

| CBA Output |  |  |
| :--- | :--- | :--- |
| BMG Output | 4 | 5 |
| 4 | 32,32 | 28,35 |
| 5 | 35,28 | 30,30 |

This is a simultaneous move game

## Prisoner's Dilemma

prisoners dilemma: two prisoners in separate cells accused of jointly committing a crime

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | don't confess | confess |
| don't confess | 32,32 | 28,35 |
| confess | 35,28 | 30,30 |

tragedy of the commons: two fishermen and a single lake

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | light fishing | intense fishing |
| light fishing | 32,32 | 28,35 |
| intense fishing | 35,28 | 30,30 |

public goods: contribute towards building a bridge

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | contribute | do not |
| contribute | 32,32 | 28,35 |
| do not | 35,28 | 30,30 |

## Pareto Dominance, Externalities and Free Riding

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | contribute | do not |
| contribute | 32,32 | 28,35 |
| do not | 35,28 | 30,30 |

- $(32,32)$ pareto dominates $(30,30)$
- all players are at least as well off, and one is strictly better off
- question: does $(32,30)$ pareto dominate $(30,30)$
- does $(32,29)$ ?

Compare $(32,32)$ to $(28,35)$. Here we say that player 2 is being a free rider; player 2 gets the benefit of the bridge, but does not have to pay for it

We also say that by switching from contributing to not contributing, player 2 generates a negative externality for player 1 by reducing player 1's payoff; player 2 does not bear the full cost of his action, part of the loss is suffered by player 1

Pareto efficiency/optimality - an outcome that is NOT pareto dominated by any other outcome

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | contribute | do not |
| contribute | 32,32 | 28,35 |
| do not | 35,28 | 30,30 |

- $(32,32),(35,28),(28,35)$ are pareto efficient
- $(30,30)$ is not
- why?


## Dominant Strategies

|  | Player 2 |  |
| :--- | :--- | :--- |
| Player 1 | contribute | do not |
| contribute | 32,32 | 28,35 |
| do not | 35,28 | 30,30 |

the strategy "do not contribute" is dominant because it gives at least as high a payoff regardless of opponent's play, and sometimes higher [warning: do not confuse the dominance of a strategy with pareto dominance]
weak dominance $=$ dominance
strict dominance a higher payoff no matter what the opponent does

- $(32,28)$ are payoff to contribute
- $(35,30)$ strictly dominates
- How about $(35,28)$ ? $(35,27)$ ?


## Tragedy of the Commons

When both players play their dominant strategies the outcome is $(30,30)$ which is Pareto dominated by $(32,32)$
basic conflict between individual and social objectives

