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Expected Utility Theory

Let Ω be a probability space

A gamble is a random variable where the quantity represents “money” or “consumption”

Suppose that x_1 and x_2 are “gambles”

Which gamble is prefered?

Von Neumann-Morgerstern Preferences

Gambles are compared using a numeric valued utility function
$u: \Re_+ \to \Re$

$u(x)$ is the utility from consuming *x*

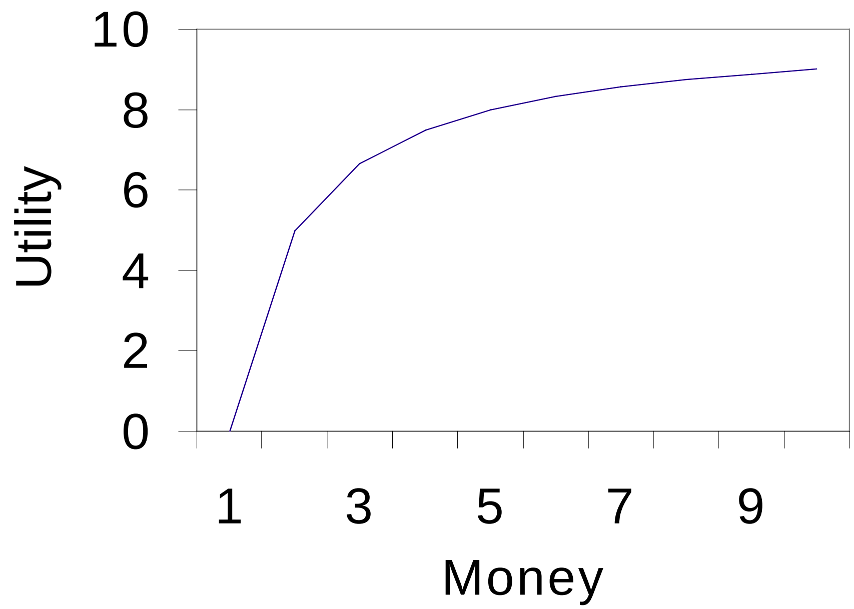
x_1 is at least as good (strictly better than) as x_2

$Eu(x_1) \geq (>) Eu(x_2)$

Expected Utility Theory

Example

$$u(x) = 10 - 10 / x$$



Money payoffs for player 1

	H	T
U	5	1
D	4	2

Utility payoffs for player 1

	H	T
U	8	0
D	7.5	5

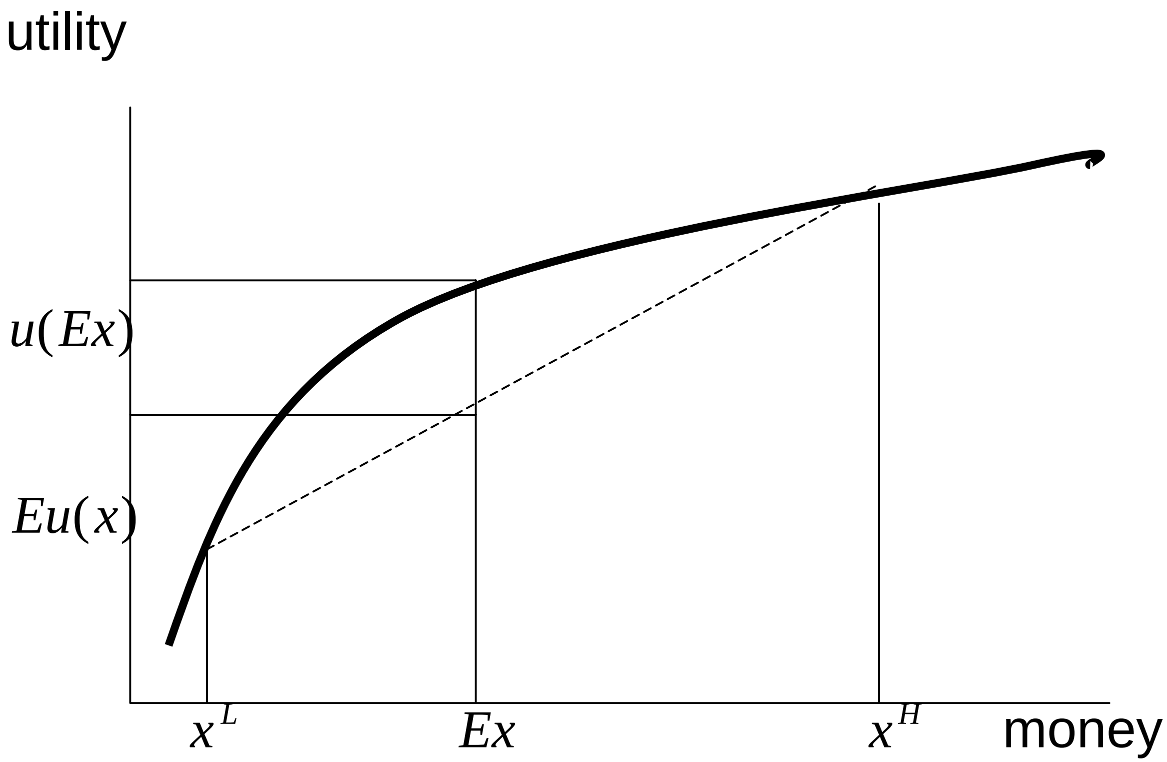
If H and T have equal probability is it better to choose U or D?

