FIRST MIDTERM SOLUTIONS

1. Here is the payoff matrix of the game;

	R	G	B
R	$\underline{10},\underline{10}$	$\underline{3},9$	$0,0$
G	$9,\underline{3}$	$1,1$	$\underline{2},2$
B	$0,0$	$2,\underline{2}$	$1,1$

(a) *Are any strategies weakly or strictly dominated? Can you apply iterated dominance of either sort?*

No, there are no weakly or strictly dominated strategy. Thus, we can not apply iterated dominance of either sort.

(b) *What are the Nash equilibria?*

The best responses of each player to other player's strategies are underlined on the payoff matrix above. Thus, we can conclude that (R, R) is the only Nash equilibrium of this game since each strategy is best response to the other.

(c) *What can you say about the Pareto efficiency of the Nash equilibria?*

Nash equilibrium (R, R) is Pareto efficient since there is no way to make someone better off without making someone worse off. In fact, the equilibrium outcome yields the highest possible payoffs to both players.

Now, the payoff matrix for the new game is given as follows;

	R	G	B
R	$10,10$	$3,\underline{12}$	$0,0$
G	$\underline{12},3$	$\underline{4},\underline{4}$	$\underline{5},2$
B	$0,0$	$2,\underline{5}$	$1,1$

(a) *Are any strategies weakly or strictly dominated? Can you apply iterated dominance of either sort?*

G strictly dominates R and B for both players. If we eliminate R and B for both players, only remains (G, G).

(b) *What are the Nash equilibria?*

The best responses of each player to other player's strategies are underlined on the payoff matrix above. Thus, we can conclude that strategy profile (G, G) is the only Nash equilibrium of this game since each strategy is best response to the other.

(c) *What can you say about the Pareto efficiency of the Nash equilibria? Are the players better off as a result of the contributions of the philanthropist? Does the philanthropist lose money as a result of her generosity?*

(G, G) is not Pareto efficient since (R, R) Pareto dominates (G, G). So, both players are worse-off by the contributions of the philanthropist, and philanthropist loses 6 as a result of her generosity.

2. (a) We need to determine whether the bidding (strategy) profile (1,2,3) is a Nash equilibrium or not. In order to do so, we need to check whether each player is playing the best response strategy to the other players' strategies. If the strategy profile (1,2,3) were played, then John would be the winner, get all six paintings (others would get nothing) and pay 1. So the payoffs would be (0,0,2). However, Bjorn could do better if he bid (played) 4. The reason is if he bid 4, he would win the auction, and get all the paintings and pay 1. So, his payoff would be 1. Thus, (1,2,3) is not a Nash equilibrium since 2 is not a best response to 1 and 3.

   (b) Let's consider the bidding profile (2,3,4). If this profile were played, John would be the winner, get all the paintings and pay 2. So the payoffs would be (0,0,1). John could not do any better if he lowered his bid. In particular, if he bid 1 or 2, he would lose the auction (remember Bjorn bid 3) and get nothing. Thus, his payoff would be 0. If he bid 3, then he would be the winner with Bjorn, get three paintings with payoff 1.5 and pay 2. The net payoff is -0.5. Hence, 4 is the best response to 2 and 3 for John.

   Similarly, Bjorn could not do any better if he lowered/raised his bid. If he bid 4, then he would be the winner with John, get 3 paintings and pay 2. So ,his net payoff would be -1. If he bid 1 or 2, he would lose the auction and get nothing. So, his net payoff would be 0. Hence, 3 is a best response to 2 and 4 for Bjorn.

   Finally, Peter could not do any better if he lowered/raised his bid. If he bid 4, then he would be the winner with John, get 3 paintings and pay 3. So ,her payoff would be -2.5.

If he bid 1 or 3, he would lose the auction and get nothing. So, his payoff would be 0. Hence, 2 is a best response to 3 and 4 for Peter.

Therefore, (2,3,4) is a Nash equilibrium since each strategy is a best response to other strategies.

(c) Yes, if the bidding profile (4,2,1) were played, Peter would win the auction, get all paintings and pay 1. So, the associated payoffs are (0,0,0). Obviously, 4 is a best response to 2 and 1 for Peter. To see why it is true, suppose he bids 1 instead of 4. Then he would lose the auction and get nothing. So his payoff would be 0. If he bids 2, then he would win the auction with Bjorn, get 3 paintings and pay 1. So his net payoff would be -0.5. If he bids 3, then he would win the auction, get all paintings and pay 1. So his payoff would be 0. Hence, 4 is a best response to 2 and 1 for Peter.

For Bjorn, there is no profitable deviation since bidding 1 or 3 instead of 2 would not change anything because he would lose the auction anyway and get nothing. If he bids 4, then he would win the auction with Peter, get 3 paintings and pay 1. So, his payoff would be 0. Hence, 2 is a best response to 4 and 1 for him.

For John, there is no profitable deviation too since bidding 2 or 3 instead of 1 would not change his payoff because he would lose the auction anyway and get nothing. If he bids 4, then he would win the auction with Peter, get 3 paintings and pay 2. So, his net payoff would be -0.5. Hence, 1 is a best response to 4 and 2 for him.

Therefore, (4,2,1) is a Nash equilibrium and Peter gets all six paintings.

3. (a) The profit functions of Unbalance and Contrapositive are given below;

$$\Pi_U(x_U, x_C) = [120 - 3(x_U + x_C)]x_U - 2x_U$$

$$\Pi_C(x_U, x_C) = [120 - 3(x_U + x_C)]x_C - 7x_C$$

where U denotes Unbalance and C denotes Contrapositive.

(b) Best response of Unbalance to Contrapositive's output is determined by

$$\max_{x_U} \Pi_U(x_U, x_C) = [120 - 3(x_U + x_C)]x_U - 2x_U$$

Since the profit is maximized when $\frac{\partial \Pi_U}{\partial x_U} = 0$, we obtain

$$\frac{\partial \Pi_U}{\partial x_U} = 120 - 6x_U - 3x_C - 2 = 0$$

$$118 - 3x_C = 6x_U$$

$$\boxed{x_U = \tfrac{59}{3} - \tfrac{x_C}{2}}$$

Similarly, Contrapositive's best response to Unbalance's output level is determined by the following;

$$\max_{x_C} \Pi_C(x_U, x_C) = [120 - 3(x_U + x_C)]x_C - 7x_C$$

Since the profit is maximized when $\frac{\partial \Pi_C}{\partial x_C} = 0$, we obtain

$$\frac{\partial \Pi_C}{\partial x_C} = 120 - 6x_C - 3x_U - 7 = 0$$

$$113 - 3x_U = 6x_C$$

$$\boxed{x_C = \tfrac{113}{6} - \tfrac{x_U}{2}}$$

(c) Nash equilibrium is the outcome where each firm produces the best response output to the other's output level. If we plug one best response to the other, we obtain

$$x_C^* = \frac{113}{6} - \frac{\frac{59}{3} - \frac{x_C^*}{2}}{2}$$

$$x_C^* = \frac{113}{6} - (\frac{59}{6} - \frac{x_C^*}{4})$$

$$\frac{3}{4}x_C^* = \frac{54}{6}$$

$$\boxed{x_C^* = 12}$$

Since $x_U^* = \frac{59}{3} - \frac{x_C^*}{2}$ and $x_C^* = 12$, then we obtain

$$x_U^* = \frac{59}{3} - \frac{12}{2}$$

$$\boxed{x_U^* = \tfrac{41}{3} \approx 13.67}$$

Hence, $(x_U^*, x_C^*) = (\frac{41}{3}, 12) \approx (13.67, 12)$ is the Nash equilibrium.