ECON 4011 04/04/2012
SPRING 2012 CEYHUN COBAN

SECOND MIDTERM SOLUTIONS

1. (a) The profit functions of Maple and Bell are given below;

$$\Pi_M(x_M, x_B) = [120 - (x_M + x_B)]x_M - 8x_M$$

$$\Pi_B(x_M, x_B) = [120 - (x_M + x_B)]x_B - 4x_B$$

where M denotes Maple and B denotes Bell.

Best response of Maple to Bell's output is determined by

$$\max_{x_M} \Pi_M(x_M, x_B) = [120 - (x_M + x_B)]x_M - 8x_M$$

Since the profit is maximized when $\frac{\partial \Pi_M}{\partial x_M} = 0$, we obtain

$$\frac{\partial \Pi_M}{\partial x_M} = 120 - 2x_M - x_B - 8 = 0$$

$$112 - x_B = 2x_M$$

$$\boxed{x_M = 56 - \tfrac{x_B}{2}}$$

Similarly, best response of Bell to Maple's output is determined by

$$\max_{x_B} \Pi_B(x_M, x_B) = [120 - (x_M + x_B)]x_B - 4x_B$$

Since the profit is maximized when $\frac{\partial \Pi_B}{\partial x_B} = 0$, we obtain

$$\frac{\partial \Pi_B}{\partial x_B} = 120 - x_M - 2x_B - 4 = 0$$

$$116 - x_M = 2x_B$$

$$\boxed{x_B = 58 - \tfrac{x_M}{2}}$$

(b) If Maple is the Stackelberg leader, then Maple knows that Bell will choose own output level as a best response to Maple's output. Thus, knowing this Maple can perfectly guess the output level of Bell in terms of its own output so that it can internalize it.

Thus, Maple's profit maximization problem can be solved by replacing x_B with $58 - \frac{x_M}{2}$ in the profit function given below;

$$\max_{x_M} \Pi_M(x_M, x_B) = [120 - (x_M + 58 - \frac{x_M}{2})]x_M - 8x_M$$

Since the profit is maximized when $\frac{\partial \Pi_M}{\partial x_M} = 0$, we obtain

$$\frac{\partial \Pi_M}{\partial x_M} = 120 - 2x_M - 58 + x_M - 8 = 0$$

$$54 - x_M = 0$$

$$\boxed{x_M = 54}$$

Since $x_M = 54$ and $x_B = 58 - \frac{x_M}{2}$, then we obtain

$$x_B = 58 - \frac{54}{2}$$

$$\boxed{x_B = 31}$$

Hence, $(x_M, x_B) = (54, 31)$ is the Stackelberg equilibrium.

2. (a) In order to determine the static Nash equilibria (NE) of this game, best responses are underlined on the payoff matrix which is given below;

	C	D
C	$40, 30$	$0, \underline{40}$
D	$\underline{50}, 10$	$\underline{10}, \underline{20}$

Hence, (D, D) is the unique static NE.

(b) Grim-trigger strategies form a Subgame Perfect Nash Equilibrium (SPNE) if there is no profitable deviation in any period for any player. So, at any period t, if Ann (row player) plays C then her average discounted payoff will be 40 (Bob also follows grim-trigger strategies and plays C unless Ann deviated in the previous period). If she chooses to deviate, then she will get 50 for the current period, and 10 in the subsequent periods (because Bob will punish Ann by playing D after observing the deviation). So, her average discounted payoff is $(1 - \delta)50 + 10\delta$. Since playing C must be optimal, then

$$40 \geq (1 - \delta)50 + 10\delta$$

$$40 \geq 50 - 40\delta$$

$$\boxed{\delta \geq 0.25}$$

However, it is not sufficient to consider optimality of Ann's strategy only (payoffs are not symmetric). Similarly, if Bob (column player) plays C in the current and subsequent periods, then his average discounted payoff is 30. If he chooses to deviate, then he receives a payoff 40 in the current period, but 20 in the subsequent periods. So, his average discounted payoff is $(1-\delta)40 + 20\delta$. Then, by optimality condition,

$$30 \geq (1-\delta)40 + 20\delta$$

$$30 \geq 40 - 20\delta$$

$$\boxed{\delta \geq 0.5}$$

Hence, the common discount factor has to be greater than both 0.25 and 0.5 (otherwise, Bob chooses to deviate). Therefore, Grim-trigger strategy forms a SPNE if $\delta \geq 0.5$.

(c) Each player plays D in every period irrespective of what has happened in the past. These strategies form an SPNE since a player can only hurt himself by deviating in any period. Given that they are playing the Nash Equilibrium in every period, deviation does not lead to any gain today. Moreover since the strategy in any period does not depend on what has happened in the past, deviating today cannot result in higher payoff in the future. By following the strategies Ann gets 10. By deviating today her average payoff becomes $(1-0.1)(0) + (0.1)10$, which is not greater than 10. A similar argument applies for the other player.

3. First we need to determine the events in this problem. Let E be the event that evidence is found, let H be the event that human error is the cause, let S be the event that spontaneous combustion is the cause and let N be the event that natural causes is the cause. It is important to note that any two of the events H, S, N cannot occur at the same time. Then, we need to determine $P(H|E)$. Since we are given the following probabilities

$$P(H) = 0.1$$

$$P(S) = 0.5$$

$$P(N) = 0.4$$

$$P(E|H) = 0.8$$

$$P(E|S) = 0.4$$

$$P(E|N) = 0.2$$

by using Baye's Law, we obtain

$$\begin{aligned} P(H|E) &= \frac{P(E|H)P(H)}{P(E|H)P(H) + P(E|S)P(S) + P(E|N)P(N)} \\ &= \frac{(0.8)(0.1)}{(0.8)(0.1) + (0.4)(0.5) + (0.2)(0.4)} \\ &= \frac{2}{9} \end{aligned}$$