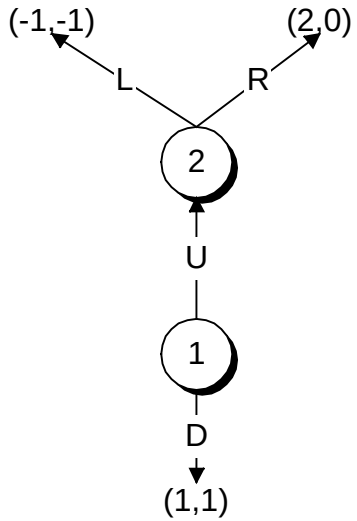
Dynamic Games and Subgame Perfection

- economic theory works some of the time
- an experimental literature argues there are gross violations of theory
- failures do not involve Nash equilibrium
- involve a variant of Nash equilibrium: subgame perfection
- introduce what game theory is really about: time and uncertainty
- captured through the notion of an extensive form game

Example: The Selten Game



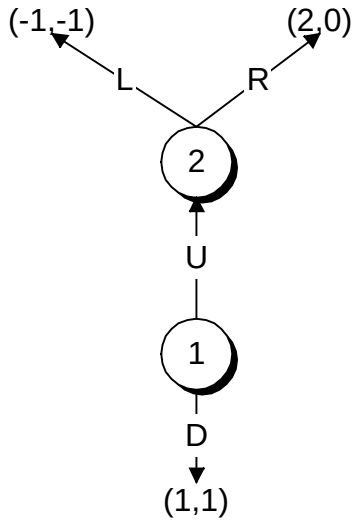
	L	R
U	-1,-1	2*,0*
D	1*,1*	1,1*

Key notion: ***Strategies***

Nash equilibria: D,L and U,R

Assessment of Nash Equilibria

Does D,L make sense?



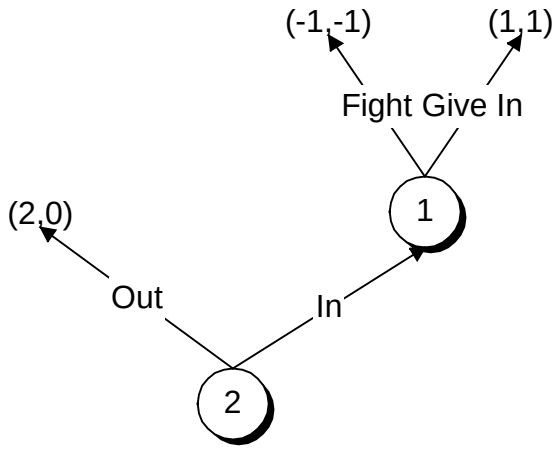
Subgame Perfection

Subgame Perfection: a Nash equilibrium in each subgame

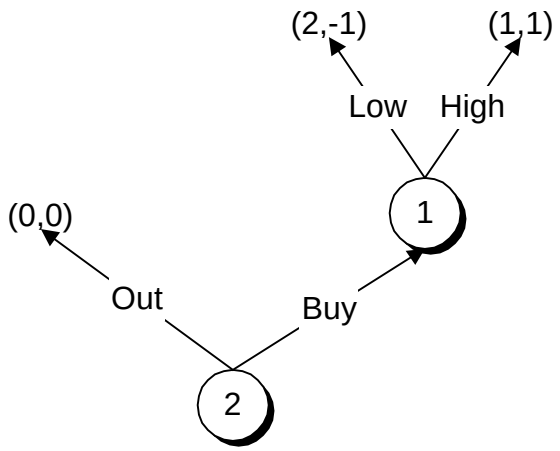
Backwards Induction: a method of finding subgame perfect equilibria by solving backwards from the end of the game, also called *recursive* method

