

Economics 401 Microtheory

David K Levine

Spring 2008

Final Exam PART I: 40 Minutes, both questions have equal weight

1. A consumer values two good x_1, x_2 , which she may purchase at prices p_1, p_2 using money income I . Her preferences can be represented by a utility function $u(x_1, x_2) = x_1^{1/3}x_2^{1/2}$. Find the demand functions of the consumer for both x_1 and x_2 . Verify that these functions are homogeneous of degree zero in prices and income.

2. Demand for a good x is given by $p = 11 - 2x$. If there are n firms that are Cournot oligopolists all producing at constant marginal cost of 1, how much does each firm produce in a symmetric Cournot equilibrium?

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Final Exam PART II: 40 Minutes, all parts have equal weight

A starving serf has ten seeds. She must decide whether to plant them or eat them. If she plants them, they will grow into forty seeds. However, if she plants the seeds, an evil emperor must decide whether to take them all, or merely take half.

- a. Draw the extensive form of this game. Find the subgame perfect equilibria.
- b. Find the normal form of this game. Find the Nash equilibria.
- c. What is the Stackelberg equilibrium if the emperor moves first?

Now, suppose the evil emperor lives forever, and discounts the future with discount factor δ . The emperor faces a sequence of different serfs, each of whom live one period.

- d. Find a discount factor δ for the emperor and Nash equilibrium strategies for both players such that the emperor gets the same utility in the equilibrium as in the Stackelberg equilibrium of the game played once. Explain why these strategies are subgame perfect.

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Final Exam PART III: 40 Minutes, all parts have equal weight

An innocent merchant must decide whether to bring his goods to market. If he chooses to do so, he must pay off two robber barons. In addition to his goods, the merchant has 100 gold coins and is risk neutral (as are the robber barons). The merchant will not know until he gets to market the value of his trade goods, they may either be zero, or worth 500 gold coins. There is a 20% chance that the goods are valueless. Everyone knows this, however, the merchant also has a noisy signal of whether the trade goods will be worth anything. The signal has a 40% chance of error.

1. If he receives the good signal, how likely does the merchant think it that his goods are worth 500 gold coins? If he gets the bad signal? (hint: use Bayes law)
2. Suppose that robber baron i sets a price p_i for travelling through his territory. Denote the combined price $p = p_1 + p_2$. If the merchant receives the good signal, for what combined prices p will he choose to bring his goods to market? If he receives the bad signal?
3. For a given combined price p what is the probability the merchant will bring the goods to market? (hint: use the answer to part 2, and consider the probability the signal is either good or bad.)
4. Suppose that the robber barons set their prices independently to maximize the expected revenue they receive from the merchant. What prices will they set? (hint: you should make use of the answer to part 3).
5. How does your answer to question 3 change if the robber barons collude with each other? (This means that they set a combined price p to maximize expected combined revenue.)