Bargaining

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Econ 4011

Haggling and its variants

- The price of vegetables at the local vendor.
- Salary of NFL players
- Divorce settlements
- International border disputes
- Trade unions and firms

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Basic Components

- Multiple players
- Surplus
  - The value of a full season of football
  - The cost of a lengthy court case
  - The cost of war
- Necessity of agreement

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- What are the shares for each player?
- Efficiency
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Axiomatic/Cooperative Approach

- S is the set of all possible outcomes in terms of utility.
- $s_0 \in S$ is the disagreement outcome.
- (S, s_0) is a *bargaining problem*.
- A bargaining solution simply chooses a possible outcome for any given bargaining problem.
- Examples
- Impose conditions which Bargaining solutions must satisfy
- Nash, Kalai Smorodinsky, Proportional Solution

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Non cooperative approach

- Explicitly model the negotiation process as a game.
- Solve for Nash Equilibria, Subgame Perfect Equilibria.
- Is the equilibrium efficient?
- Does it favour some players over others?

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Discrete Nash Demand Game

- 2 players, John and Oskar
- Simultaneously announce demands from $\{0, 1/3, 1/2, 2/3, 1\}$
- If $x_j + x_o \leq 1$ then each player gets his demand.
- Otherwise they both get a payoff of 0
- What are the Nash Equilibria of this game?
- Efficiency?

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Shrinking Cake Alternating Offers

- 2 players, Ingolf and Ariel
- First period: Ingolf proposes a division $(x_i, 1 - x_i)$, where $x_i \in \{0, 1/3, 1/2, 2/3, 1\}$.
- If Ariel accepts the division, he gets $1 - x_i$, while Ingolf gets x_i and the game ends.
- If Ariel rejects then the cake shrinks to size δ and the second period starts.
- Second period: Ariel proposes a division $(1 - x_a, x_a)$, where $x_a \in \{0, 1/3, 1/2, 2/3, 1\}$.
- If Ingolf accepts the division, he gets $\delta(1 - x_a)$, while Ariel gets $\delta(x_a)$ and the game ends.
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Commitment in Bargaining

- John and Oskar announce their demands x_J and x_O simultaneously from the set $\{0, 1/3, 1/2, 2/3, 1\}$.
- If the sum doesn't exceed 1 they get their own demands.
- Otherwise they play the game depicted below.
- What does subgame perfection predict?

	Accept	*Stick*
Accept	$1 - x_O, 1 - x_J$	$1 - x_O - 0.49, x_O$
Stick	$x_J, 1 - x_J - 0.49$	$0, 0$

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