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## Dominance and the Second Price Auction

- A central question in economics: how are prices set.
- In monopoly the question is how much money can the monopolist extract from buyers?
- A common method of price setting is to sell items by means of an auction.


## Types of Auctions

- English auction-announced bids, sold to highest bidder at the price bid (oral, first-price)
- Sealed bid (first-price)
- Descending bid
- Sealed bid second price - each buyer submits a single bid at the same time, sold to highest bidder at the second highest bid.
- Sealed bid second price = English auction - why?


## A Simple Sealed Bid Second Price Auction Model

a single item is to be auctioned.
value to the seller is zero.
two buyers
value $v_{i}>0$ to buyer $i$
possible values 2 or 4
"independent private values"
(compare: common value auction - oil field)
each buyer submits a bid $b_{i}$ equal to one of the possible values
the item is sold to the highest bidder at the second highest bid

## Solution

suppose that the second highest bid is $\hat{b}$ and that there are $M(=1,2$ obviously) winning bidders
then a winning bidder gets $\frac{v^{i}-\hat{b}}{M}$
all other players get 0

## Dominance

weak dominance never a lower payoff no matter what the opponent does, and sometimes a higher payoff
strict dominance a higher payoff no matter what the opponent does admissibility: never use a weakly dominated strategy

## Application of Weak Dominance to Second Price Auction

the strategy of bidding $b_{i}=v_{i}$ weakly dominates all other strategies
Calculate utility. Let $\hat{b}$ be the bid by the other player.

| Your value $=2$ | Bid 2 | Bid 4 |
| :--- | :--- | :--- |
| $\hat{b}=2$ | 0 | 0 |
| $\hat{b}=4$ | 0 | -1 |


| Your value $=4$ | Bid 2 | Bid 4 |
| :--- | :--- | :--- |
| $\hat{b}=2$ | 1 | 2 |
| $\hat{b}=4$ | 0 | 0 |

## Theory of Second Price Auctions

The highest valued buyer wins the auction and pays the second highest value.

- What happens with many possible values? Bids? More bidders? [discussed in section]
- What happens in a first price auction?
- Can the seller design an auction that gathers more revenue?

If the seller knows the buyers values, then he should just charge the highest value (minus a penny, perhaps): this yields more revenue

- What happens when the seller does not know in advance what the buyer values will be.

Theory of choice under uncertainty, to be discussed later in the course

## BDM Mechanism

This ticket is worth $\$ 2.00$ to you.
You can sell it.
Name your offer price.
A price will be posted shortly
The posted price was drawn randomly between:

## [\$0 and \$ 6 ]

If your offer price is below the posted price then you sell your ticket at the posted price.

If your offer price is above the posted price then you do not sell your ticket but you do collect the $\$ 2.00$ value of the ticket.

You can view the posted price after you have named your price.

## Price Posting



## The Becker DeGroot Marschak (BDM) Mechanism

- Willingness to pay versus
- Willingness to accept

Buy low sell high?
Widely used, for example in public goods surveys: how much would you pay to avoid polution; how much would we need to pay you

Endowment effect?

