Economics 401 Microtheory

Mechanism Design

1. Compensation

A board of directors must design a compensation scheme for a CEO. The CEO must decide between providing a high level of effort H and a low level of effort L. As a consequence of the CEO's decision, the company is either successful yielding a profit of $4v$ or goes bankrupt yielding a profit of 0. If the CEO provides a high level of effort, the probability of success is 3/4, if the CEO provides a low level of effort the probability of success is 1/4. The CEO has an expected utility function $\log(1+w)$ if she provides low effort, and $\log(1+w)-\log 3$ in case of high effort. The amount paid by the board of directors cannot be negative, and the board of directors acts to maximize the expected profit of the firm net of the cost of paying the CEO.

First suppose that the board can observe the effort of the CEO. The compensation scheme has the form of a wage w_H paid in case of high effort, and w_L paid in case of low effort.

a. Write the objective function of the board of directors in case of high effort. Then again in case of low effort.

b. You may assume that the optimal compensation scheme pays $w_L = 0$, that is, no payment in case of low effort. Write down the condition for the CEO to be willing to provide high effort (the incentive constraint for the CEO). If the board chooses to induce the CEO to provide high effort, what wage must it pay in case of high effort?

c. When will the board prefer to induce the CEO to provide high effort?

Now suppose that the board can only observe the whether the firm is successful or not. The compensation scheme has the form of a wage w_v paid in case of success and a wage w_0 paid in case of bankruptcy.

d. Write the objective function of the board in case of low effort. Then again in case of high effort.

e. You may assume that the optimal compensation scheme pays $w_0 = 0$, that is, no payment in case of bankruptcy. Write down the condition for the CEO to be willing to

provide high effort (the incentive constraint for the CEO). If the board chooses to induce the CEO to provide high effort, what wage must it pay in case of success?
f. When will the board prefer to induce the CEO to provide high effort?

2. Price Discrimination
A firm wishes to sell either 1 or 2 units of a good to a consumer with uncertain demand. (The firm is not allowed to sell 0 units; there is no production cost.) With probability ½ the utility function of the consumer is $(1-p)x$ where p is the price paid per unit and x is the number of units purchased. With
probability 1/2 the utility is instead $(3-p)x$. Only the consumer knows his utility function. Formulate this as a mechanism design problem for the seller, and determine how he can best maximize his expected revenue

Demand Theory

1. If the utility for flounder x_1 and flour x_2 is $\log x_1 + \log x_2$ what is the demand function for flounder? By what approximately what percent does the demand for flounder change if the price rises by 10% ? What happens to the demand for flour?

2. In each case explain why the function might or might not be a demand function, or pair of such functions:
a. $x_1 = mp_2 / p_1$
b. $x_1 = x_2 = m/(p_1 + p_2)$

Partial Equilibrium

A consumer values two good x_1, x_2, which she may purchase at prices p_1, p_2 using money income I. Her preferences can be represented by a utility function $u(x_1, x_2) = x_1 - (x_2 - 12)^2$.
a. Find the demand functions of the consumer for both x_1 and x_2.
b. Verify that these functions are homogeneous of degree zero in prices and income.
c. What happens if $x_2 > 12$?
Suppose that there is a firm that produces the good x_2 at constant marginal cost c.
d. Find the competitive equilibrium output and price in the market for x_2.

e. Fine the optimal output and price of a monopolist in the market for x_2.

General Equilibrium

1. Suppose that Orca and Dorca are dolphins who like to consume herring x_1 and tuna x_2. Both have utility functions given by $x_1^{1/2}x_2^{1/2}$. Orca has one tuna and Dorca one herring. Draw a careful picture of the Edgeworth box, showing the endowment, the competitive equilibrium (and competitive equilibrium price line), the contract curve and the core.

2. Suppose that Orca and Dorca's utility is given by $x_1^\alpha + x_2^\alpha$ where $1 > \alpha > 0$. Orca has $\overline{x}_1^1$ herrings and no tuna; Dorca has $\overline{x}_2^2$ tunas and no herring.

a. Find Orca and Dorca's individual excess demand for tuna and herring.

b. Find competitive equilibrium prices as a function of $\overline{x}_1^1, \overline{x}_2^2, \alpha$. If good 1 is numeraire, what happens to the price of good 2 when $\overline{x}_2^2$ is increased?

3. In Lalaland, crystals are the only good. Three people live in Lalaland, Mr. Yuppie, Ms. Yuppie and a starving student. Mr. Yuppie's consumption of crystals is x_1; Ms. Yuppie's x_2; and the starving student's x_3. Mr. Yuppie and Ms. Yuppie each have 14 crystals, while the student has only 8. Each person is not only concerned about his or her own crystal consumption, but feels morally obligated to see that his or her fellows are adequately supplied with crystals. Mr. Yuppie gets utility $(x_1)^2 x_2 x_3$; Ms. Yuppie gets $x_1(x_2)^2 x_3$ and the student gets $x_1 x_2 (x_3)^2$.

a. Show that neither Mr. nor Ms. Yuppie will donate a crystal to the student.

b. Show that everyone will be better off if both Mr. and Ms. Yuppie give a crystal to the starving student. What does this imply about the efficiency of the competitive equilibrium?

c. Suppose that Ms. Yuppie offers to match any donations of crystals made by Mr. Yuppie to the student. Will Mr. Yuppie donate a crystal? Why? Will Ms. Yuppie make the offer? Why?