1. Bellman’s equation

\[ v_{\text{bankrupt}} = 0 \]

\[ v_{\text{wealthy}} = \max \left\{ (1 - \delta) + \delta v_{\text{wealthy}} \right\} \]

if max is bond then \( v_{\text{wealthy}} = 1 \)
if max is stock then \( v_{\text{wealthy}} = 2(1 - \delta) + (1 - p)\delta v_{\text{wealthy}} \)

Solve second equation for \( v_{\text{wealthy}} \) to find

\[ v_{\text{wealthy}} = \frac{2(1 - \delta)}{1 - \delta(1 - p)} \]

Stocks better for \( \frac{2(1 - \delta)}{1 - \delta(1 - p)} \geq 1 \) or rewrite as \( 1 - \delta \geq \delta p \)

2. a)
(c) since minmax = static nash=0 this is also the worst equilibrium; the set of equilibrium payoffs is the line segment from 0 to \( v \)

(d) best for lr is to have giving; requires at least a .5 chance of paying; if lr pays and sr gives then lr receives 3, so \( v = 3 \);
also from incentive constraint \( v \geq (1 - \delta)5 + \delta 0 \), so \( 3 \geq (1 - \delta)5, \delta \geq 2 / 5 \)

(e) incentive constraints
\[
\bar{v} = (1 - \delta)3 + \delta (5w(p) + .5w(n))
\]
\[
\bar{v} \geq (1 - \delta)5 + \delta w(n)
\]
maximization of \( \bar{v} \) requires that second hold with equality and that \( w(p) = \bar{v} \);
solving yields \( \bar{v} = 1 \); \( w(n) = \frac{1 - (1 - \delta)5}{\delta} \leq 1, \delta \geq 4 / 5 \)

3)

<table>
<thead>
<tr>
<th>2*,2*</th>
<th>1,0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

a) Static nash is 2,2; also the unique pareto efficient point
Minmax is 1,1
b)

c) bot for k periods, then top forever, provided no deviation; if deviation, start over again. Utility is \( 2\delta^k \)
\[
2\delta^k = 1.5
\]
\[
\delta^k = 3 / 4
\]
if deviate in initial period get \( (1 - \delta) + 2\delta^{k+1} \).
condition for equilibrium is
\[2\delta^i \geq (1 - \delta) + 2\delta^{i+1}\]

\[0 \geq (1 - \delta) + 2\delta(3/4) - 2(3/4)\]

\[= 1 - \delta + 3\delta / 2 - 3/2 = \delta / 2 - 1/2\]

so this works for any \(\delta, k\) combination with \(\delta^i = 3/4\)

d) pick \(\delta, k\) as above. \(\eta \in I = (0, 1, 2, \ldots, k)\). If you both have flag 0 play top; if either has flag \(\eta > 0\) play bot. If you both have flag 0 and you play top you get flag \(\max\{\eta - 1, 0\}\). If you play bot you get flag \(k\). If either has flag \(\eta > 0\) and you play top you get flag \(k\); if you play bot you get flag \(\max\{\eta - 1, 0\}\). Everyone starts with flag \(k\).