Copyright (C) 2001 David K. Levine

This document is an open textbook; you can redistribute it and/or modify it under the terms of version 1 of the open text license amendment to version 2 of the GNU General Public License. The open text license amendment is published by Michele Boldrin et al at http://levine.sscnet.ucla.edu/general/gpl.htm; the GPL is published by the Free Software Foundation at http://www.gnu.org/copyleft/gpl.html.

Cournot and Bertrand

The Cournot Model

- a market with *n* identical firms facing constant marginal cost *c*
- demand given by p = a bx

so that the competitive solution is (a - c) / b units of output and the monopoly solution is (a - c) / 2b units of output

let \overline{x} be output of representative firm profits of a representative firm $\pi_i = [a - b(x_i + (n - 1)\overline{x})]x_i - cx_i$

Reaction Function

$$\frac{d\pi_i}{dx_i} = [a - b(2x_i + (n-1)\overline{x})] - c = 0$$

in a symmetric equilibrium $x_i = \overline{x}$, so

 $a - b(n+1)\overline{x} = c$ giving the result

$$\overline{x} = rac{a-c}{b(n+1)}$$
 per firm

$$\overline{x} = rac{a-c}{b(n+1)}$$
 per firm
industry output $rac{n}{(n+1)} rac{a-c}{b}$

when n = 1 this gives the usual monopoly solution as $n \to \infty$ this approaches the competitive solution

Bertrand Competition

Firms choose prices rather than quantities

- two facing constant marginal cost *c*
- demand given by p = a bx

so that the competitive solution is (a - c) / b units of output and the monopoly solution is (a - c) / 2b units of output

consumers buy from the lowest price firm: demand for firm *i*

$$x_{i} = \begin{cases} 0 & p_{i} > p_{-i} \\ \frac{a - p_{i}}{2b} & p_{i} = p_{-i} \\ \frac{a - p_{i}}{b} & p_{i} < p_{-i} \end{cases}$$

Suppose in equilibrium $p_{-i} > c$ profits are

$$\pi_{i} = \begin{cases} 0 & p_{i} > p_{-i} \\ (p_{i} - c) \frac{a - p_{i}}{2b} & p_{i} = p_{-i} \\ (p_{i} - c) \frac{a - p_{i}}{b} & p_{i} < p_{-i} \end{cases}$$

this problem does not have a solution

as $p_i \uparrow p_{-i}$ profits approach

$$(p_i-c)\frac{a-p_i}{b}$$

• always undercut by a little bit

if $p_i = p_{-i} = c$ then we have a Nash equilibrium

Bertrand competition between two firms is competitive

Bertrand vs. Cournot

- choosing output is a commitment not to produce more
- in Bertrand competition firms will provide whatever amount the market wants

Bertrand Competition in the Hotelling Model

$$\begin{array}{c} \text{firm 1} & \text{firm 2} \\ \hline 0 & 1 \end{array}$$

- consumers are located on the line between 0 and 1
- firms are located on each edge
- a consumer gets b units of satisfaction from buying 1 unit, minus x where x is the distance traveled to purchase the good, minus the price
- both firms have constant marginal cost c

indifference of a consumer between stores

$$b - x^* - p_1 = b - (1 - x^*) - p_2$$

solving for x gives demand for firm 1

firm 2 demand is 1 - x

$$x^* = rac{1 - p_1 + p_2}{2}$$

Reaction Function

profit
$$\pi_1 = (p_1 - c) \frac{1 - p_1 + p_2}{2}$$

differentiate

$$rac{d\pi_1}{dp_1} = rac{1-p_1+p_2-p_1+c}{2} = 0$$

in symmetric equilibrium $p_1 = p_2$

 $p_1 = c + 1$

this is valid provided

 $b-1/2-c-1\geq 0$