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Economics 401
Spring 2008

Final Exam Answers

Part I

1) This is Cobb-Douglas utility of the form
$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$ where $\alpha = 1/3$, $\beta = 1/2$.

Remember that Cobb-Douglas utility function's derived demand is:

$$x_1 = \frac{\alpha}{\alpha+\beta} \cdot \frac{I}{P_1} \quad \text{and} \quad x_2 = \frac{\beta}{\alpha+\beta} \cdot \frac{I}{P_2}$$

Therefore:

$$x_1 = \frac{2}{5} \cdot \frac{I}{P_1} \quad \text{and} \quad x_2 = \frac{3}{5} \cdot \frac{I}{P_2}$$

Check that x_1 is HOD zero:

$$x_1(I, p) = \frac{2}{5} \cdot \frac{I}{P_1} = \frac{2}{5} \cdot \frac{\lambda I}{\lambda P_1} = x_1(\lambda I, \lambda p)$$

Same is true for x_2.

2) $$\pi_i = \left[11 - 2(x_i + (n-1)\bar{x})\right] x_i - x_i$$

where $\bar{x}$ is the output of a representative firm.

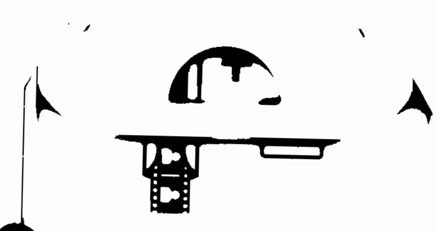
First order condition:

$$\frac{\partial \pi_i}{\partial x_i} = \left[11 - 2(2x_i + (n-1)\bar{x})\right] - 1 = 0$$

Since in a symmetric equilibrium $\bar{x} = x_i$;

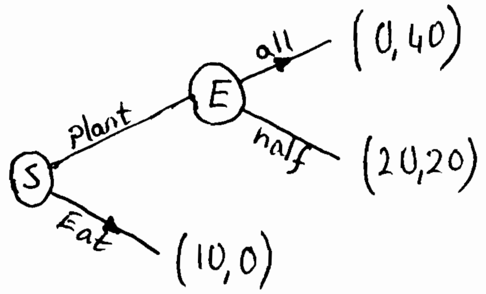
$$11 - 2(n+1)\bar{x} = 1$$

$$\Rightarrow \bar{x} = \frac{10}{2(n+1)} = \underline{\underline{\frac{5}{(n+1)}}}$$



Part II

a)



Unique SPE is (eat, all).

b)

	all	half
Plant	$0,40^*$	$20^*,20$
Eat	$10^*,0^*$	$10,0^*$

Unique pure strategy NE = (eat, all)

c) With the emperor as the Stackleberg leader, the Stackleberg equilibrium becomes:

1) Emperor commits to half
2) Serf plants

d) We need a δ such that it is more profitable for the Emperor to stick to the Stackleberg equilibrium rather than going back to the static Nash equilibrium of (eat, all).

$$\Rightarrow \quad 20 \geq (1-\delta)(40) + (\delta)(0)$$

$$\Rightarrow \quad \delta \geq 1/2$$

For $\delta \geq 1/2$ we just showed that playing the grim trigger strategy of playing half first period and every period if (plant, half) has been played until then is better than the static Nash equilibrium (eat, all). Since (eat, all) is a SPE, the grim trigger strategies are SPE given $\delta \geq 1/2$.

Note: This is trivially true for the short-run player since he only plays a best response.

Part III

1) $P(500 \mid good) = \dfrac{P(good \mid 500)\, P(500)}{P(good)}$

$$= \frac{(0.6)\cdot(0.8)}{(0.8)(0.6)+(0.2)(0.4)} = \frac{0.48}{0.56} \approx 0.86$$

$$P(500 \mid bad) = \frac{P(bad \mid 500)\, P(500)}{P(bad)}$$

$$= \frac{(0.4)\,(0.8)}{(0.2)(0.6)+(0.8)(0.4)} = \frac{0.32}{0.44} \approx 0.73$$

2) <u>Good Signal:</u>

Expected utility from bringing goods to market $= (500)(0.86) + (0)(0.14)$

$= 430$

So, merchant will bring his goods to market if

$p \leq 430$

<u>Bad Signal:</u>

Expected utility $= (500)(0.73) + (0)(0.27)$

$= 365$

Merchant will bring goods to market if

$p \leq 365$

3) $$P(\text{bring goods} \mid p) = \begin{cases} 1 & \text{if } p \le 365 \\ 0{,}56 & \text{if } 365 < p \le 430 \\ 0 & \text{if } p > 430 \end{cases}$$

If prices are less than 365, then the merchant will bring the goods to market regardless of the signal (hence probability 1). If prices are between 365 and 430, only the merchant who receives a good signal will bring the goods to market. Probability of a good signal is 0.56 as found in part 1. If prices are greater than 430, the merchant will <u>not</u> bring goods to market regardless of the signal.

4) Given the result above, it doesn't make sense for the barons to set combined price strictly less than 365. Let us conjecture $P_1 = P_2 = \frac{365}{2} = 182.5$. In order for this to be an equilibrium we need to show that neither baron has an incentive to deviate. Since $P(\text{bring goods} \mid p)$ is constant in p over the interval $(365, 430]$, baron 1 can increase his price until $p = 430$ without reducing the mentioned probability further from 0.56. So, the decision is: Should I stick to 182.5 or increase to $430 - 182.5 = 247.5$. Expected utility for baron 1 for a combined price of 430 is: $(247.5)(0.56) + (0)(0.44) = 138.6$ which is lower than a guaranteed 182.5. Therefore, neither baron has an incentive to deviate $\Rightarrow P_1 = P_2 = 182.5$.

5) Same as above, the expected utility from setting $p = 430$ is:

$$(430)(0.56) + (0)(0.44) = 240.8$$

This is lower than a guaranteed revenue of 365 if prices were set to $p = 365$. Therefore, barons will collude to set the combined price to 365.