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Introductory Lecture

What this class is about

- economic science as it exists today
- intrinsically a mathematical subject
- the heart of economics is mechanism design theory
- the goal of the class is to give you a limited working knowledge of mechanism design theory



The Problem of the Consumer

$$\max u(x_1, x_2)$$

subject to $p_1x_1 + p_2x_2 \leq I$ (budget set)

x_1, x_2 quantity of good 1,2 consumed respectively

p_1, p_2 money prices of goods 1 and 2

I money income

Utility Theory

- utility we take as given in this class
- utility is not primitive, preferences are
- under mild assumptions preferences can be *represented* by a utility function
- the cocktail party approach to science
 - all things are relative
 - economists assume people are lightning calculators of pleasure and pain

Demand Theory

solution of the consumer problem

$$\max u(x_1, x_2)$$

subject to $p_1x_1 + p_2x_2 \leq I$ (budget set)

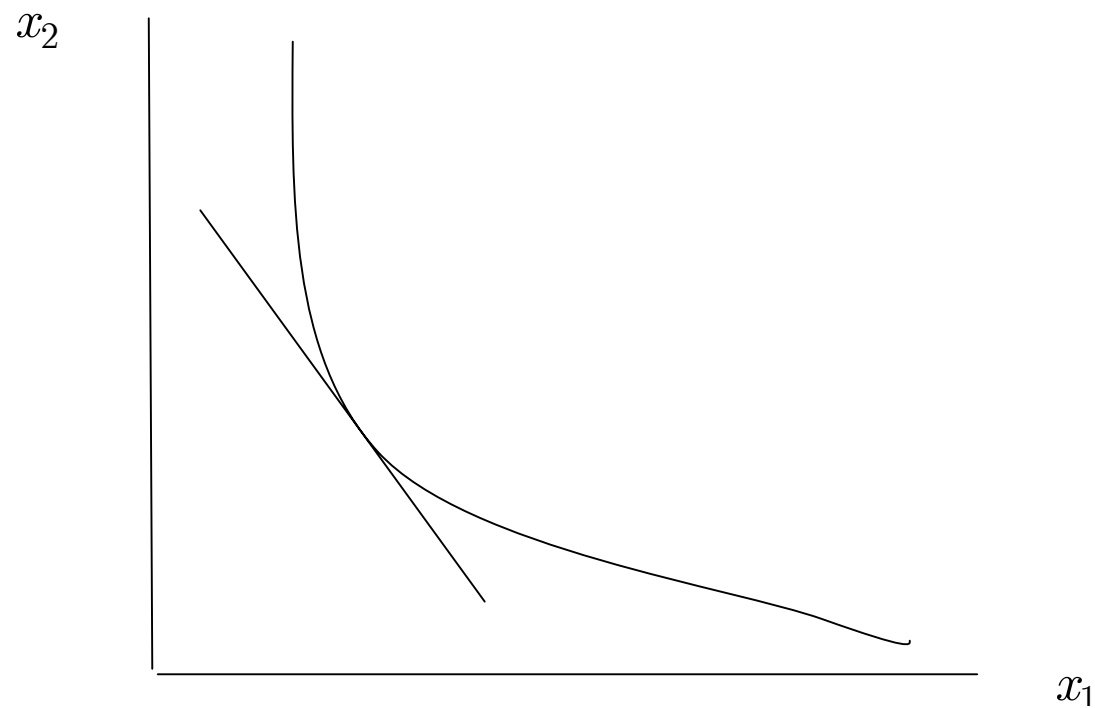
indifference curve $u(x_1, x_2) = \bar{u}$

find the slope: $\frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2 = d\bar{u} = 0$

