Competitive Equilibrium with Pure Exchange

- Simplest example
- The economy exists for one period
- There is no production
- People are endowed with goods and services
- They trade them voluntarily
- Preferences and endowments are private information

Questions

- What is the competitive mechanism?
- What happens if people reveal the truth about their preferences and endowments?
- Is the mechanism incentive compatible?
- Implications for free trade

Economic Fundamentals

n traders

k goods

x^i_j consumption by trader i of good j

x^i denote the vector, bundle or basket of goods consumed by trader j

trader i's preferences for consuming different goods given by her utility function $u^i(x^i)$

trader i endowed with $\overline{x}^i_j$ of good j

- there is no production in this economy, it is a *pure exchange* economy
- traders simply exchange goods with each other
- the economy lasts only one period

The Market Mechanism

p_j the price of good j

p list of all prices of all goods, or the price vector

competitive equilibrium prices $\hat{p}$ and quantities $\hat{x}$ are such that based on the announced endowments and preferences every trader can simultaneously satisfy her desire to trade at those prices

Implementation Issues

Many possible implementations: double-oral auctions, posted price mechanisms

Two key elements:

- traders scope out opportunities to such an extent that each good is sold (and purchased) at only one price: gives rise to “the law of one price”
- incentive compatibility: arises from **competitive behavior** traders do not perceive that they have any influence over market prices

(competitive equilibrium is not the Nash equilibrium of the competitive game…)

Tools of the Trade: Demand Theory

$x_j^i(p, m)$ demand by trader i for good j when prices are p and money income is m

the solution to the problem

$\max_{x^i} u^i(x^i)$

subject to $\sum_{j=1}^{k} p_j x_j^i \leq m$ or $p \cdot x^i \leq m$

Excess Demand

in pure exchange economy money income generated by selling endowment

[doesn't cost anything extra to sell your endowment then buy it back, since the prices at which you buy and sell are the same]

demand to buy or net, or excess demand

$$z_j^i(p, \overline{x}^i) \equiv x_j^i(p, \sum_{j=1}^{m} p_j \cdot \overline{x}_j^i) - \overline{x}_j^i.$$

this can be negative, as big as $-\overline{x}_j^i$

Cobb Douglas Example

$u(x_1, x_2) = Ax_1^{\alpha}x_2^{\beta}$

demand from Lagrangean

$Ax_1^{\alpha}x_2^{\beta} - \lambda(p_1x_1 + p_2x_2)$

first order conditions

$A\alpha x_1^{\alpha-1}x_2^{\beta} - \lambda p_1 = 0$

$A\beta x_1^{\alpha}x_2^{\beta-1} - \lambda p_2 = 0$

rearrange and divide

$\frac{A\alpha x_1^{\alpha-1}x_2^{\beta}}{A\beta x_1^{\alpha}x_2^{\beta-1}} = \frac{\lambda p_1}{\lambda p_2}$ cancelling terms $\frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$

$\frac{\alpha x_2}{\beta x_1} = \frac{p_1}{p_2}$ cross multiply $\alpha p_2 x_2 = \beta p_1 x_1$

plug into the budget constraint $p_1 x_1 + p_2 x_2 = m$ to get

$p_1 x_1 + (\beta / \alpha) p_1 x_1 = m$

or $x_1 = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}$, $x_2 = \frac{\alpha}{\alpha + \beta} \frac{m}{p_2}$

excess demand

$$z_1 = \frac{\alpha}{\alpha + \beta} \frac{p_1 \bar{x}_1 + p_2 \bar{x}_2}{p_1} - \bar{x}_1, \; x_2 = \frac{\beta}{\alpha + \beta} \frac{p_1 \bar{x}_1 + p_2 \bar{x}_2}{p_2} - \bar{x}_2$$

for simplicity take $\alpha + \beta = 1$ then

$$z_1 = \alpha \frac{p_2}{p_1} \bar{x}_2 - (1 - \alpha) \bar{x}_1, \; z_2 = (1 - \alpha) \frac{p_1 \bar{x}_1}{p_2} - \alpha \bar{x}_2$$

Aggregate or Market Excess Demand

$$z_j(p) \equiv \sum_{i=1}^{n} z_j^i(p)$$

in market for each good demand to buy cannot exceed zero

there is no production or outsider to provide supply to the market

one traders excess demand must be anothers excess supply

$\hat{p}$ competitive equilibrium prices are determined by

$z_j(\hat{p}) \leq 0$ for every good $j = 1, 2, \ldots, k$

Cobb-Douglas Economy

Two consumers both have identical preferences with $\alpha = 1/2$

Endowments are $\overline{x}^1 = (1,0), \overline{x}^2 = (0,1)$

$$z_1^1 = -\frac{1}{2}$$

$$z_1^2 = \frac{1}{2}\frac{p_2}{p_1} \text{ so } z_1 = -\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}$$

$$z_2^1 = \frac{1}{2}\frac{p_1}{p_2}$$

$$z_2^2 = -\frac{1}{2} \text{ so } z_2 = \frac{1}{2}\frac{p_1}{p_2} - \frac{1}{2}$$

we aren't going to solve this yet

Properties of Demand

Individual demand: two key properties

Homogeneous of degree zero

$$x_j^i(\lambda p, \lambda m) = x_j^i(p, m)$$

(relationship to inflation, dollars versus quarters)

$$x_1 = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}$$

Satisfies the budget constraint

$$\sum_{j=1}^{k} p_j x_j^i(p_j, m) = m$$

$$p_1 \frac{\alpha}{\alpha + \beta} \frac{m}{p_1} + p_2 \frac{\beta}{\alpha + \beta} \frac{m}{p_2} = ??$$

Individual excess demand: two key properties

Homogeneous of degree zero

$$z_j^i(\lambda p) = z_j^i(p)$$

$$z_1 = \alpha \frac{p_2}{p_1} \overline{x}_2 - (1-\alpha)\overline{x}_1$$

Walras's Law

$$\sum_{j=1}^{k} p_j z_j^i(p) = 0$$

proof:

$$\begin{aligned} m &= \sum_{j=1}^{k} p_j x_j^i(p_j, m) \\ &= \sum_{j=1}^{k} p_j x_j^i(p_j, p \cdot \overline{x}^i) - p_j \overline{x}_j^i + p_j \overline{x}_j^i \\ &= \sum_{j=1}^{k} p_j z_j^i(p_j) + \sum_{j=1}^{k} p_j \overline{x}_j^i \\ &= \sum_{j=1}^{k} p_j z_j^i(p_j) + m \end{aligned}$$

Aggregate excess demand: two key properties

Homogeneous of degree zero

$z_j(\lambda p) = z_j(p)$

$z_1 = -\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}$

Walras's Law

$\sum_{j=1}^{k} p_j z_j(p) = 0$

$$p_1\left[-\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}\right] + p_2\left[\frac{1}{2}\frac{p_1}{p_2} - \frac{1}{2}\right] = ??$$