ECON 4011 02/06/2012

SPRING 2012 CEYHUN COBAN

PROBLEM SET #1 SOLUTIONS

1. If the monopolist uses Guava technology, then the profit maximization problem of the monopolist is stated as follows;

$$\max \quad \pi_G(x) = (15 - x)x - (3x + 2x^2)$$

Since the profit is maximized when $\frac{d\pi_G}{dx} = 0$, we obtain

$$\frac{d\pi_G}{dx} = 15 - 2x - 3 - 4x = 0$$

$$12 = 6x$$

$$\boxed{\text{x=2}}$$

On the other hand, if the monopolist uses Froid technology, then the profit maximization problem of the monopolist becomes;

$$\max \quad \pi_F(x) = (15 - x)x - (6x + \tfrac{x^2}{2})$$

Since the profit is maximized when $\frac{d\pi_F}{dx} = 0$, we obtain

$$\frac{d\pi_F}{dx} = 15 - 2x - 6 - x = 0$$

$$9 = 3x$$

$$\boxed{\text{x=3}}$$

Monopolist will choose the technology that leads to more profit, so we need to compare the profits;

$$\pi_G(2) = (15 - 2)2 - (3(2) + 2(2)^2) = 12$$

$$\pi_F(3) = (15 - 3)3 - (6(3) + \frac{3^2}{2}) = 13.5$$

Since Froid technology yields higher profit than Guava technology, monopolist will prefer to use the Froid technology.

2. Here is the payoff matrix for this game;

		Bob	
		S	W
Ann	S	$4,1$	$10,5$
	W	$2,10$	$2,0$

where S denotes the action of going to Steinberg Cafe and W denotes the action of going to Whispers.

-Are there any strategies that are strongly or weakly dominated in this game?

W is strongly (strictly) dominated by S for Ann since

$$u_A(S,S) = 4 > 2 = u_A(W,S)$$

and

$$u_A(S,W) = 10 > 2 = u_A(W,W)$$

-Find the reaction (best response) functions.

The best responses are underlined in the payoff matrix that is given below;

		Bob	
		S	W
Ann	S	$\underline{4},1$	$\underline{10},\underline{5}$
	W	$2,\underline{10}$	$2,0$

<u>OPTIONAL</u>: The formal way of writing down the best response functions are given as follows;
Ann's best response function: $BR(a) = S$, *for all* $a \in \{S, W\}$ *where* a *is the action of Bob.*
Bob's best response function:

$$BR(a) = \begin{cases} W & \text{if } a = S \\ \\ S & \text{if } a = W \end{cases}$$

where a *is the action of Ann.*

-*What are the Pareto efficient outcomes of the game?*

(W, S) and (S, W) are the Pareto efficient outcomes of this game since there is no other outcome that makes someone better off without making the other worse off.

-*Is this game dominance solvable? If so, then what is the dominance solvable outcome?*

Yes, it is. First eliminate Ann's action W since it is strongly dominated by S. Then, the the payoff matrix for the smaller game is given below;

		Bob	
		S	W
Ann	S	$4, 1$	$10, 5$

Now, eliminate Bob's strategy S since it is strongly dominated by W. Hence, we obtain the dominance solvable outcome, which is (10,5).

3. Here is the payoff matrix for this game;

		James					
		0	250	500	5000	10000	20000
	0	$5000, 250$	$0, 250$	$0, 0$	$0, -4500$	$0, -9500$	$0, -19500$
	250	$9750, 0$	$4875, 125$	$0, 0$	$0, -4500$	$0, -9500$	$0, -19500$
	500	$9500, 0$	$9500, 0$	$4750, 0$	$0, -4500$	$0, -9500$	$0, -19500$
Slyvia	5000	$5000, 0$	$5000, 0$	$5000, 0$	$2500, -2250$	$0, -9500$	$0, -19500$
	10000	$0, 0$	$0, 0$	$0, 0$	$0, 0$	$0, -4750$	$0, -19500$
	20000	$-10000, 0$	$-10000, 0$	$-10000, 0$	$-10000, 0$	$-10000, 0$	$-5000, -9750$

-*Which strategies are weakly or strongly dominated. Eliminate weakly dominated strategies, then apply iterated strong dominance: which actions survive?*

20000 is strongly dominated by any other strategy for both Slyvia and James.
0 and 10000 is weakly dominated by 250, 500 and 5000 for Slyvia, and 0, 500, 5000 and 10000 are weakly dominated by 500 for James.

If we eliminate weakly dominated strategies, we obtain the following payoff matrix;

		James
		250
	250	$4875, 125$
Slyvia	500	$9500, 0$
	5000	$5000, 0$

Now, we can eliminate Slyvia's actions 250 and 5000 since both of them are strongly dominated by 500. Hence, 500 for Slyvia and 250 for James survive.

-Only apply iterated weak dominance : which actions survive?

