

Sample answer to an exam question

Question: Suppose that the demand for a quantity x in a market is given by price $p = 17 - x$ and that there are two firms. One firm has a high marginal cost of 3 and one has a low marginal cost of 1. Find the Cournot (same meaning as Nash) equilibrium in which both firms choose quantities.

Answer – note that there are no elaborate explanations, but it is clearly explained what the equations are; if you introduce notation that was not in the original question, you need to explain that too.

find the Nash equilibrium

first find payoffs, in this question the profits

profits are revenue minus cost $\pi_i = px_i - c_i x_i$

using price from demand $\pi_i = (17 - x_i - x_{-i})x_i - c_i x_i$

from payoffs find the best-response by maximizing payoffs

here by first order condition for maximum

differentiate profits with respect to output and equate to zero $17 - x_{-i} - 2x_i - c_i = 0$

solve this FOC for best-response

$$x_i = \frac{17 - x_{-i} - c_i}{2}$$

notation: use $i = 1$ for firm with $c_i = 1$ and $i = 3$ for firm with $c_i = 3$

sanity check: does this work in our usual case $c_1 = 1$?

$c_1 = 1$ then $x_1 = 8 - x_3 / 2$, the usual answer

for $c_3 = 3$ the best response becomes $x_3 = 7 - x_1 / 2$

solve for the intersection of the two best-response function by substituting the solution

for $i = 3$ into the solution for $i = 1$ $x_1 = 8 - (7 - x_1 / 2) / 2 = 9 / 2 + x_1 / 4$

solve that to find $x_1 = 6$

plug the solution for low-cost firm back in to high-cost firm reaction function to get the rest of the solution

$$x_3 = 7 - x_1 / 2 = 4$$