3. Information in a Long-Run Short-Run Player Game

Mr. Investor has one million dollars. He may consume it resulting in a utility of 1 million for himself and nothing for Mr. Manager, or he may give it to Mr. Manager to operate a flower business. With probability 90% the flower business is a great success and earns a profit of ten million dollars. With probability 10% it is a huge flop and the investment is lost completely (a profit of zero). If the flower business is a flop both players receive zero. If it is a success, Mr. Manager must decide whether to keep all the profit for himself, or whether to keep a management fee of one million dollars. Assume first that Mr. Investor can tell whether the business is a flop.

a) Find the extensive and normal forms of the game, and all Nash and subgame perfect equilibria.

Normal form:

	Keep money	invest
Keep profit	0,0	9,-1
Share profit	0,0	1.8,6.2

b) If Mr. Manager is a long-run player with discount factor δ facing a series of short-run investors, what is his set of perfect public equilibrium payoffs?

Static Nash = minmax = 0 is worst equilibrium

Best long-run equilibrium is pure stackelberg (no benefit here from randomization) = 1.8

The discount factor needed for these, supposes that the gain of cheating $(1-\delta)7.2$ equals the long term cost $\delta 1.8$ for which we need $\delta \geq 7.2/9.0 = 4/5$.

Now suppose that Mr. Investor cannot tell whether the business is a success or failure, but learns only whether or not he gets paid (7 million).

c) If Mr. Manager is a long-run player with discount factor δ facing a series of short-run investors, what is his set of perfect public equilibrium payoffs?

Worst equilibrium obviously doesn't change. Best equilibrium

$\bar{v} = (1-\delta)1.8 + \delta(.9\bar{v} + .1w)$

$\bar{v} = (1-\delta)9.0 + \delta w$

solve to find

$\bar{v} = (1-\delta)1.8 + (\delta .9\bar{v} + .1(\bar{v} - (1-\delta)9.0))$
$.9(1-\delta)\bar{v} = (1-\delta)(1.8 - .9)$
$\bar{v} = 1.0$
$w = \frac{1}{\delta}(\bar{v} - (1-\delta)9.0) = \frac{1}{\delta}(1.0 - (1-\delta)9.0)$
$w = 0$
$1-\delta = 1/9$
$\delta = 8/9$

so that the best equilibrium give only 1.0, and a discount factor of at least 8/9 is needed

d) Explain how a reputational model would work in this setting.

Strictly speaking, reputation doesn't help here, since if the money is kept, the outcome of the project isn't known. If the outcome could be learned after the fact, even if the investment isn't made (for example, whether a particular market turns out to be a big one or not), a reputational model with mixed strategy types enables the long-run player to essentially build a reputation for mixing giving the mixed Stackelberg payoff. Mixing that makes short-run indifferent is
$1.0x = 6.2(1-x)$
$x = 6.2/7.2 = 31/36$

This give long-run a payoff of $9.0 \times 31/36 + 1.8 \times 5/36 = 8.0$

4. Evolution and Learning

There are three players. Each chooses one of two actions: nice (n) or mean (m). Each player cares only about his own action and that take by one opponent. Player 1 cares about the action taken by player 2; player 2 about the action taken by player 3 and player 3 about the action taken by player 1. Payoffs are the sum of the cost of your own action plus the benefit of the opponent action. Nice costs 1; mean costs 0; if the opponent is nice you receive 5; if your opponent is mean you receive 0. Players may only play one of two strategies: (1) always be mean; (2) be mean if the player you don't care about was mean last period and be nice if the player you don't care about was nice last period.

a) Show that all players playing (1) is an equilibrium.

If you deviate you get a cost of 1.

b) Find a discount factor such that all players playing (2) is an equilibrium.

Cost of $(1-\delta)1$ today versus $\delta 5$ forever, so $\delta \geq 1/6$

Fix the discount factor as in part (b). Consider the following Markov process: the state are the strategies used by all three players. Player 1 can change strategies only in periods 1, 4, etc.; player 2 in periods 2, 5 etc. and player 3 in periods 3, 6 etc. In a period where a player can move, with probability $1-\varepsilon$ he believes his opponents will continue to play the same strategy forever, and he chooses whichever of (1) or (2) is a best response. With probability ε he plays the opposite of the best response.

c) What is the relative frequency of the two equilibria in the evolutionary model?

If only one player is playing (2) then the player behind him will want to play (2) and no one else. So there is a steady state with one player playing (2) but it rotates around the circle. Similarly, with one player playing (1) it is a steady state cycle that rotates around the circle. So the quickest way to move from all (1) to all (2) (or vice versa) is by passing through the intermediate steady states. This takes three non-simultaneous mutations. Since the number of mutations in each direction is equal, both steady states are equally frequent. However, the best response cycles are both reachable with at most two mutations from anywhere, so they are equally likely and about $1/\varepsilon$ more likely than the steady states. So in the very long-run we should primarily see the two best response cycles.

d) What happens to the system in the very long-run?

As indicated above, the system rotates between the two best response cycles.