The Competitive Mechanism

Cobb-Douglas Aggregate Excess Demans:

$$z_1 = -\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}$$

$$z_2 = \frac{1}{2}\frac{p_1}{p_2} - \frac{1}{2}$$

Aggregate excess demand: two key properties

Homogeneous of degree zero

$z_j(\lambda p) = z_j(p)$

$z_1 = -\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}$

Walras's Law

$\sum_{j=1}^{k} p_j z_j(p) = 0$

$$p_1\left[-\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1}\right] + p_2\left[\frac{1}{2}\frac{p_1}{p_2} - \frac{1}{2}\right] = ??$$

Solving for Equilibrium

there are k different excess demand conditions $z_j(p) = 0$ and there are k different prices $p_1, \ldots, p_k$

but one excess demand condition is redundant

suppose $z_j(p) = 0$ for $j = 1, \ldots, k-1$, then from Walras's law $z_k(p) = 0$

on the other hand, if $z_j(p) = 0$ for all $j = 1, \ldots, k$ then so does $z_j(\lambda p) = 0$

so many competitive equilibria

can solve only for relative prices using $k-1$ excess demand equations

Existence of competitive equilibrium?

The Numeraire

may arbitrarily set the price of one good to 1

called the numeraire good, all prices are measured relative to that good

(for example – money is numeraire)

Cobb-Douglas Example

Pick one equation

$$z_1 = -\frac{1}{2} + \frac{1}{2}\frac{p_2}{p_1} = 0$$

so $p_2 / p_1 = 1$

pick the other equation

$$z_2 = \frac{1}{2}\frac{p_1}{p_2} - \frac{1}{2}$$

get the same answer of course

if we choose good 1 as numeraire then we have $p_1 = 1, p_2 = 1$

how do we find individual demands?

The First Welfare Theorem

Suppose we have a competitive equilibrium with prices p and individual demands x_i^j

is this pareto efficient?

That is: can we find $\tilde{x}_j^i$ socially feasible that makes nobody worse off and at least one person better off?

That is: can we find $\sum_{i=1}^n \tilde{x}_j^i \leq \sum_{i=1}^n \overline{x}_j^i$ so that $u^i(\tilde{x}^i) \geq u^i(\overline{x}^i)$ for everybody (all i) and for somebody (some i) $u^i(\tilde{x}^i) > u^i(\overline{x}^i)$?

Observation: if $u^i(\tilde{x}^i) > u^i(\overline{x}^i)$ then $p \cdot \tilde{x}^i > p \cdot \overline{x}^i$

Why??

Further observation: $u^i(\tilde{x}^i) \geq u^i(\overline{x}^i)$ then $p \cdot \tilde{x}^i \geq p \cdot \overline{x}^i$ (otherwise spend your extra income to buy more)

Our conclusion: if $u^i(\tilde{x}^i) \geq u^i(\overline{x}^i)$ for everybody (all i) and for somebody (some i) $u^i(\tilde{x}^i) > u^i(\overline{x}^i)$, then

$p \cdot \tilde{x}^i \geq p \cdot \overline{x}^i$ for all i and $p \cdot \tilde{x}^i > p \cdot \overline{x}^i$ for some i

add these together:

$\sum_{i=1}^n p \cdot \tilde{x}^i > \sum_{i=1}^n p \cdot \overline{x}^i$

on the other hand

$\sum_{i=1}^n \tilde{x}_j^i \leq \sum_{i=1}^n \overline{x}_j^i$, so adding over different goods

$\sum_{j=1}^k p_j \sum_{i=1}^n \tilde{x}_j^i \leq \sum_{j=1}^k p_j \sum_{i=1}^n \overline{x}_j^i$

which says that $\sum_{i=1}^n p \cdot \tilde{x}^i \leq \sum_{i=1}^n p \cdot \overline{x}^i$

- relationship to the core
- implications for international trade
- the edgeworth box
- the second welfare theorem
- the competitive mechanism

Finance

Trade in period 0 claims to consumption in period 1

k different states of nature in period 1, probability of state j is π_j

consumption in state j is c_j

time 0 price of consumption in state j is p_j

budget constraint $\sum_{j=1}^{k} p_j c_j \leq m$

"martingale" prices $\tilde{p}_j = p_j / \pi_j$

budget constraint in martingale prices

$E\tilde{p}c = \sum_{j=1}^{k} \pi_j \tilde{p}_j c_j \leq m$

Securities

Security a pays r_j in state j

Examples:

Arrow security on state j pays 1 in state j 0 in all other states

Price of an arrow security p_j or $\tilde{p}_j$

Bond pays 1 in all states

Price of a bond $\sum_{j=1}^{k} p_j$ or $E\tilde{p}$

“arbitrage pricing” = law of one price

Spanning

Two states $j = 1{,}2$, $k = 2$

Stock 1 pays $(2{,}1)$ price q_1, Stock 2 pays $(1{,}2)$ price q_2

What is the price of a bond?

Buy both stocks: get $(3,3)$ bond pays $(1{,}1)$ so 1/3$^{\text{rd}}$ of both stocks

$q_b = (q_1 + q_2)/3$

what does spanning mean?

k different assets that are “independent”

can determine all asset prices in terms of a spanning set of assets

Short Sales

Stock 1 pays $(2,1)$ price q_1, Stock 2 pays $(3,1)$ price q_2

What is the price of a bond?

$$a(2,1) + b(3,1) = (1,1)$$

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= (-1) \begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

check $2(2,1) - 1(3,1) = (4,2) - (3,1) = (1,1)$

so buying 2 units of stock 1 and -1 units of stock 2 is the same as buying a bond: cost $2q_1 - q_2$

what does it mean to buy -1 unit of stock 2?

observation $2q_1 - q_2 > 0$ so we can conclude that $q_1 > q_2/2$

The Holdup Problem

"entrepreneur" (inventor, merchant) creates value of ρ

ρ is drawn from a uniform distribution over $[0,1]$ and is private information to the entrepreneur

case 1: the innovator receives a fraction of the social total $\phi\rho$

case 2: the innovator receives the entire social total ρ but must pay N existing "rights holders" for the right to create value

examples:

the silk road

patents and copyrights

pollution

efficiency = the good is always produced

in case 1 the good is always produced

in case 2

rights holder i set price p_i for his right and gets an expected revenue of

$$\left(1-(N-1)p-p_i\right)p_i$$

$p = 1/(N+1)$

entrepreneur pays $\frac{N}{N+1}$ to clear the needed rights, so creation if $\frac{N}{N+1} < \rho$

what happens as $N \to \infty$

as technologies grow more and more complex requiring more and more specialized inputs,monopoly power induced by patents and copyright becomes more and more socially damaging