Mechanism Design

Rohan Dutta/David Levine
Washington University in St. Louis

Econ 4011

- Two people in the economy, Eric and Roger.
- Should we build the bridge?
- How should it be paid for?
- Costs 40 to build the bridge.
- Eric and Roger are both one of two types(value for the bridge).
  - High(H) 100.
  - Low(L) 0.
- Each person's type is private information.
- Commonly known probability of being a high type $\pi^h = 0.8$
- No one can be forced to pay.
- Once built, anyone can use the bridge.

Objective: To maximize the sum of Eric and Roger's expected payoffs.

Complete Information Benchmark

Suppose the types of Eric and Roger were observable (not private information).
Example of an allocation and payment rule.

- If both are high type the bridge is built if and only if each pays 20.
- If one is a high type and the other low, the bridge is built if and only if the high type pays 40.
- If both are low type the bridge is not built and nobody pays anything.

Incentives and Payoffs

Incentives

If $v_E = 100$ and $v_R = 0$, then Eric would want to pay 40 and get the payoff of 60.

Not paying 40 would give Eric a payoff of 0.

If $v_E = v_R = 100$

Better to pay 20 and get a payoff of 80 than the 0 from not paying.

Payoffs

Expected payoff to Eric (without knowing his type):

$(0.64)(80) + (0.16)(60) = 60.8$

Sum of expected payoffs to Eric and Roger $= 121.6$.

Incomplete Information

- Cannot devise allocation and payment rule conditional on types.
- How about a constant fee?
- The bridge is built if and only if Eric and Roger pays 20 each.
- If only one pays 20, he simply gets it back and no bridge is built.

Incentives and Payoffs

Incentives

A high type always prefers to pay 20.

The low type always prefers to pay nothing.

Payoffs

The device is installed only when both players have high type.

Expected payoff to Eric (without knowing his type): $0.64(80) = 51.2$.

Sum of expected payoffs to Eric and Roger $= 102.4$.

Other Mechanisms?

- Build or don't build.
- Probability of building $q \in [0, 1]$.
- Specify strategies available to Eric and Roger.
- Each strategy profile results in an outcome.
- Outcomes comprise of payment rules (who pays what) and chances of the bridge being built.
- Specifying available strategies and outcome functions results in a game between Eric and Roger.
- Eric and Roger choose strategies that constitute a Bayesian Nash Equilibrium of this game.

Revelation Game

- Strategies are simply announcements of types.
- Each type of Eric and Roger chooses to announce H or L.
- $s_i^j \in \{H, L\}$, $i \in \{E, R\}$ and $j \in \{H, L\}$.
- The probability of building the bridge is a function of the two announcements.
  - q^{hh}, Both announce H
  - q^{lh}, One announces H and the other L
  - q^{ll}, Both announce L

Revelation Game

- If the bridge is built then the payments are also determined by announcements.
- HH, then both players pay p^{hh}.
- LL, then both pay p^{ll}.
- LH, Eric pays p^{lh} while Roger pays p^{hl}.
- HL, Eric pays p^{hl} while Roger pays p^{lh}.

Truthful Equilibrium

A truthful equilibrium of a revelation game is a Bayes Nash Equilibrium in which each type of each player announces their true type.

Payoffs

Suppose each type of each player did report truthfully.

Eric's high type's expected payoff:

$$u_E^h = \pi^h q^{hh}[v^h - p^{hh}] + (1 - \pi^h)q^{lh}[v^h - p^{hl}]$$

Eric's low type's expected payoff:

$$u_E^l = \pi^h q^{lh}[v^l - p^{lh}] + (1 - \pi^h)q^{ll}[v^l - p^{ll}]$$

Eric's expected payoff (without knowing his type)

$$u_E = \pi^h u_E^h + (1 - \pi^h)u_E^l$$

Sum of expected payoffs for Eric and Roger (without knowing their types)

$$\begin{aligned} u_E + u_R = &(\pi^h)^2 q^{hh} 2[v^h - p^{hh}] + 2(\pi^h)(1 - \pi^h)q^{lh}(v^h - p^{hl} + v^l - p^{lh}) \\ &+ (1 - \pi^h)^2 q^{ll} 2[v^l - p^{ll}] \end{aligned}$$

Goal: Find the truthful equilibrium that yields the highest sum of expected payoffs to Eric and George.

$$
\begin{aligned}
u_E + u_R =& (\pi^h)^2 q^{hh} 2[v^h - p^{hh}] + 2(\pi^h)(1-\pi^h) q^{lh} (v^h - p^{hl} + v^l - p^{lh}) \\
& + (1-\pi^h)^2 q^{ll} 2[v^l - p^{ll}] \\
=& (1.28) q^{hh} [100 - p^{hh}] + (0.32) q^{lh} (100 - p^{hl} + 0 - p^{lh}) \\
& + (0.08) q^{ll} [0 - p^{ll}]
\end{aligned}
$$

Why not set prices to 0 and always build the bridge?

If the bridge is made, Eric and Roger's payments must cover the cost.

