Answers to Problem Set 3: Dynamic Game Theory

February 20, 1997 © David K. Levine

1. The Folk Theorem

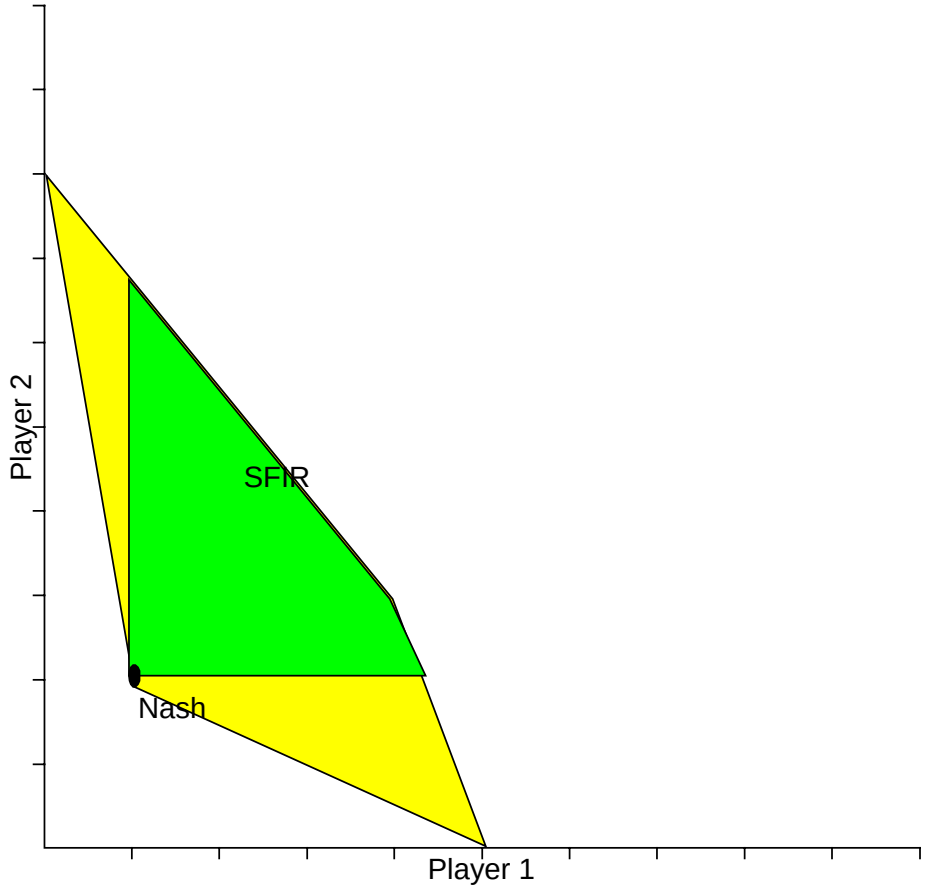
a)

