PROBLEM SET #2 SOLUTIONS

1. Here is the payoff matrix of the game where best responses are underlined (20000 is eliminated since it is a strictly dominated strategy for each player);

<div align="center">James</div>

	0	250	500	5000	10000
0	$5000, \underline{250}$	$0, \underline{250}$	$0, 0$	$0, -4500$	$\underline{0}, -9500$
250	$\underline{9750}, 0$	$4875, \underline{125}$	$0, 0$	$0, -4500$	$\underline{0}, -9500$
500	$9500, \underline{0}$	$\underline{9500}, \underline{0}$	$4750, \underline{0}$	$0, -4500$	$\underline{0}, -9500$
5000	$5000, \underline{0}$	$5000, \underline{0}$	$\underline{5000}, \underline{0}$	$\underline{2500}, -2250$	$\underline{0}, -9500$
10000	$0, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$0, \underline{0}$	$\underline{0}, -4750$

Slyvia (row labels on left: 0, 250, 500, 5000, 10000)

Hence, (500,250) and (5000,500) are Nash equilibria since strategies are mutually best-response to each other.

2. Here is the payoff matrix for this game;

<div align="center">John</div>

	1	2
1	$1, 1$	$3, 2$
2	$2, 3$	$2, 2$

Paul (row labels: 1, 2)

-Which outcomes are Pareto efficient?

Strategy profiles (1,2) and (2,1) are the Pareto efficient outcomes of this game since there is no way to make someone better of without making the other worse off.

-What is predicted by the iterated elimination of strictly dominated strategies? What is predicted by the iterated elimination of weakly dominated strategies? Find the reaction (best response) functions.

Any strategy is neither strictly nor weakly dominated for both players. The best response function of Paul and John is underlined in the payoff matrix as follows;

John

		1	2
Paul	1	$1,1$	$\underline{3},2$
	2	$2,\underline{3}$	$2,2$

-State all Nash Equilibria of the game.
Strategy profiles (1,2) and (2,1) are Nash equilibria since strategies are mutually best-response to each other.

Now suppose the utility functions for Paul and John are given by $2min\{x+y\} - x$ and $2min\{x+y\} - y$. Then, the payoff matrix is given as follows;

John

		1	2
Paul	1	$1,1$	$1,0$
	2	$0,1$	$2,2$

-Which outcomes are Pareto efficient?
Strategy profile (2,2) is the only Pareto efficient outcome of this game since there is no way to make someone better of without making the other worse off.

-What is predicted by the iterated elimination of strictly dominated strategies? What is predicted by the iterated elimination of weakly dominated strategies? Find the reaction (best response) functions.
Any strategy is neither strictly nor weakly dominated for both players. The best response function of Paul and John is underlined in the payoff matrix as follows;

John

	1	2
Paul 1	$\underline{1}, \underline{1}$	$1, 0$
2	$0, 1$	$\underline{2}, \underline{2}$

-State all Nash Equilibria of the game.

Strategy profiles (1,1) and (2,2) are Nash equilibria since strategies are mutually best-response to each other.

3. Here is the payoff matrix for this game;

Bill

	L	M	R
Alice U	$8, 1$	$0, 4$	$2, 9$
C	$6, 0$	$1, 1$	$3, 0$
D	$2, 2$	$1, 3$	$4, 4$

-What would iterated elimination of strictly dominated strategies predict for this game?

Since L is strictly dominated by M, we can eliminate L and we obtain the following smaller matrix;

Bill

	M	R
Alice U	$0, 4$	$2, 9$
C	$1, 1$	$3, 0$
D	$1, 3$	$4, 4$

Now, U is strictly dominated by both C and D implying that U is eliminated, and we obtain the following payoff matrix;

	Bill	
	M	R
C	1, 1	3, 0
D	1, 3	4, 4

(Alice is the row player labeled on the left: C, D)

There is no more strict dominance relation between the remaining actions. Hence, iteration ends here.

-What would the prediction of iterated elimination of weakly dominant strategies be?

First, we follow the steps above. Then, observe that C is weakly dominated by D implying that C is eliminated and we obtain a smaller payoff matrix;

	Bill	
	M	R
Alice D	1, 3	4, 4

Now, M is strictly dominated by R. Thus, M is eliminated, and hence the outcome of iterated elimination of weakly dominated strategies is (D, R).

-Find the reaction (best response) functions.

The best response function of Alice and Bill is underlined in the payoff matrix as follows;

		Bill	
	L	M	R
U	$\underline{8}, 1$	0, 4	2, $\underline{9}$
C	6, 0	$\underline{1}, \underline{1}$	3, 0
D	2, 2	$\underline{1}, 3$	$\underline{4}, \underline{4}$

(Alice is the row player: U, C, D)

-What are the Nash Equilibria of this game?

