MIDTERM 1: SOLUTIONS

February 23, 2012

1. BDM (Real Number Case)

   (a) Bidding 5 is not weakly dominated. Let x and a be the participant's bidding number and the realization of random number respectively. **Assume both are real numbers.**

   - Case1: $x < 5$
     - If $a \in [4, 5)$, the participant receive a which is less than 5, the payoff in the case of bidding 5. Otherwise, the payoff of bidding x and that of bidding 5 are the same.
   - Case2: $x > 5$
     - Similarly, if $a \in (5, x]$, the participant gets 5 which is less than what he could have received if he had bid 5. Otherwise, the participant gets the same payoff as the case of bidding 5.

   (b) Suppose the participant is maximizing the expected payoffs. Then the participant solves the following problem.

   $$max_{x\in[0,\infty)} P(a \leq x) \cdot 5 + [1 - P(a \leq x)] \cdot x$$

   If $x < 4$ then $P(a \leq x) = 0$ and hence the expected payoff is x. Likewise, if $x > 8$ then $P(a \leq x) = 1$ and the expected payoff is 5. Consider $x \in [4, 8]$. The problem reduces as below.

   $$max_{x\in[4,8]} \frac{x-4}{4} \cdot 5 + \frac{8-x}{4} \cdot x$$

   A simple calculation shows the optimal x is 6.5 and the maximized value is greater than 5. Therefore, the participant should bid $x = 6.5$.

2. BDM (Integer Case)

   (a) Bidding 5 and bidding 6 are not weakly dominated. Let x and a be the participant's bidding number and the realization of random number respectively. **Assume both**

are integers. Note that the payoffs from bidding 5 and bidding 6 are the same regardless of the realization of the random number.

- Case1: $x < 5$
  - If $a = 4$, the participant receive a which is less than 5, the payoff in the case of bidding 5 or 6. Otherwise, the payoff of bidding x and that of bidding 5 or 6 are the same.
- Case2: $x = 7$
  - Similarly, if $a = 6$, the participant gets 5 which is less than what he could have received if he had bid 5 or 6. Otherwise, the participant gets the same payoff as the case of bidding 5 or 6.
- Case3: $x = 8$
  - Similarly, if $a \in \{6, 7\}$, the participant gets 5 which is less than what he could have received if he had bid 5 or 6. Otherwise, the participant gets the same payoff as the case of bidding 5 or 6.
- Case3: $x > 8$
  - Similarly, if $a \in \{6, 7, 8\}$, the participant gets 5 which is less than what he could have received if he had bid 5 or 6. Otherwise, the participant gets the same payoff as the case of bidding 5 or 6.

(b) Suppose the participant is maximizing the expected payoffs. Then the participant solves the following problem.

$$max_{x \in [0,\infty)} P(a \leq x) \cdot 5 + [1 - P(a \leq x)] \cdot x$$

If $x < 4$ then $P(a \leq x) = 0$ and hence the expected payoff is x. Likewise, if $x > 8$ then $P(a \leq x) = 1$ and the expected payoff is 5. Consider $x \in \{4, 5, 6, 7, 8\}$. The corresponding expected payoffs are $\{\frac{21}{5}, \frac{25}{5}, \frac{28}{5}, \frac{29}{5}, \frac{28}{5}\}$. Therefore, the participant should bid $x = 7$.

3. Extensive Form Game

(a) The normal form

		Serf	
		C	R
Emperor	N	0,2	0,2
	T	2,0	-1,-1

- N: Not tax
- T: Tax
- C: Comply
- R: Run away

(b) {Not tax, Run away} and {Tax, Comply} are Nash equilibria since the agents do not have any incentive to deviate in either situation.

(c) They are all Pareto efficient since there is no other payoff in which everyone is better off.

(d) Yes. For {Not tax, Run away}, Run away is weakly dominated by Comply.

(e) The subgame perfect equilibrium is the Nash equilibrium in every subgame. There are two subgames: one is the original game and the other is the second stage of the game, that is, the stage where the serf considers either complying or running away. {Not tax, Run away} is not subgame perfect equilibrium since Run away is not a Nash equilibrium in this subgame. Therefore, {Tax, Comply} is the only subgame perfect equilibrium.

4. Cournot Duopoly

(a) The profit functions:

- Unbalance: $\Pi_u = px_u - x_u = [60 - 2(x_u + x_c)]x_u - x_u$
- Contrapositive: $\Pi_c = px_c - 4x_c = [60 - 2(x_u + x_c)]x_c - 4x_c$

(b) The best response functions for each agent is determined by maximizing the profit given the other's output:

- Unbalance: $max_{\{x_u\}}\Pi_u = [60 - 2(x_u + x_c)]x_u - x_u$
- Contrapositive: $max_{\{x_c\}}\Pi_c = [60 - 2(x_u + x_c)]x_c - 4x_c$
- Since the profit is maximized when $\frac{\partial \pi_u}{\partial x_u} = 0$ and $\frac{\partial \pi_c}{\partial x_c} = 0$, we can get the reaction functions as below:
  - Unbalance: $x_u = \frac{59-2x_c}{4}$
  - Contrapositive: $x_c = \frac{56-2x_u}{4}$

(c) Nash equilibrium is the pair of outcomes of each firm in which each firm produces the best response output to the other's output level. Therefore, we can get Nash equilibrium by plugging one's best response to the other's. The resulting equilibrium is

$$(x_u^*, x_c^*) = (\frac{31}{3}, \frac{53}{6})$$