1. Information and Reputation

A long-run player 1 against a short-run player 2 plays a coordination game with payoff matrix

$$\begin{bmatrix} 1,1 & 0,0 \\ 0,0 & 1,1 \end{bmatrix}$$

a) Find the two pure and one mixed Nash equilibrium, and the pure and mixed Stackelberg equilibrium.

b) Find the set of equilibrium payoffs for the long-run player as a function of his discount factor.

c) How does this set change if there is a positive probability of a type of long-run player who always plays “up”?

d) Suppose that the player 2’s are a sequence of different players and that each player 2 does not get to directly observer the previous play of either player 1 ***or the earlier player 2’s*** but instead observes only a signal “H” or “T” of the outcome of each earlier match.. The probability of “H” depends on the actual play of both players and is given by the matrix

$$\begin{bmatrix} .75 & .25 \\ .25 & .75 \end{bmatrix}$$

How does your answer to part c) above change?

e) Continuing to consider the reputational case (positive probability of long-run player who always plays “up”), and imperfect observability with only “H” and “T” observed with probabilities given above, suppose that the payoff matrix is

$$\begin{bmatrix} 2,2 & 0,0 \\ 0,0 & 1,1 \end{bmatrix}.$$

Be sure to distinguish between what you could actually prove based upon known theorems and what you suspect to be true based on intuition.

2. Long Run vs. Short Run with Moral Hazard

Consider a game between a manager and an employee. The manager must decide whether or not to provide an effort, and the employee must decide whether or not to work for the manager. The manager is patient, the employee is not. The manager’s effort is not observed by the employee, but the employee’s decision whether or not to work is observed by everyone. If the employee decides not to work for the manager, both players get zero. If the employee does work, the manager and employee each receive the amount of profit given below, with probabilities depending on the managers effort.

probabilities	profit=0	profit=6	profit=12
effort=0	2/3	1/6	1/6
effort=1	0	1/2	1/2

The manager deducts 5 from his profit if he makes an effort, the employee always deducts 5 from his profit: since he is supervised he must make an effort.

(a) find the normal form of the game and the pure strategy Stackelberg payoff to the manager

(b) what is the worst equilibrium of the repeated game?

(c) what is the best equilibrium of the repeated game as $\delta \to 1$ and for what discount factors is this an equilibrium?

3. Learning

Consider the Jordan three-person simultaneous move matching pennies game. Player 1 gets 1 if he matches player 2 and zero if he does not; player 2 gets 1 if he matches player 3 and zero if he does not, while player 3 gets 1 if he ***does not match*** player 1 and zero if he ***does***. Suppose that this game is played by randomly choosing three player from each of a large population to play the game each period, with results observed by all players in all populations. Suppose that all players use fictitious play based upon their observations of the past play in all matches.

a) Describe how play evolves over time.

b) Is an individual player better off using fictitious play or best-responding to his opponents play last period? What is your criterion for "better" (discounting, time-averaging, etc)?