The Folk Theorem

Review of Long Run vs. Short run

- $\max u^1(a)$
- mixed precommitment/Stackelberg
- $\bar{v}^1$ best dynamic equilibrium
- best dynamic equilibrium w/ moral hazard
- pure precommitment/Stackelberg

Set of dynamic equilibria

- static Nash
- worst dynamic equilibrium w/ moral hazard
- $v^1$ worst dynamic equilibrium

- $\minmax$

- $\min u^1(a)$

- structure of an equilibrium
- role of reputation (can do strictly better when there is moral hazard)
**Simple Folk Theorems**

- socially feasible
- individually rational

Statement of Folk Theorem

Prisoner’s Dilemma Game

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>U</strong></td>
<td>2,2</td>
<td>0,3</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>3,0</td>
<td>1,1</td>
</tr>
</tbody>
</table>

- Nash with time averaging
- perfect Nash threats with discounting

Public randomization vs. discount factors near one

\[ v_t = (1 - \delta)u_t + \delta v_{t+1} \]
\[ v_{t+1} = \delta^{-1}v_t - (1 - \delta)\delta^{-1}u_t \]

note that coefficient add up to one
Fudenberg Maskin Theorem

issue: perfection and minmaxing

minmax followed by reversion to another equilibrium
note simultaneous determination of equilibria
Matching and Information Systems (Kandori)

\[ u^i(a) \]

\( I \) a finite set of information states
\( \eta: A \times I^2 \rightarrow I \) an information system
if at \( t \) you and your opponent played \( a_t \) and had
states \( \eta^i_t, \eta^{-i}_t \), then your next state is
\( \eta^i_{t+1} = \eta(a_t, \eta^i_t, \eta^{-i}_t) \)

players randomly matched in a population
observe their current opponents current state

Ellison: even without information states could have
cooperation due to contagion effects

But contagion effects diminish as population size
grows

Folk Theorem for information systems: socially
feasible individually rational payoff – exists an
information system that supports it
Example

Prisoner’s dilemma

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>$x, x$</td>
<td>$0, x + 1$</td>
</tr>
<tr>
<td>D</td>
<td>$x + 1, 0$</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

$I = \{r, g\}$

$$\eta(a^i, \eta^{-i}) = \begin{cases} 
G & (a^i, \eta^{-i}) = C, G \\
R & (a^i, \eta^{-i}) = C, R \\
R & (a^i, \eta^{-i}) = D, G \\
G & (a^i, \eta^{-i}) = D, R 
\end{cases}$$
"green team strategy"
defect on red
cooperate on green

\[ V(g) = x \]
\[ V(r) = \delta x \]

C \( (1 - \delta)x + \delta V(g) = x \)

D \( (1 - \delta)(x + 1) + \delta V(r) = (1 - \delta)(x + 1) + \delta^2 x = \)
\( (1 - \delta) + (1 - \delta + \delta^2)x \)

\[ x \geq (1 - \delta) + (1 - \delta + \delta^2)x \]

So \( \delta(1 - \delta)x \geq (1 - \delta) \)
\[ \delta \geq \frac{1}{x} \]

Remark: this method works also with finitely-lived individuals, although naturally towards the end of their life, you may not be able to punish them very much