

SECOND MIDTERM EXAM (DRAFT)  
Econ 4011, Fall 2011

Do all questions. The questions have equal weight. You have 1hr and 23 minutes.

### 1. *Two Period Bargaining*

Ingolf and Ariel are bargaining over how to divide a surplus of size 1. The procedure they use to determine their shares is as follows:

Period 1 starts with Ingolf making a demand of  $x_I$ , where  $x_I \in \left\{ \frac{1}{3}, \frac{1}{2}, \frac{2}{3} \right\}$ . Following

this, Ariel may choose to *Accept* or *Reject* the proposed division. Should Ariel accept, Ingolf gets a payoff of  $x_I$  while Ariel gets  $1 - x_I$ , and the game ends. If Ariel rejects then the game moves into Period 2.

Period 2 starts with Ariel making a demand of  $x_A$ , where  $x_A \in \left\{ \frac{1}{4}, \frac{3}{4} \right\}$ . Following this,

Ingolf may choose to *Accept* or *Reject* the proposed division. Should Ingolf accept, Ingolf gets a payoff of  $1 - x_A$  while Ariel gets  $x_A$ , in Period 2. If Ariel rejects then both players get 0 in Period 2.

Ingolf and Ariel have a common discount factor of  $\delta$ . In other words, a payoff of  $z$  to a player in Period 2 is valued at  $\delta z$  in period 1.

- Write down the extensive form of this game.
- Find a subgame perfect equilibrium of this game when  $\delta = \frac{1}{3}$
- Find a subgame perfect equilibrium of this game when  $\delta = \frac{5}{6}$
- When  $\delta = \frac{1}{3}$ , find a Nash Equilibrium of this game that is *not* a Subgame Perfect Equilibrium.

### 2. *Stackelberg Equilibrium*

Suppose that the demand for quantity  $x$  in a market is given by the function  $p=30-x$  and there are two firms, Intendo and CCube. Intendo faces a marginal cost of 9 while CCube faces a marginal cost of 3. Note that if Intendo chooses  $x_I$  and CCube chooses  $x_C$ , then  $x = x_I + x_C$ . (Remember firms can only choose a non negative quantity of output.)

- Write down the best response functions for each firm.
- What is the Stackelberg Equilibrium of this game if Intendo is the leader?
- What is the Stackelberg Equilibrium of this game if CCube is the leader?

### 3. Repeated Game Equilibria

Consider the simultaneous move stage game which is repeated infinitely.

	C	D
C	15, 10	-5, 50
D	30, -20	5, 0

Jack is the row player while Sparrow is the column player, with a common discount factor of  $\delta$ .

- What is the Static Nash Equilibrium of this game?
- Write down strategies for the repeated game which give rise to a subgame perfect equilibrium, *irrespective of the value of the discount factor,  $\delta$* .
- Consider the “grim trigger” strategy of playing C in period 1; playing C as long as both players have played only C in the past, and playing D otherwise. For what values of the common discount factor,  $\delta$ , do these strategies form a subgame perfect equilibrium?