	L	R
U	4,3	0,7*
D	5*,0	1*,2*

Dominant strategies so no mixed equilibrium

Minmax for 1 is 1 by playing D

Minmax for 2 is 2 by playing R

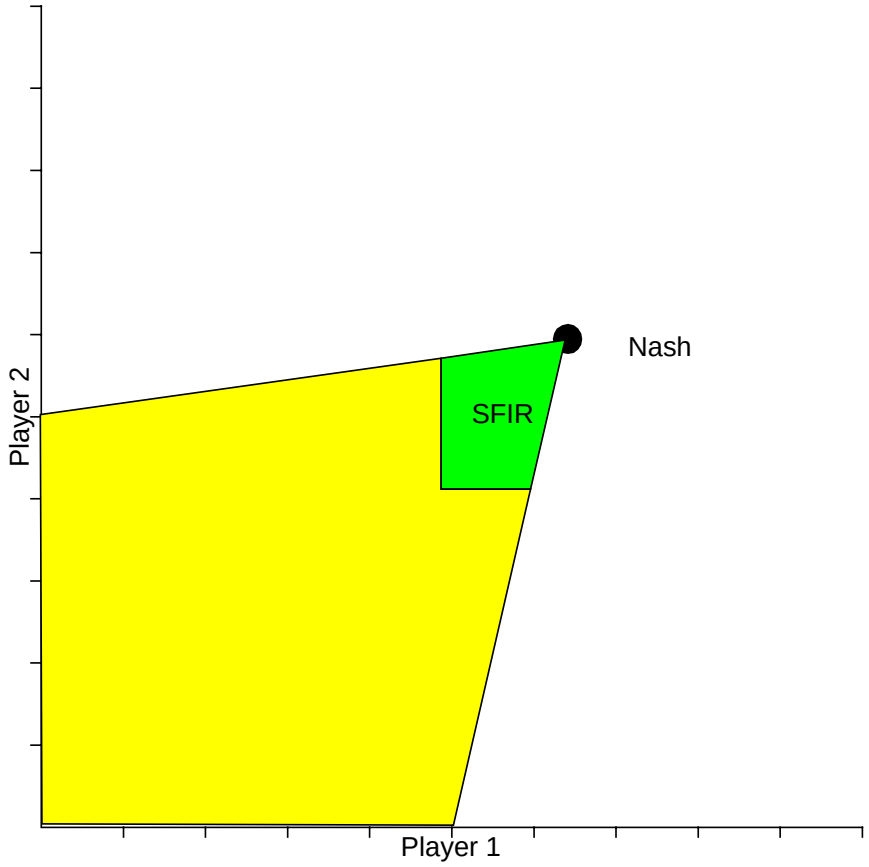


b)

	L	R
U	6*,6*	5*,0
D	0,5*	0,0

Dominant strategy so no mixed equilibrium

Minmax for both players is 0



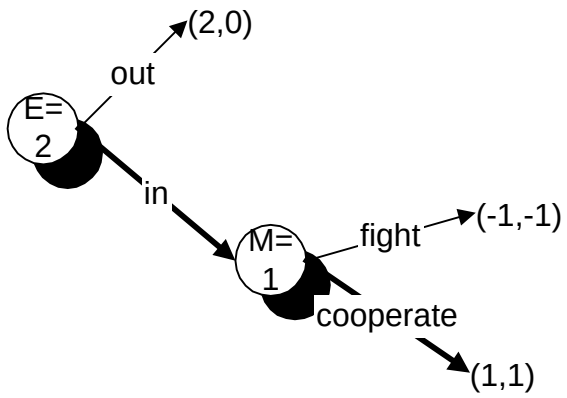
2. Equilibrium in a Repeated Game

	U	D
U	1,1	-1,100
D	100,-1	0,0

If you play U against grim always you get an average present value of 1

If you play D against grim you get $(1-\delta)100$ in the first period and 0 (or –1) in every subsequent period. So it must be that $1 \geq (1-\delta)100$ or $\delta \geq .99$.

3. Long Run versus Short Run



subgame perfect equilibrium as marked

	out	in
fight	2*,0*	-1,-1
cooperate	2*,0	1*,1*

Out/fight is Nash, but isn't plausible because the incumbent wouldn't really fight.

Enter/cooperate is subgame perfect in the infinitely repeated game because it is subgame perfect in the stage game.

For the "out" equilibrium in the repeated game, note that after a failure to fight, the equilibrium is the subgame perfect "enter/cooperate" equilibrium. We must find the value of δ for which it is actually optimal for the incumbent to fight if there is entry. (Obviously if he does so, the entrants won't wish to enter.) That is

$$(1-\delta)(-1)+\delta 2 \geq 1$$
$$3\delta \geq 2$$
$$\delta \geq 2/3$$

Unlike the non-perfect equilibrium of the stage game, this makes sense, since the incumbent is actually willing to fight, when the penalty is entry forever afterwards when he does not.