ECON 4011 11/20/2011

FALL 2011 CEYHUN COBAN

PROBLEM SET #5 SOLUTIONS

1. First we need to determine the events in this problem. Let E be the event that evidence is found and let B be the event that vase is broken due to bad packaging. Then, we need to determine $P(B|E)$. Since we are given the following probabilities

$$P(B) = 0.1$$

$$P(E|B) = 0.9$$

$$P(E|B^c) = 0.3$$

by using Baye's Law, we obtain

$$\begin{aligned} P(B|E) &= \frac{P(E|B)P(B)}{P(E|B)P(B) + P(E|B^c)P(B^c)} \\ &= \frac{(0.9)(0.1)}{(0.9)(0.1) + (0.3)(0.9)} \\ &= 0.25 \end{aligned}$$

2. Expected utility that is gained from Gamble A is calculated as follows;

$$\begin{aligned} Eu(A) &= P(x=5)u(5) + P(x=2)u(2) \\ &= \frac{1}{2}(20 - \frac{20}{5}) + \frac{1}{2}(20 - \frac{20}{2}) \\ &= 13 \end{aligned}$$

Similarly, expected utility that is gained from Gamble B is

$$\begin{aligned} Eu(B) &= P(x=10)u(10) + P(x=1)u(1) \\ &= \frac{1}{2}(20 - \frac{20}{10}) + \frac{1}{2}(20 - \frac{20}{1}) \\ &= 9 \end{aligned}$$

Hence, gamble A is chosen.

3. (a) In order to determine pure strategy Nash equilibria (PSNE) of this game, best responses are underlined on the payoff matrix which is given below;

	L	R
U	$\underline{2}, 4$	$3, \underline{7}$
D	$1, 5$	$\underline{4}, \underline{6}$

Hence, (D, R) is the unique PSNE.

Since L is strictly dominated by R for column player, column player never randomizes between L and R. Knowing this, it is always optimal to play D for row player. Hence, there is no mixed strategy Nash equilibria (MSNE).

(b) Similarly, best responses are underlined on the payoff matrix which is given below;

	L	R
U	$0, 0$	$\underline{30}, \underline{60}$
D	$\underline{90}, \underline{10}$	$20, 0$

Hence, (D, L) and (U, R) are the PSNE.

Let p be the probability of playing U for row player, and then obviously $1 - p$ be the probability of playing D (i.e,, $\sigma_1 = (p, 1 - p)$). So, column player randomizes if the expected payoff of playing L and R are the same. Thus,

$$u_2(\sigma_1, L) = u_2(\sigma_1, R)$$

$$(p)(0) + (1 - p)(10) = (p)(60) + (1 - p)(0)$$

$$(1 - p)10 = 60p$$

$$\boxed{p = 1/7}$$

Similarly, let q be the probability of playing L for column player, then obviously $1 - q$ be the probability of playing R (i.e,, $\sigma_2 = (q, 1 - q)$). So, row player randomizes if the expected payoff of playing U and D are the same. Thus,

$$u_1(U, \sigma_2) = u_1(D, \sigma_2)$$

$$(q)(0) + (1 - q)(30) = (q)(90) + (1 - q)(20)$$

$$(1 - q)10 = 90q$$

$$\boxed{q = 1/10}$$

Hence, $((\frac{1}{7}, \frac{6}{7}), (\frac{1}{10}, \frac{9}{10}))$ is the MSNE.

(c) Similarly, best responses are underlined on the payoff matrix which is given below;

	L	R
U	$\underline{8}, 4$	$6, \underline{10}$
D	$4, \underline{8}$	$\underline{8}, 4$

Hence, there is no PSNE.

Let p be the probability of playing U for row player, and then obviously $1-p$ be the probability of playing D (i.e,, $\sigma_1 = (p, 1-p)$). So, column player randomizes if the expected payoff of playing L and R are the same. Thus,

$$u_2(\sigma_1, L) = u_2(\sigma_1, R)$$

$$(p)(4) + (1-p)(8) = (p)(10) + (1-p)(4)$$

$$(1-p)4 = 6p$$

$$\boxed{p = 2/5}$$

Similarly, let q be the probability of playing L for column player, then obviously $1-q$ be the probability of playing R (i.e,, $\sigma_2 = (q, 1-q)$). So, row player randomizes if the expected payoff of playing U and D are the same. Thus,

$$u_1(U, \sigma_2) = u_1(D, \sigma_2)$$

$$(q)(8) + (1-q)(6) = (q)(4) + (1-q)(8)$$

$$4q = (1-q)2$$

$$\boxed{q = 1/3}$$

Hence, $((\frac{2}{5}, \frac{3}{5}), (\frac{1}{3}, \frac{2}{3}))$ is the MSNE.