Problem Set 2

1. *First Price Auction (Reprise)*

For Q3 on Problem set 1, state all the Nash Equilibria.

2. *Collaborating??*

Paul and John are working on a project for which they will receive the same grade. The utility to each of them would be their grade minus their own effort. If Paul puts in x units of effort and John puts in x units of effort their grade will be equal to the higher of the two effort levels, x and y. So for a given choice of x and x Paul's utility is given by $2\max\{x,\ y\} - x$ while John gets a utility of $2\max\{x,y\} - y$. The possible choices for effort level are 1 and 2.

Write down the payoff matrix for this game. Which outcomes are Pareto efficient? What is predicted by the iterated elimination of strictly dominated strategies? What is predicted by the iterated elimination of weakly dominated strategies? Find the reaction (best response) functions. State all Nash Equilibria of the game.

Now suppose the utility functions for Paul and John are given by $2\min\{x,y\} - x$ and $2\min\{x,y\} - y$. In other words the common grade is equal to the lower of the two effort levels, x and y.

Again, write down the payoff matrix for this game. Which outcomes are Pareto efficient? What is predicted by the iterated elimination of strictly dominated strategies? What is predicted by the iterated elimination of weakly dominated strategies? Find the reaction (best response) functions. State all Nash Equilibria of the game.

3. *Nash Equilibrium*

	L	M	R
U	8,1	0,4	2,9
C	6,0	1,1	3,0
D	2,2	1,3	4,4

In the game depicted above, Alice is the row player while Bill is the column player. What would iterated elimination of strictly dominated strategies predict for this game? What would the prediction of iterated elimination of weakly dominant strategies be? Find the reaction (best reponse) functions. What are the Nash Equilibria of this game?

4. *Cournot Duopoly*

Bell and HPhi are the leading producers of abaci. The market demand function for abaci is given by $p = 90 - 2x$. The number of abaci produced by Bell and HPhi are denoted by x_B and x_H, respectively. Remember, $x = x_B + x_H$. The cost functions for Bell and HPhi are $2x_B$ and $4x_H$, respectively. The two firms can choose any positive level of production.

(a) Write down the profit equations for Bell and HPhi.

(b) Write down the reaction (best response) functions.

(c) Write down two choices of x_B that are *strictly dominated* and two that are not.

(d) What are the Nash Equilibria choices of x_B and x_H.

(P.S. Assume, for now, that fractions of an abacus can in fact be produced and sold!)

5. *Three Player Game*

There are three players. Players 1 and 2 must each choose whether to engage or drop. Player 3 must choose whether to favor player 1 or player 2. If Players 1 and 2 both drop, all three players get a payoff of 1. If either Player 1 or Player 2 engages then the player favored by Player 3 gets 3 and the other player gets 0, while Player 3 himself gets 0. Find all the pure strategy Nash equilibria.