## Answers to Problem Set 1: Static Game Theory

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## 1. Chicken

|  | lose face | fight |
| :--- | :--- | :--- |
| lose face | 6,6 | $2^{*}, 7^{*}$ |
| fight | $7^{*}, 2^{*}$ | 0,0 |

No strategies are dominated weakly or strictly. The reaction functions are marked in the payoff matrix. There are two Nash equilibria: in each one one player fights and the other loses face.

## 2. First Price Auction

Seagull $=$ row player, VandeCamp $=$ column player

|  | 0 | 500 | 1000 | 10000 | 20000 | 25000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | $10000,500^{*}$ | $0,500^{*}$ | 0,0 | $0,-9000$ | $0^{*},-19000$ | $0^{*},-24000$ |
| 500 | $19500^{*}, 0$ | $9750,250^{*}$ | 0,0 | $0,-9000$ | $0^{*},-19000$ | $0^{*},-24000$ |
| 1000 | $19000,0^{*}$ | $19000^{*}, 0^{*}$ | $9500^{*}, 0^{*}$ | $0,-9000$ | $0^{*},-19000$ | $0^{*},-24000$ |
| 10000 | $10000,0^{*}$ | $10000,0^{*}$ | $10000^{*}, 0^{*}$ | $5000^{*},-4500$ | $0^{*},-19000$ | $0^{*},-24000$ |
| 20000 | $0,0^{*}$ | $0,0^{*}$ | $0,0^{*}$ | $0,0^{*}$ | $0^{*},-9500$ | $0^{*},-24000$ |
| 25000 | $-5000,0^{*}$ | $-5000,0^{*}$ | $-5000,0^{*}$ | $-5000,0^{*}$ | $-5000,0^{*}$ | $-2500,-12000$ |

row player: 25000 is strictly dominated; 0 and 20000 are weakly dominated
column player: -25000 is strictly dominated, 0,20000 , 10000 and 1000 are all weakly dominated
game after elimination of weakly dominated strategies

|  | 500 |
| :--- | :--- |
| 500 | 9750,250 |
| 1000 | 19000,0 |
| 10000 | 10000,0 |

for row player 500 and 10000 are strictly dominated, so we conclude that the column player bids 500, and the row player bids 1000. So the row player wins and gets 19000 .

For best responses correspondences, see the payoff matrix: there are two Nash equilibria, one is the same solution derived by iterated weak dominance, the other is both players bid 1000 , and each has a $50 \%$ chance of winning. The column player gets 0 , the row player a $50 \%$ chance of 19000 .

## 3. Duopoly

profits are

$$
\begin{aligned}
& \pi_{i}=a+(b-c) x_{i}-e\left(x_{i}\right)^{2}-f x_{i} x_{-i} \\
& \frac{d \pi_{i}}{d x_{i}}=(b-c)-2 e x_{i}-f x_{-i}=0
\end{aligned}
$$

in the symmetric equilibrium $x_{i}=x_{-i}$

$$
x_{i}=\frac{b-c}{2 e+f}
$$

As $f$ increases the equilibrium level of film violence goes down.

