Suggested Solutions of Second Midterm Exam

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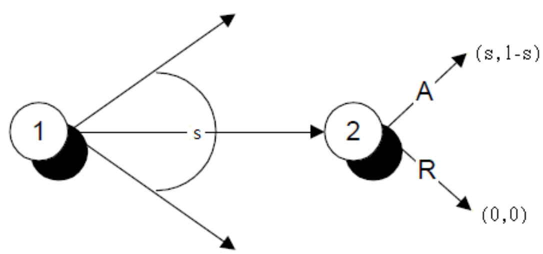
1 Ultimatum Bargaining

a) The game tree is



b) By backward induction, given s, a best response[1] of player2 is

$$\begin{cases} \text{Accept} & if\ s \leq 1 \\ \text{Reject} & otherwise \end{cases}$$

Given the best response, player1 will make the offer $s = 1$, which gives him the highest share. Hence, the subgame perfect equilibrium is ($s = 1$, Accept $\forall s \in [0, 1]$).

c) Yes, there is this kind of Nash equilibrium. Consider the following strategy profile,

$$\left(s = \frac{1}{2}, \begin{cases} \text{Accept} & if\ s = \frac{1}{2} \\ \text{Reject} & otherwise \end{cases} \right)$$

It is a Nash equilibrium. For player1, he can only get a positive share when $s = \frac{1}{2}$. For player2, he will accept and get a positive share when $s = \frac{1}{2}$, and feels indifferent between acceptance and rejection when $s \neq \frac{1}{2}$. Hence, given the other's strategy, everyone's strategy is his best response.

[1] Note that we pick up a best response that player2 will accept when $s = 1$. If player2 will reject when $s = 1$, player1 will make an offer $s \to 1$. At that time, there is no SPE.

2 Long Run-Short Run/Repeated Game

a) The best response is

	C	D
C	$3, 2^*$	$0, 0$
D	$^*5, 0$	$^*1, 1^*$

Hence, the static Nash Equilibrium is (D, D).

b) Yes it is a subgame perfect equilibrium outcome path when $\delta = \frac{1}{2}$. Consider the following grim trigger strategy: In the first period, both long-run and short-run player play C. After that, the long-run player and sequential short-run ones play C if the outcome was always (C, C) in every previous period; otherwise, they play D. We then show that given the trigger strategy, everyone's strategy is his best response in every subgame:

For the long-run player, in the first period and those subgames that the outcome was always (C, C) in every previous period, playing C is a best response since

$$\begin{aligned} C &: (1-\delta)[3 + 3\delta + 3\delta^2 + ...] = 3 \\ D &: (1-\delta)[5 + \delta + \delta^2 + ...] = 5 - 4\delta = 3 \leq 3 \end{aligned}$$

And in the subgames that someone played D in some previous periods, playing D is a best response since

$$\begin{aligned} C &: (1-\delta)[0 + \delta + \delta^2 + ...] = \delta = \frac{1}{2} \\ D &: (1-\delta)[1 + \delta + \delta^2 + ...] = 1 \geq \frac{1}{2} \end{aligned}$$

For short-run players, in the first period and those subgames that the outcome was always (C, C) in every previous period, playing C is a static best response since $2 \geq 0$. And in the subgames that someone played D in some previous periods, playing D is a static best response since $1 \geq 0$.

From the argument above, we conclude that the trigger strategy is a subgame perfect equilibrium.

c) Yes, there is a subgame perfect equilibrium in which row player's average payoff is 4. Note that $(4, 1) = \frac{1}{2}(3, 2) + \frac{1}{2}(5, 0)$, and the min max is 1 for both players. Hence, $(4, 1)$ is in the socially feasible individually rational set. By the Folk Theorem, when players are patient enough (δ is large enough), the average payoff 4 can be supported as a subgame perfect equilibrium.

3 Stackelberg Equilibrium

a) Define (x_n, x_m) are the innovator and imitator's production levels. By backward induction, given x_n, the imitator's profit function is:

$$\pi_m = (20 - x_n - x_m)x_m - 3x_m$$

F.O.C.

$$17 - x_n - 2x_m = 0$$

The best response of the imitator is

$$x_m = \frac{17 - x_n}{2}$$

Given the best response, the innovator's profit function is:

$$\pi_n = (20 - x_n - \frac{17 - x_n}{2})x_n - 3x_n - R$$

F.O.C.

$$\frac{17}{2} - x_n = 0$$

The best response of the innovator is

$$x_n = \frac{17}{2}$$

Hence, the Stackelberg Equilibrium is $(x_n, x_m) = (\frac{17}{2}, \frac{17-x_n}{2})$.

b) The equilibrium outcome is $(\frac{17}{2}, \frac{17}{4})$. Hence, the equilibrium profits of the innovator and imitator are

$$\begin{aligned} \pi_n &= \frac{289}{8} - R \\ \pi_m &= \frac{289}{16} \end{aligned}$$

Being imitator is preferable to being innovator if

$$\begin{aligned} \frac{289}{8} - R &\leq \frac{289}{16} \\ R &\geq \frac{289}{16} \end{aligned}$$