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# Mechanism Design

## *An “auction” problem*

- Single seller has a single item
- Seller does not value item
- Two buyers with independent valuations

$0 \leq v^l < v^h$  low and high valuations

$\pi^l + \pi^h = 1$  probabilities of low and high valuations

what is the best way to sell the object

- Auction
- Fixed price
- Other

## ***The Revelation Principle***

Design a game for the buyers to play

- Auction game
- Poker game
- Etc.

Design the game so that there is a Nash equilibrium that yields highest possible revenue to the seller

The revelation principle says that it is enough to consider a special game

- strategies are “announcements” of types
- the game has a “truthful revelation” equilibrium

## ***In the Auction Environment***

$q^l, q^h$  probability of getting item when low and high

$p^h, p^l$  expected payment when low and high

*individual rationality constraint*

$$(IR) \quad q^i v^i - p^i \geq 0$$

- if you announce truthfully, you get at least the utility from not playing the game

*incentive compatibility constraint*

$$(IC) \quad q^i v^i - p^i \geq q^{-i} v^i - p^{-i}$$

- you gain no benefit from lying about your type

the incentive compatibility constraint is the key to equilibrium

## *Other constraints*

$q^l, q^h$  probability of getting item when low and high

they can't be anything at all:

probability constraints

$$(1) 0 \leq q^i \leq \pi^{-i} + \pi^i / 2$$

(win against other type, 50% chance of winning against self)

$$(2) \pi^l q^l + \pi^h q^h \leq 1/2$$

(probability of getting the good before knowing type less than 50%)



## *Seller Problem*

Maximize seller utility  $U = \pi^l p^l + \pi^h p^h$

Subject to IC and IR

To solve the problem we make a guess:

IR binds for low value

$$q^l v^l - p^l = 0$$

IC binds for high value

$$q^h v^h - p^h = q^l v^h - p^l$$

## *Solution Method*

- Use the guess, plus the probability constraints
- Solve the problem
- Show that the other IR, IC constraints don't bind
- Show that there exists a mechanism that gives the correct probabilities so that the other "probability" constraints don't matter

## *The solution*

$$p^l = q^l v^l \text{ from low IR}$$

substitute into high IC

$$p^h = (q^h - q^l)v^h + q^l v^l$$

plug into utility of seller

$$U = \pi^l q^l v^l + \pi^h ((q^h - q^l)v^h + q^l v^l)$$

$$U = q^l (\pi^l v^l - \pi^h v^h + \pi^h v^l) + \pi^h q^h v^h$$

$$\pi^l + \pi^h = 1 \text{ so}$$

$$U = q^l (v^l - \pi^h v^h) + \pi^h q^h v^h$$

*Two cases:*

$v^l > \pi^h v^h$  want to sell to low value

$v^l < \pi^h v^h$  don't want to sell to low value