First start eliminating Slyvia's weakly dominated strategies (actions) 0, 10000 and 20000, then we obtain the following payoff matrix;

		James					
		0	250	500	5000	10000	20000
	250	$9750, 0$	$4875, 125$	$0, 0$	$0, -4500$	$0, -9500$	$0, -19500$
Slyvia	500	$9500, 0$	$9500, 0$	$4750, 0$	$0, -4500$	$0, -9500$	$0, -19500$
	5000	$5000, 0$	$5000, 0$	$5000, 0$	$2500, -2250$	$0, -9500$	$0, -19500$

Then, eliminate James's weakly dominated strategies (actions) 0, 5000, 10000 and 20000, then we obtain the following payoff matrix;

		James	
		250	500
	250	$4875, 125$	$0, 0$
Slyvia	500	$9500, 0$	$4750, 0$
	5000	$5000, 0$	$5000, 0$

Now, eliminate 250 which is weakly dominated by 500 for Slyvia so that we obtain the following payoff matrix;

Slyvia \ James	250	500
500	$9500, 0$	$4750, 0$
5000	$5000, 0$	$5000, 0$

Hence, there is no available elimination further. Thus, actions 500 and 5000 of Slyvia and actions 250 and 500 of James will survive. Observe that the outcomes of two elimination methods are different.

-Find the reaction (best response) functions.

The best responses are underlined in the payoff matrix that is given below;

Slyvia \ James	0	250	500	5000	10000	20000
0	$5000, \underline{250}$	$0, \underline{250}$	$0, 0$	$0, -4500$	$\underline{0}, -9500$	$\underline{0}, -19500$
250	$\underline{9750}, 0$	$4875, \underline{125}$	$0, 0$	$0, -4500$	$\underline{0}, -9500$	$\underline{0}, -19500$
500	$9500, \underline{0}$	$\underline{9500}, \underline{0}$	$4750, \underline{0}$	$0, -4500$	$\underline{0}, -9500$	$\underline{0}, -19500$
5000	$5000, \underline{0}$	$5000, \underline{0}$	$\underline{5000}, \underline{0}$	$\underline{2500}, -2250$	$\underline{0}, -9500$	$\underline{0}, -19500$
10000	$0, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$\underline{0}, -4750$	$\underline{0}, -19500$
20000	$-10000, \underline{0}$	$-10000, \underline{0}$	$-10000, \underline{0}$	$-10000, \underline{0}$	$-10000, \underline{0}$	$-5000, -9750$

<u>OPTIONAL</u>: The best response function for Slyvia is given as follows;

$$BR(a) = \begin{cases} 250 & \textit{if } a = 0 \\ 500 & \textit{if } a = 250 \\ 5000 & \textit{if } a = 500, 5000 \\ 0, 250, 500, 5000, 10000 & \textit{if } a = 10000, 20000 \end{cases}$$

where a is the action of James.

The best response function for James is given as follows;

$$BR(a) = \begin{cases} 0, 250 & \textit{if } a = 0 \\ 250 & \textit{if } a = 250 \\ 0, 250, 500 & \textit{if } a = 500, 5000 \\ 0, 250, 500, 5000 & \textit{if } a = 10000 \\ 0, 250, 500, 5000, 10000 & \textit{if } a = 20000 \end{cases}$$

where a is the action of Slyvia.

4. Here is the payoff matrix for this game;

Keith \ Mick	0	1	2
0	$0, 0$	$6, -3$	$12, -6$
1	$-3, 6$	$3, 3$	$9, 0$
2	$-6, 12$	$0, 9$	$6, 6$

-Which outcomes are Pareto efficient?
(-6,12), (12,-6), (0,9), (9,0) and (6,6) are the Pareto efficient outcomes of this game since there is no way to make someone better of without making the other worse off.

-What is predicted by the theory of dominant strategy equilibrium? Find the reaction (best response) functions.
Since 1 and 2 are strongly dominated by 0 for both Keith and Mick, we can conclude that strategy profile (0,0) is the dominant strategy equilibrium. The best responses of Keith and Mick are underlined in the payoff matrix that is given as below;

		Mick 0	1	2
	0	$\underline{0}, \underline{0}$	$\underline{6}, -3$	$\underline{12}, -6$
Keith	1	$-3, \underline{6}$	$3, 3$	$9, 0$
	2	$-6, \underline{12}$	$0, 9$	$6, 6$

<u>OPTIONAL</u>: The best response function of both Keith and Mick is stated formally as follows; $BR(a) = 0$*, for all* $a \in \{0, 1, 2\}$.

Now suppose the utility functions for Keith and Mick are given by $2(x+y)-x$ and $2(x+y)-y$. Then, the payoff matrix is given as follows;

		Mick 0	1	2
	0	$0, 0$	$2, 1$	$4, 2$
Keith	1	$1, 2$	$3, 3$	$5, 4$
	2	$2, 4$	$4, 5$	$6, 6$

-*Which outcomes are Pareto efficient?*
(6,6) is the only Pareto efficient outcome of this game since there is no way to make someone better of without making the other worse off.

-*What is predicted by the theory of dominant strategy equilibrium? Find the reaction (best response) functions.*
Since 0 and 1 are strongly dominated by 2 for both Keith and Mick, we can conclude that strategy profile (2,2) is the dominant strategy equilibrium. The best responses of Keith and Mick are underlined in the payoff matrix that is given as below;

		Mick		
		0	1	2
	0	$0,0$	$2,1$	$4,\underline{2}$
Keith	1	$1,2$	$3,3$	$5,\underline{4}$
	2	$\underline{2},4$	$\underline{4},5$	$\underline{6},\underline{6}$

OPTIONAL: The best response function of Keith and Mick is given formally as follows; $BR(a) = 2$, *for all* $a \in \{0, 1, 2\}$ *where* a *is the action of the other player.*