Problem Set 3

1. Forward Induction

Stephen J. Seagull must decide whether or not to start a new movie project. If he decides not to, he and Clod VandeCamp both get a utility of ten. If he decides to begin the project, both he and Clod must simultaneously decide who the director should be: George Spellbinder, or Ed Tree. If they disagree on the director, the movie isn't made, but both have wasted time, so they get only a utility of zero. If they agree on George Spellbinder, the movie will be a roaring success, and each gets a utility of 20. If they agree on Ed Tree, the movie will be terrible. and they will only get a utility of 5.

Draw the extensive form of this game. Find the normal form. Find all the Nash equilibria. Find all the subgame perfect equilibria. Apply the theory of iterated elimination of weakly dominated strategies and state its prediction.

2. Stackelberg Equilibrium

Suppose that demand for quantity x in a market is given by the price $p=17-x$ and there are two firms, Savannah and Frontier. Savannah has a marginal cost of 3 while Frontier has a marginal cost of 1. What is the Stackelberg Equilibrium if Savannah is the Stackelberg leader? What is the Stackelberg Equilibrium if Frontier is the Stackelberg leader?

3. The Pirates and the Equilibrium

There are five pirates, named One, Two, Three, Four and Five. Pirate One is highest in the chain of command, followed by Pirate Two, then by Pirate Three and so on. The treasure they have on board of 50 gold coins need to be distributed among them. To do so they use the following procedure.

The highest ranking pirate still on the ship proposes a division. If *more than half* the pirates on board accept the proposal, the treasure is divided according to the said proposal. However, if *at least* half the pirates on board vote against the division, the proposer is made to walk the plank to meet his death. Following this, the next highest ranking pirate makes his proposal.

Each pirate gets a utility of x from getting x gold coins. Pirates prefer getting 0 gold coins to ending up dead. Proposal cannot involve fractions of gold coins. Conditional on being alive and getting as many gold coins as he can get each pirate wants to see as many superiors meet their death as possible.

Construct a Subgame Perfect Equilibrium of this game.