	Expected money	Expected utility
U	3	4
D	3	6.25

Choose D

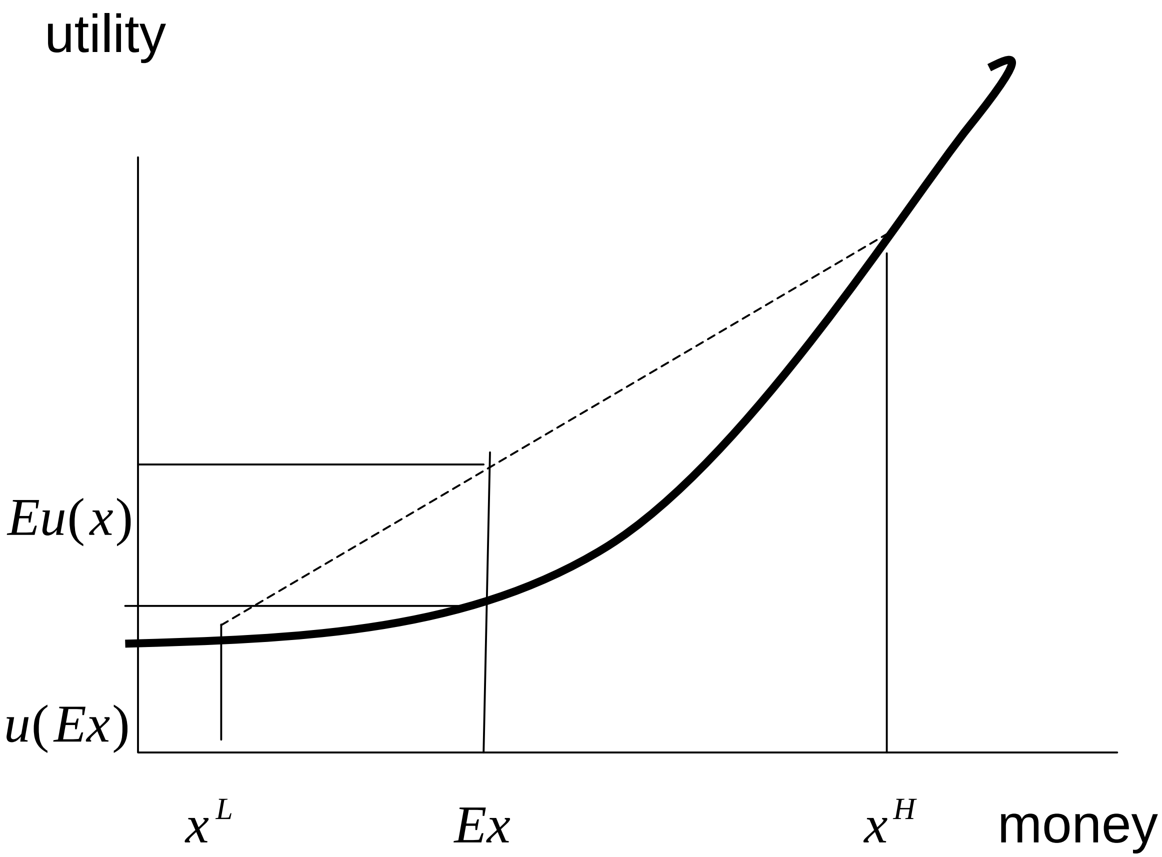
Risk Aversion

Would you rather get a gamble *x* or get the expected value of the gamble E*x* for sure? Suppose that the gamble is x^L with probability *p* and x^H with probability 1-*p*



What happens as *p* changes?

Risk Loving



- Insurance: auto insurance company charges a premium
- Investment: risky portfolio? Stocks or bonds?
- Gambling

Allais Paradox

Case 1, choose between:

Gamble 1

.33 chance of $27.5 billion

.66 chance of $24.0 billion

.01 chance of nothing

Gamble 2

$24.0 billion for sure

Case 2, choose between:

Gamble 1

.33 chance of $27.5 billion

.67 chance of nothing

Gamble 2

.34 chance of $24.0 billion

.66 chance of nothing

Case 1

$$.33u(27.5b)+.66u(24.0b)+.01u(0)-u(24.0b)=$$
$$.33u(27.5b)-.34u(24.0b)+.01u(0)$$

Case 1

$$.33u(27.5b)+.67u(0)-(.34u(24b)+.66u(0))=$$
$$.33u(27.5b)-.34u(24b)+.01u(0)$$

Expected utility predicts the same choice between gambles in the two cases.