(C, M) and (D, R) are the Nash equilibria of this game since strategies are mutually best-response to each other.

4. (a) The profit functions of Bell and HPhi are given below;

$$\Pi_B(x_B, x_H) = [90 - 2(x_B + x_H)]x_B - 2x_B$$

$$\Pi_H(x_B, x_H) = [90 - 2(x_B + x_H)]x_H - 4x_H$$

where B denotes Bell and H denotes HPhi.

(b) Best response function of a firm is the output level that maximizes the profit for a given level of other firm's output. Formally, best response of Bell to HPhi's output is determined by

$$\max_{x_B} \Pi_B(x_B, x_H) = [90 - 2(x_B + x_H)]x_B - 2x_B$$

Since the profit is maximized when $\frac{\partial \Pi_B}{\partial x_B} = 0$, we obtain

$$\frac{\partial \Pi_B}{\partial x_B} = 90 - 4x_B - 2x_H - 2 = 0$$

$$88 - 2x_H = 4x_B$$

$$\boxed{x_B = 22 - \frac{x_H}{2}}$$

Similarly, HPhi's best response to Bell's output level is determined by the following;

$$\max_{x_H} \Pi_H(x_B, x_H) = [90 - 2(x_B + x_H)]x_H - 4x_H$$

Since the profit is maximized when $\frac{\partial \Pi_H}{\partial x_H} = 0$, we obtain

$$\frac{\partial \Pi_H}{\partial x_H} = 90 - 4x_H - 2x_B - 4 = 0$$

$$86 - 2x_B = 4x_H$$

$$\boxed{x_H = 21.5 - \frac{x_B}{2}}$$

(c) Take $x_B = 30$ and $x_B = 40$. We claim that these two particular choices of x_B are strictly dominated by 20. All we need to do is to compare profits for any given value of x_H.

$$\Pi_B(20, x_H) = [88 - 2(20 + x_H)]20 = 960 - 40x_H$$

$$\Pi_B(30, x_H) = [88 - 2(30 + x_H)]30 = 840 - 60x_H$$

$$\Pi_B(40, x_H) = [88 - 2(40 + x_H)]40 = 320 - 80x_H$$

Hence,

$$960 - 40x_H > 840 - 60x_H > 320 - 80x_H$$

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for any $x_H \geq 0$. Thus, 30 and 40 are strictly dominated by 20.

Now, take two possible choices of x_H; 20 and 30. We can determine the best choice of x_B in response to 20 and 30 by using the best response function obtained in part (a);

$$x_B = 22 - \frac{20}{2} = 12$$

$$x_B = 22 - \frac{30}{2} = 7$$

So, we can conclude that 7 and 12 are never strictly dominated by any other choice of x_B since there is no other output level that yields higher profit than 7 and 12 when the other firm HPhi chooses 20 and 30, respectively.

(d) Nash equilibrium is the outcome where each firm produces the best response output to the other's output level. If we plug one best response to the other, we obtain

$$x_H^* = 21.5 - \frac{22 - \frac{x_H^*}{2}}{2}$$

$$x_H^* = 21.5 - (11 - \frac{x_H^*}{4})$$

$$\frac{3}{4}x_H^* = 10.5$$

$$\boxed{x_H^* = 14}$$

Since $x_B^* = 22 - \frac{x_H^*}{2}$ and $x_H^* = 14$, then we obtain

$$x_B^* = 22 - \frac{14}{2}$$

$$\boxed{x_B^* = 15}$$

Hence, $(x_B^*, x_H^*) = (15, 14)$ is the Nash equilibrium.

5. Since there are three players in this game, third player's strategies are represented by matrices. So, there will be 2 payoff matrices corresponding to strategies of player3. They are given below with the underlined best responses;

$favor Player 1 (fP1)$

	E	D
E	$\underline{3}, \underline{0}, 0$	$\underline{3}, 0, 0$
D	$\underline{3}, 0, \underline{0}$	$1, 1, 1$

$favor Player 2 (fP2)$

	E	D
E	$0, \underline{3}, 0$	$0, \underline{3}, \underline{0}$
D	$0, \underline{3}, \underline{0}$	$\underline{1}, 1, \underline{1}$

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where matrix on the left represents the strategy $favorPlayer1(fP1)$ of player3 and the one on the right represents the strategy $favorPlayer2(fP2)$. As usual player1 is the row player and player2 is the column player. Player3's best responses are determined by comparing his payoffs in two matrices. For instance, both $fP1$ and $fP2$ are best responses to (E, E) since they both yield 0. Hence, there are 4 Nash equilibria of this game; $(E, E, fP1)$, $(E, E, fP2)$, $(E, D, fP1)$ and $(D, E, fP2)$.