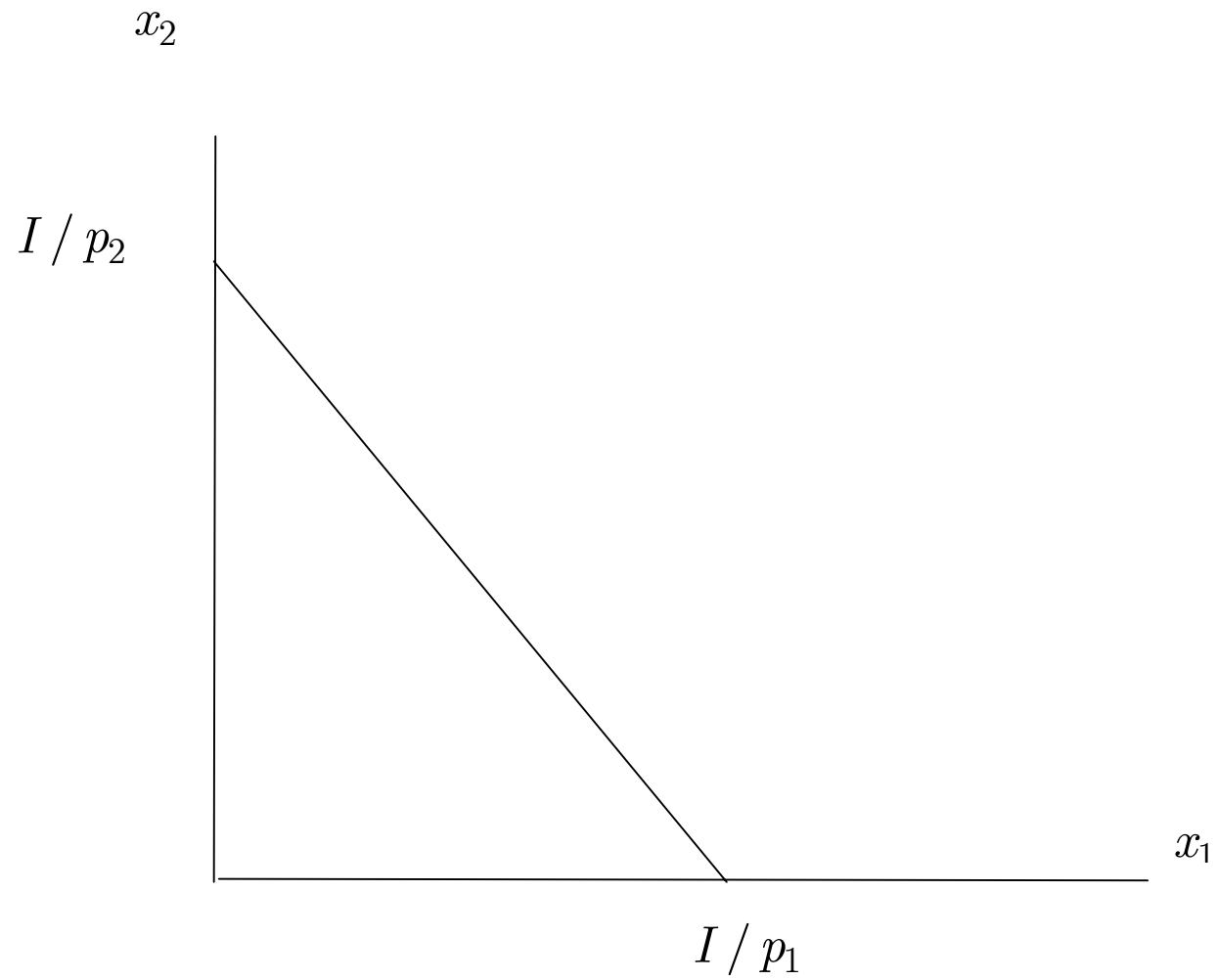
solution: $\frac{dx_2}{dx_1} = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2}$

example: Cobb-Douglas utility $u(x_1, x_2) = x_1^\alpha x_2^\beta$, $\alpha, \beta > 0$

$$\frac{dx_2}{dx_1} = -\frac{\alpha x_1^{\alpha-1} x_2^\beta}{\beta x_1^\alpha x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1}$$

