Mathematical Economics Comprehensive Exam
Spring 1997

You have 4 hours plus an extra ½ hour if English is not your native language.

Do 4 of the 8 questions.

Good luck.

1. LP and WE

In a quasilinear model of an exchange economy let V be a collection of functions $v : \mathcal{Z}^\ell \to [\infty, \infty)$ where $\mathcal{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$; hence (trades in) non-money commodities are indivisible. Define the economy by $\mathcal{E} \in M[V]$ (where $M[X]$ is a positive measure on a Borel σ-algebra of subsets of X).

(a) Letting $\psi(v, z) = v(z)$ and $\mu \in M[V \times \mathcal{Z}^\ell]$, describe the linear programming problem obtained by maximizing $\langle \psi, \mu \rangle$ subject to the constraint that μ is a feasible allocation for $\mathcal{E}$ and describe the resulting dual.

(b) Show that a solution to the LP problem defined by $\mathcal{E}$ and its dual is equivalent to a Walrasian equilibrium for $\mathcal{E}$.

(c) Suppose $\mathcal{E} \sim (1, 1, \ldots, 1) \in \mathbf{R}^n$. What is the distinction in terms of the restrictions on the feasible μ between the interpretation of $\mathcal{E}$ as (i) a finite economy having one individual each of n different types and (ii) a continuum economy with mass one of each of the n types?

(d) Show that if a WE exists for the finite economy interpretation of $\mathcal{E}$, then a WE exists for the continuum interpretation?

(e) Explain why the converse of (d) does not hold and if time permits give an example. Also, give conditions under which the converse does hold.

2. Equilibrium through arbitrage

Suppose a continuum economy defined by a finite number of types $v = (\succeq, \omega)$, where α_v is the mass of v. Let $t = (\succeq, \omega, x) = (v, x) \in T$ denote the preferences, initial allocation and final allocation of an individual, where the relevant commodity space is $\mathbf{R}^\ell_+$ and preferences are monotonic and convex. Define $S(t) = \{z : x - z \succeq x\}$ to be the set of changes to the allocation x — as seen by the outsider making the changes — that would be at least as desirable for t as no change.

(a) As a measure on T, define a feasible allocation μ for the continuum economy.

(b) If μ is a measure with finite support, describe the set of changes (trading opportunities) available to an arbitrager who can avail himself of each $S(t)$ in the support of μ. Call this set $K(\mu)$.

(c) After defining the condition on $K(\mu)$ eliminating arbitrage opportunities, i.e., opportunities for the arbitrager to obtain something for nothing, show that if μ eliminates arbitrage opportunities, then there exists a non-null price vector p such that $K(\mu)$ lies on one side of the hyperplane through the origin defined by p.

(d) Define an arbitrage-free equilibrium as a μ such that for each $t = (\succeq, \omega, x)$ in the support of μ, if $z \in K(\mu)$, then $x \succeq \omega + z$. Under what conditions is an arbitrage-free equilibrium a Walrasian equilibrium?

(e) Briefly explain why the implications of arbitrage are so much weaker in a model with a finite number of individuals than in a continuum economy.

3. Job Search

An individual is qualified for two types of jobs A and B. All firms pay $40 per period for a type A job and $60 per period for a type B job. While unemployed the worker receives nothing and may apply for either type of job (but not both at the same time). An able worker has a 2/3 chance of getting an A job if she applies for it, and a 1/3 chance of the B job. A less able worker has a 1/2 at the A job and a 1/10 chance at the B job. For what discount factors δ should the more and less able worker apply for the A job? What are the expected unemployment durations?

4. Refinements of Equilibrium

In the Kreps Beer/Quiche game, a stranger walks into a bar. He may be one of two types: a wimp or tough. Wimps like to eat quiche (utility +1) and tough guys like to drink beer (utility +1). In the bar is a redneck who likes to fight with wimps (utility +1) but who prefers not to fight with tough guys (utility –1). Neither the wimp nor the tough guy like to fight (utility –1). Suppose that there is a 45% chance of tough

(a) Find the extensive and normal forms of this game.
(b) Find all pure strategy Nash equilibria.
Which of these pure strategy Nash equilibria
(c) survive the iterated removal of weakly dominated strategies?
(d) are subgame perfect equilibria?
(e) are sequential?

(f) is there a heterogeneous self-confirming equilibrium that has a non-Nash equilibrium path?

5. Repeated Games

Every year a peasant must decide whether to plant his corn or eat it. If he eats the corn he gets 1 unit of utility and no one else gets anything. If he plants the corn there will be three units of corn. What happens next depends on the King. If the King instructs his Sheriffs to be tough, the peasant gets nothing and the King gets 3. If the King instructs his Sheriffs to be kind there are two possibilities: with probability p the Sheriffs are kind, and the peasant gets 2 and the King 1. With probability 1-p the Sheriffs are corrupt and take all 3 units of corn, giving only 1 to the King. The peasant does not see whether or not the Sheriffs are corrupt, only what he has to pay them.

(a) Find the extensive and normal forms of this game.

(b) Find the static Nash and the pure and mixed strategy Stackelberg equilibria of the game played once.

(c) If the game is repeated describe the set of public perfect equilibrium payoffs as a function of the discount factor.

(d) What would happen in a reputational model with “types” of King player? Which types of King would make a difference here?

(e) How would your answer differ if there was only one peasant who is as patient as the King?

6. Learning in Games

Consider the Jordan three-person simultaneous move matching pennies game. Player 1 gets 1 if he matches player 2 and zero if he does not; player 2 gets 1 if he matches player 3 and zero if he does not, while player 3 gets 1 if he ***does not match*** player 1 and zero if he ***does***. Suppose that this game is played by randomly choosing three players from each of a large population to play the game each period, with results observed by all players in all populations. Suppose that all players use fictitious play based upon their observations of the past play in all matches.

a) Characterize the evolution of play over time.

b) Is an individual player better off using fictitious play or best-responding to his opponents play last period? What is your criterion for “better” (discounting, time-averaging, etc)?

c) Show that play is “calibrated” in the sense that in those periods in which he chose H it was a best-response for each player, and similarly in those period when he chose T.