ECON 4011 12/02/2011
FALL 2011 CEYHUN COBAN

PROBLEM SET #6 SOLUTIONS

1. (a) Let's start determining the events in this problem. Let U be the event that sport figure uses a performance enhancing drug, let $+A$ be the event that result is positive if test A is taken. Moreover, let's denote the complements of events U and $+A$ as NU and $-A$, respectively (i.e $P(U) + P(NU) = 1$). Since we are given the following probabilities

$$P(U) = 0.4$$

$$P(+A|NU) = 0.4$$

$$P(-A|U) = 0.1$$

, then we need to determine $P(U| + A)$. By using Baye's Law, we obtain

$$\begin{aligned} P(U| + A) &= \frac{P(+A|U)P(U)}{P(+A|U)P(U) + P(+A|NU)P(NU)} \\ &= \frac{(0.9)(0.4)}{(0.9)(0.4) + (0.4)(0.6)} \\ &= 0.6 \end{aligned}$$

Then, the expected payoff from banning a tested positive athlete is

$$\begin{aligned} Eu(ban) &= P(U| + A)(10) + P(NU| + A)(-50) \\ &= (0.6)(10) + (0.4)(-50) \\ &= -14 \end{aligned}$$

On the other hand, the expected payoff from not banning a tested positive athlete is

$$\begin{aligned} Eu(notban) &= P(U| + A)(-10) + P(NU| + A)(0) \\ &= (0.6)(-10) + (0.4)(0) \\ &= -6 \end{aligned}$$

Hence, the correct decision is to not ban the sports figure.

(b) Similarly, let $+B$ be the event that result is positive if test B is taken. Then, we are given the following probabilities

$$P(U) = 0.4$$

$$P(+B|NU) = 0.2$$

$$P(-B|U) = 0.1$$

By using Baye's Law, we obtain $P(U|+B)$ as follows;

$$\begin{aligned} P(U|+B) &= \frac{P(+B|U)P(U)}{P(+B|U)P(U) + P(+B|NU)P(NU)} \\ &= \frac{(0.9)(0.4)}{(0.9)(0.4) + (0.2)(0.6)} \\ &= 0.75 \end{aligned}$$

Then, the expected payoff from banning a tested positive sports figure is

$$\begin{aligned} Eu(ban) &= P(U|+B)(10) + P(NU|+B)(-50) \\ &= (0.75)(10) + (0.25)(-50) \\ &= -5 \end{aligned}$$

On the other hand, the expected payoff from not banning a tested positive sports figure is

$$\begin{aligned} Eu(notban) &= P(U|+B)(-10) + P(NU|+B)(0) \\ &= (0.75)(-10) + (0.25)(0) \\ &= -7.5 \end{aligned}$$

Hence, the correct decision is to ban the sports figure.

(c) We already found that sport authorities are not able to ban a sports figure even though the result of test A is positive. Thus, a sports figure using the drug will strictly prefer to take test A.

(d) No, a nonuser sports figure will also strictly prefer to take Test A since he knows that he is not going to be banned even if the test result is wrong (positive). Moreover, there is a possibility that the result will come wrong (positive) if he takes test B and as a consequence, he will be banned.

2. There are two types of firms; low-cost firm and high-cost firm. Each firm knows its own type, but not the other. In the symmetric Bayesian Nash equilibrium, there is only one strategy that corresponds to each type. So, let's denote the equilibrium strategy of low-cost firm and high-cost firm as x^3 and x^5, respectively. Thus, we need to determine x^3 and x^5.