$2p^{hh} \geq 40$ (If made when HH)

$p^{hl} + p^{lh} \geq 40$ (If made when HL and LH)

$2p^{ll} \geq 40$ (If made when LL)

Individual Rationality

No one can be forced to pay.
Each type of each person must choose to participate in the game.

$$\pi^h q^{hh}[v^h - p^{hh}] + (1 - \pi^h) q^{lh}[v^h - p^{hl}] \geq 0 \quad (1)$$

$$\pi^h q^{lh}[v^l - p^{lh}] + (1 - \pi^h) q^{ll}[v^l - p^{ll}] \geq 0 \quad (2)$$

Inequality (1) is the Individual Rationality (IR) constraint for the high type of both Eric and Roger.

Inequality (2) is the IR constraint for the low type.

Incentive Compatibility

Suppose each type of each player did report truthfully.

Each type must have the incentive to report truthfully.

Incentive Compatibility(IC) constraint for high type.

$$\pi^h q^{hh}[v^h - p^{hh}] + (1-\pi^h) q^{lh}[v^h - p^{hl}] \geq \pi^h q^{lh}[v^h - p^{lh}] + (1-\pi^h) q^{ll}[v^h - p^{ll}] \quad (3)$$

IC constraint for low type.

$$\pi^h q^{lh}[v^l - p^{lh}] + (1-\pi^h) q^{ll}[v^l - p^{ll}] \geq \pi^h q^{hh}[v^l - p^{hh}] + (1-\pi^h) q^{lh}[v^l - p^{hl}] \quad (4)$$

Assumptions

- $q^{hh} = 1$ and $p^{hh} = 20$.
- IC constraint binds for the high type.

$$\pi^h q^{hh}[v^h - p^{hh}] + (1-\pi^h)q^{lh}[v^h - p^{hl}] = \pi^h q^{lh}[v^h - p^{lh}] + (1-\pi^h)q^{ll}[v^h - p^{ll}]$$

IR FOR LOW TYPE

$$\pi^h q^{lh}[v^l - p^{lh}] + (1 - \pi^h) q^{ll}[v^l - p^{ll}] \geq 0$$

$$(0.8) q^{lh}[0 - p^{lh}] + (0.2) q^{ll}[0 - p^{ll}] \geq 0$$

It must be that $p^{lh} = p^{ll} = 0$.
If nobody pays when both announce L then $q^{ll} = 0$.
Whoever announces L in the truthful equilibrium must not pay anything.
Still need to figure out q^{lh} and p^{hl}.

If q^{lh} is positive then $p^{hl} = 40$.

IC for high type

$$\pi^h q^{hh}[v^h - p^{hh}] + (1-\pi^h)q^{lh}[v^h - p^{hl}] = \pi^h q^{lh}[v^h - p^{lh}] + (1-\pi^h)q^{ll}[v^h - p^{ll}]$$

$$(0.8)[100-20] + (0.2)q^{lh}[100 - p^{hl}] = (0.8)q^{lh}[100] + (0.2)(0)[100]$$

q^{lh} has to be positive for the equation to hold.

$$(0.8)[80] + (0.2)q^{lh}[100-40] = (0.8)q^{lh}[100]$$

$q^{lh} = \frac{16}{17}$

Other Constraints

Are IR for high type and IC for the low type satisfied?
IR for high type

$$\pi^h q^{hh}[v^h - p^{hh}] + (1-\pi^h)q^{lh}[v^h - p^{hl}] \geq 0$$

$L.H.S. = (0.8)[80] + (0.2)\frac{16}{17}[60] > 0.$

IC for low type

$$\pi^h q^{lh}[v^l - p^{lh}] + (1-\pi^h)q^{ll}[v^l - p^{ll}] \geq \pi^h q^{hh}[v^l - p^{hh}] + (1-\pi^h)q^{lh}[v^l - p^{hl}] \quad (5)$$

$L.H.S. = 0$
$R.H.S. = (0.8)[-20] + (0.2)\frac{16}{17}[-40] < 0$
$L.H.S. > R.H.S.$

The Assumptions??

Why did we make those assumptions?
Were they implied by the maximization exercise?

$$
\begin{aligned}
u_E + u_R =& (1.28)q^{hh}[100 - p^{hh}] + (0.32)q^{lh}(100 - p^{hl} + 0 - p^{lh}) \\
& + (0.08)q^{ll}[0 - p^{ll}]
\end{aligned}
$$

$$
\begin{gathered}
\pi^h q^{hh}[v^h - p^{hh}] + (1 - \pi^h)q^{lh}[v^h - p^{hl}] \geq 0 \\
\pi^h q^{lh}[v^l - p^{lh}] + (1 - \pi^h)q^{ll}[v^l - p^{ll}] \geq 0 \\
\pi^h q^{hh}[v^h - p^{hh}] + (1 - \pi^h)q^{lh}[v^h - p^{hl}] \geq \pi^h q^{lh}[v^h - p^{lh}] + (1 - \pi^h)q^{ll}[v^h - p^{ll}] \\
\pi^h q^{lh}[v^l - p^{lh}] + (1 - \pi^h)q^{ll}[v^l - p^{ll}] \geq \pi^h q^{hh}[v^l - p^{hh}] + (1 - \pi^h)q^{lh}[v^l - p^{hl}]
\end{gathered}
$$

Welfare

$u_E + u_R = (1.28)(80) + 0.32\frac{16}{17}(60) = 120.47$
Constant fee, 102.4
Complete Information, 121.6