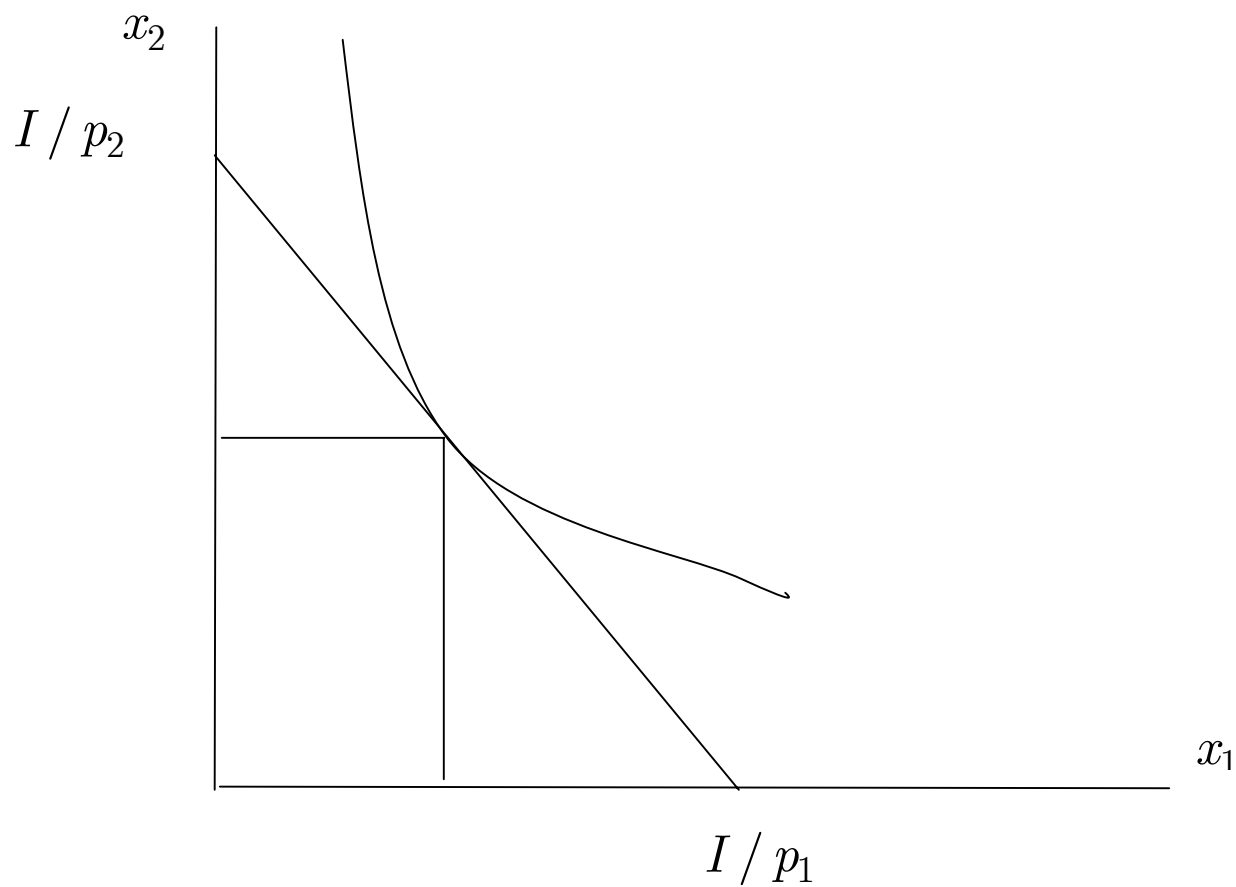


slope of the budget line: $-p_1 / p_2$



The Optimum

tangency between the budget line and indifference curve



Cobb-Douglas

$$-\frac{\alpha x_2}{\beta x_1} = -\frac{p_1}{p_2} \text{ tangency condition of equal slopes}$$