The profit function of a typical firm i is

$$\Pi_i(x_i, x_{-i}) = [60 - (x_i + x_{-i})]x_i - c_i x_i$$

Since firm i knows that with probability q its opponent is a low-cost firm, then by rearranging the profit function we obtain

$$\Pi_i(x_i, x_{-i}) = [60 - c_i - x_i - qx^3 - (1-q)x^5)]x_i$$

Best-response function is obtained by taking the derivative of profit function with respect to x_i and setting it to equal to 0 as follows;

$$60 - c_i - 2x_i - qx^3 - (1-q)x^5) = 0$$

$$x_i = \frac{60 - c_i - qx^3 - (1-q)x^5}{2}$$

Then, depending on the type of firm i, we obtain two equations;

$$x^3 = \frac{57 - qx^3 - (1-q)x^5}{2} \tag{1}$$

and

$$x^5 = \frac{55 - qx^3 - (1-q)x^5}{2} \tag{2}$$

By adding equations (1) and (2) side by side, we obtain

$$\begin{aligned} x^3 + x^5 &= 56 - qx^3 - (1-q)x^5 \\ x^5 &= \frac{56 - (1+q)x^3}{(2-q)} \end{aligned} \tag{3}$$

If we plug equation (3) into equation (1), we obtain

$$2x^3 = 57 - qx^3 - (1-q)(\frac{56 - (1+q)x^3}{(2-q)})$$

$$(4 - q^2)x^3 = 57(2-q) - 56(1-q) + (1-q^2)x^3$$

$$3x^3 = 58 - q$$

$$x^3 = \frac{58 - q}{3} \tag{4}$$

Finally, we determine x^5 by plugging equation (4) into equation (3) as follows;

$$\begin{aligned} x^5 &= \frac{168 - (1+q)(58-q)}{3(2-q)} \\ &= \frac{110 - 57q + q^2}{3(2-q)} \\ &= \frac{(55-q)(2-q)}{3(2-q)} \\ &= \frac{55-q}{3} \end{aligned}$$

3. A Bayesian Nash equilibrium of this game is a bidding profile where each player bids the valuation of his/her own type (i.e., $b_i^k = v_i^k, i \in \{E, J\}, k \in \{L, H\}$). In other words, bidding an amount equivalent to the valuation of your own type is a best response to your opponent's bid which is equivalent to the valuation of his/her type. To see why, start with Edward. If Edward is a low type than he bids 50. Then, Edward knows that with probability 0.6 Jackson is a high type and bids 110, and with probability 0.4 Jackson is low type and bids 40. Thus, Edward's expected payoff is $0.6(0) + 0.4(50 - 40) = 4$. If Edward bids less than 50, his expected payoff will not increase (why?) so that it is not a profitable deviation. If he chooses to bid more than 50, then his expected payoff will not change up to 110, and if he bids 110 or higher, he wins the auction but he has to pay 110 which lowers his expected payoff. Thus, bidding more than 50 is not profitable either. Hence, bidding 50 is optimal for Edward when his type is low. Similarly, if Edward is high type, then he bids 100 and his expected payoff is $0.6(0)+0.4(100-40) = 24$. If Edward bids less than 100, his expected payoff will not increase implying that it is not a profitable deviation. If he chooses to bid more than 100, then his expected payoff will not change up to 110, and if he bids 110 or higher, he wins the auction but he has to pay 110 which makes his expected payoff negative. Thus, bidding more than 100 is not profitable either. Hence, bidding 100 is optimal for Edward when he is high type.

   The same argument follows for Jackson. If he is a low type, then he bids 40. Knowing that with probability 0.4 Edward is a high type and bids 100, and with probability 0.6 Edward is low type and bids 50, Jackson will loose the auction in any case so that his payoff is 0. Bidding less than 40 does not change anything so that it is not a profitable deviation. If he chooses to bid more than 40, then his expected payoff will not change up to 50, and if he bids more than 50, with probability 0.6 he wins the auction but he has to pay 50 leading to a negative expected payoff. Thus, bidding more than 40 is not profitable either. Hence, bidding 40 is optimal for Jackson if he is a low type. Similarly, if Jackson is high type, then he bids 110 and then regardless of Edward's type Jackson always win the auction and his expected payoff is $0.4(110-100)+0.6(110-50) = 40$. Since Jackson always wins the auction by bidding 110, then bidding less or more than 110 does not increase his expected payoff. Thus, there is no profitable deviation for Jackson implying that bidding 110 is optimal.

   Therefore, bidding own valuation for each type constitutes a Bayesian Nash Equilibrium.