1. **Principal Agent**

a)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay</td>
<td>0,0</td>
<td>1,1</td>
</tr>
<tr>
<td>Not Pay</td>
<td>0,0 [unique pure Nash]</td>
<td>5,-1</td>
</tr>
</tbody>
</table>

b) 1 can mix up to 50% on pay, 2 chooses 0
c) pay is weakly dominated for 1
d) pure is (pay,1); mixed is (50-50,1)
e) minmax for player 1 is 0; minmax for player 2 is 0
f) worst equilibrium is 0; best equilibrium is 1 provided
   \[(1 - \delta)4 - \delta 1 \leq 0\]
   \[4/5 \leq \delta\]
g) many answers: most obvious – play pay, 1 unless there has been a deviation, then switch to 0
h) convex hull of 0,0; 1,1; 5,-1 above the point 0,0

2. **Auto Repair**

a) normal form

<table>
<thead>
<tr>
<th></th>
<th>Repair</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repair</td>
<td>[p - c, \theta v - p]</td>
<td>0, [\pi v]</td>
</tr>
<tr>
<td>Not</td>
<td>[p, \pi v - p]</td>
<td>0, [\pi v [unique pure Nash]]</td>
</tr>
</tbody>
</table>

b) some mixed Nash but all yield payoff 0; minmax for 1 is 0; pure precommitment is repair \( p - c \); mixed precommitment mix so that 2 is indifferent
\[
\alpha \theta v + (1 - \alpha)\pi v = \pi v + p
\]
\[
\alpha = \frac{p}{(\theta - \pi) v}
\]
so payoff is \( p - \alpha c \).
c) worst is obviously 0
d) \[
v = (1 - \delta)(p - c) + \delta(\theta v + (1 - \theta)w(n))
\]
\[
v = (1 - \delta)p + \delta(\pi v + (1 - \pi)w(n))
\]
\[ v = (1 - \delta)(p - c) + \delta(\theta v + (1 - \theta)w(n)) \]
\[ \frac{(1 - \delta \pi)}{(1 - \pi)} v - \frac{(1 - \delta)}{(1 - \pi)} p = \delta w(n) \]

\[ v = (1 - \delta)(p - c) + \delta \theta v + (1 - \theta) \left[ \frac{(1 - \delta \pi)}{(1 - \pi)} v - \frac{(1 - \delta)}{(1 - \pi)} p \right] \]
\[ v \left[ 1 - \frac{1 - \theta + \delta \theta - \delta \pi}{(1 - \pi)} \right] = (1 - \delta)(p - c) - (1 - \theta) \frac{(1 - \delta)}{(1 - \pi)} p \]
\[ v(\theta - \pi) = (1 - \pi)(p - c) - (1 - \theta)p = (\theta - \pi)p - (1 - \pi)c \]
\[ v = p - \frac{1 - \pi}{\theta - \pi} c \]

\[ w(n) = \frac{(1 - \delta \pi)}{\delta (1 - \pi)} v - \frac{(1 - \delta)}{\delta (1 - \pi)} p \]
\[ = \frac{(1 - \delta \pi)}{\delta (1 - \pi)} \left[ p - \frac{1 - \pi}{\theta - \pi} c \right] - \frac{(1 - \delta)}{\delta (1 - \pi)} p \]
\[ = p - \frac{(1 - \delta \pi)}{\delta (\theta - \pi)} c \geq 0 \]

\[ \delta \geq \frac{c}{p(\theta - \pi) + \pi c} < 1 \]
\[ (\theta - \pi)p - c > -\pi c \]

3. **Auction**

a) dominant strategy to announce truthfully; \( \frac{3}{4} \) chance one person has low value and get revenue $8; \frac{1}{4} \) chance both have high value and get revenue $12, so expected revenue is 9.

b) this is a 2x2 symmetric game; strategy is what to bid when type is $12

calculate payoff to $12 type only, since only that type has a choice

both bid $12
\( \frac{1}{2} \) opponent bids 12 and is 12: you win \( \frac{1}{2} \) and get 0; lose \( \frac{1}{2} \) and get 6, expected value 3
\( \frac{1}{2} \) opponent bids 8 and is 8; you win always and get 4, expected value 4

you bid 8, he bids 12
\( \frac{1}{2} \) opponent bids 12 and is 12; you lose and get 6; expected value 6
\( \frac{1}{2} \) opponent bids 8 and is 8; you win \( \frac{1}{2} \) and get 4; lose \( \frac{1}{2} \) and get 4; expected value 4

you bid 12 he bids 8
you always win and get 4

both bid $8
\( \frac{1}{4} \) chance win versus 12, getting 4
\( \frac{1}{4} \) chance lose versus 12, getting 6
\( \frac{1}{4} \) chance win versus 8, getting 4
¼ chance lose versus 8, getting 4

<table>
<thead>
<tr>
<th></th>
<th>$12</th>
<th>$8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12</td>
<td>3.5,3.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$8</td>
<td>5.4</td>
<td>4.25, 4.25</td>
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</tbody>
</table>

So bidding 8 is dominant strategy equilibrium, and expected revenue to the seller is $8.

4) Mechanism Design

\[
\frac{1}{3}[u(H,H;H) + u(L,H;L) + u(H,L;H)] + \frac{1}{3}[t(H,H) + t(L,H) + t(H,L)] = \\
(x - 1)/3 + \frac{1}{3}[t(H,H) + t(L,H) + t(H,L)] \geq \\
\frac{1}{3}[u(H,H;H) + u(L,H;H) + u(H,L;H)] + \frac{1}{3}[t(H,H) + t(L,H) + t(H,L)] = \\
x/3 + \frac{1}{3}[\alpha(H,H) + t(H,L)] \\
(x - 1)/3 + \frac{1}{3}[t(H,H) + t(L,H) + t(H,L)] \geq \\
x/3 + \frac{1}{3}[\alpha(H,H) + t(H,L)]
\]

\[t(L,H) - t(H,H) \geq 1\]

\[t(H,H) = 0 \Rightarrow t(L,H) = 1\] and then choosing \(t(H,L) = -1\) give the other constraint

these constraints can be interpreted as “budget balance” when there are two players.