$p_1 x_1 + p_2 x_2 = I$ budget constraint holds with equality

two equations in two unknowns

solve the tangency $p_2 x_2 = (\beta / \alpha) p_1 x_1$

plug in the budget constraint $p_1 x_1 + (\beta / \alpha) p_1 x_1 = I$

solve for the demand function

$$x_1 = \frac{\alpha}{\alpha + \beta} \frac{I}{p_1}$$

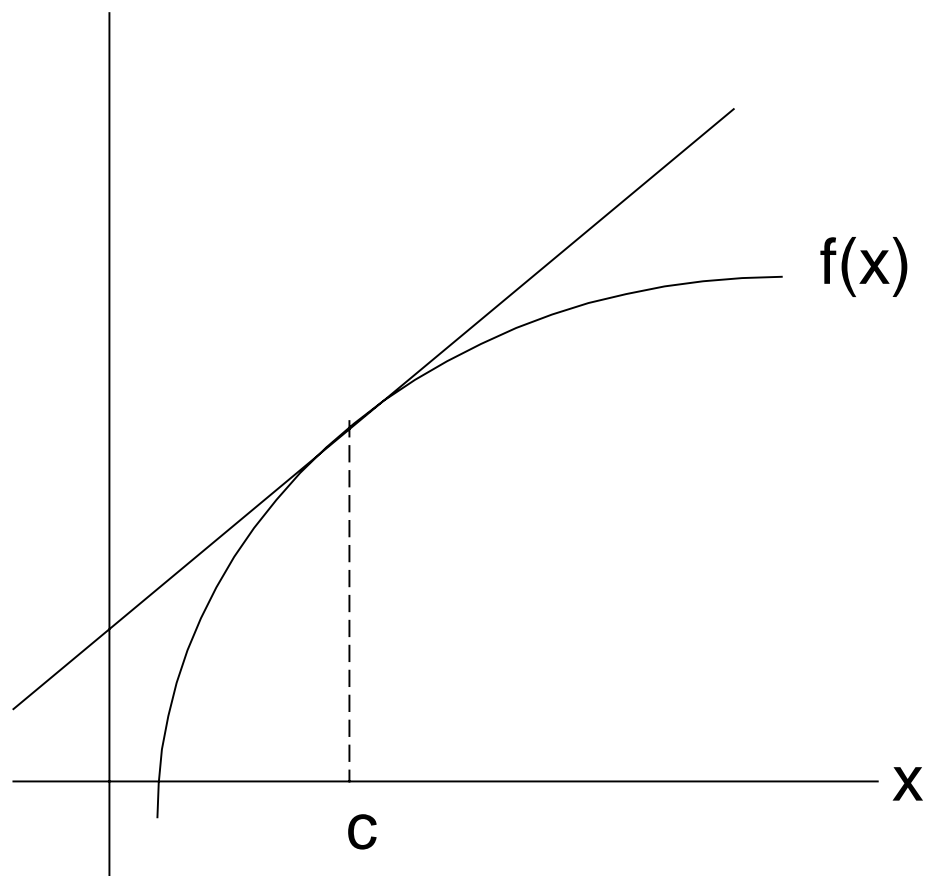
bigger α the more you like x_1 and the more you demand

homogeneity of degree zero: depends only on the ration I / p_1

Lagrange Multipliers

$\max f(x)$ subject to $x \leq c$

obvious answer: if f is concave and $f'(c) > 0$ take $x = c$



Lagrange Multipliers

$$L = f(x) - \lambda(x - c)$$

first order conditions for a maximum

$$\frac{\partial L}{\partial x} = f'(x) - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = -(x - c) = 0$$

solution: $x = c, \lambda = f'(x)$

what is measured by the multiplier λ ?

the increase in the objective when the constraint c is relaxed

Example

$$\max (x_1^\rho + x_2^\rho + x_3^\rho)^{1/\rho}$$

$$\text{subject to } p_1x_1 + p_2x_2 + p_3x_3 = I$$

Lagrangian

$$L = (x_1^\rho + x_2^\rho + x_3^\rho)^{1/\rho} - \lambda(p_1x_1 + p_2x_2 + p_3x_3 - I)$$

solution:

$$\frac{\partial L}{\partial x_i} = \frac{1}{\rho} (x_1^\rho + x_2^\rho + x_3^\rho)^{(1/\rho)-1} \rho x_i^{\rho-1} - \lambda p_i = 0$$

solve:

$$\begin{aligned} x_i &= \left[\frac{\lambda p_i}{(x_1^\rho + x_2^\rho + x_3^\rho)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} \\ &= \left[\frac{p_i}{(x_1^\rho + x_2^\rho + x_3^\rho)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} [\lambda]^{\frac{1}{\rho-1}} \end{aligned}$$

substitute in constraint

$$\sum_i p_i^{1+\frac{1}{\rho-1}} \left[\frac{\lambda}{(x_1^\rho + x_2^\rho + x_3^\rho)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} = I$$

$$[\lambda]^{\frac{1}{\rho-1}} = \frac{I}{\sum_i p_i^{\frac{\rho}{\rho-1}} (x_1^\rho + x_2^\rho + x_3^\rho)^{\frac{1}{\rho}}}$$

substitute back into solution of FOC

$$\begin{aligned} x_i &= \left[\frac{p_i}{(x_1^\rho + x_2^\rho + x_3^\rho)^{(1/\rho)-1}} \right]^{\frac{1}{\rho-1}} \frac{I}{\sum_j p_j^{\frac{\rho}{\rho-1}} (x_1^\rho + x_2^\rho + x_3^\rho)^{\frac{1}{\rho}}} \\ &= [p_i]^{\frac{1}{\rho-1}} \frac{I}{\sum_j p_j^{\frac{\rho}{\rho-1}}} \end{aligned}$$

this is the *CES* demand function widely used in empirical research

note that it satisfies homogeneity of degree zero in prices and income