Other Applications

Chain Store

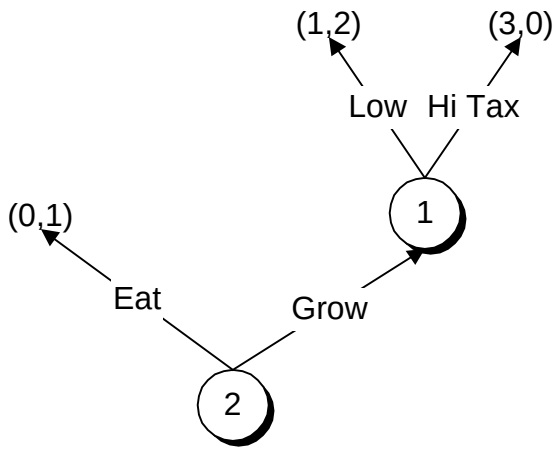


Quality Game



- what do you learn if you stay out?

Peasant Dictator



Commitment and Stackelberg Equilibrium

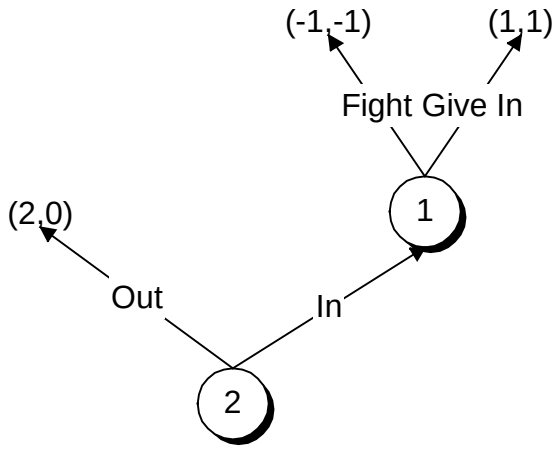
- precommitment

to be effective a precommitment must be

- public
- credible

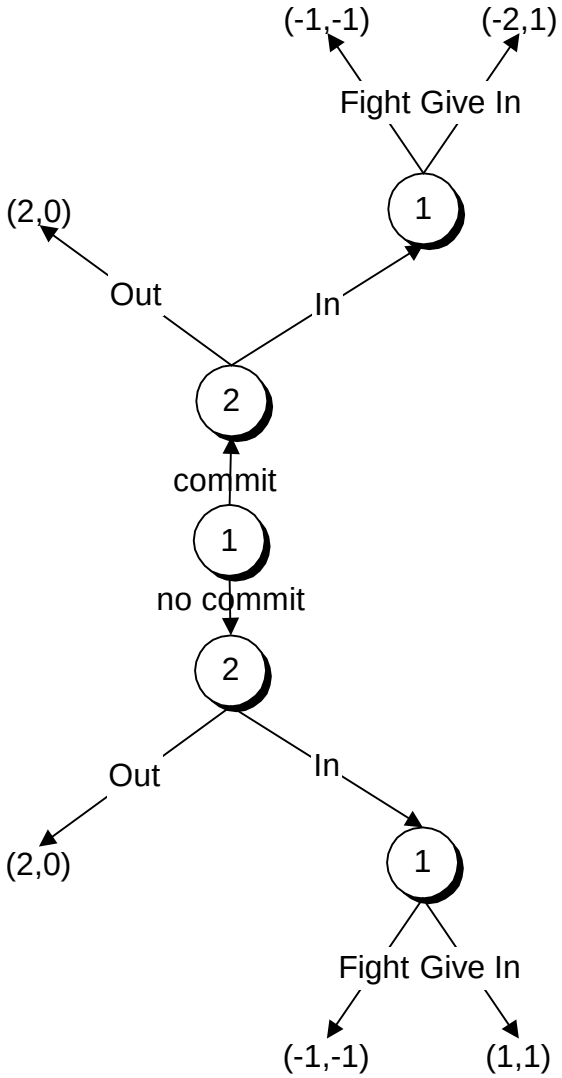
- Dr. Strangelove

The Chain Store Game



player 1 is the *Stackelberg leader*

Commitment Game

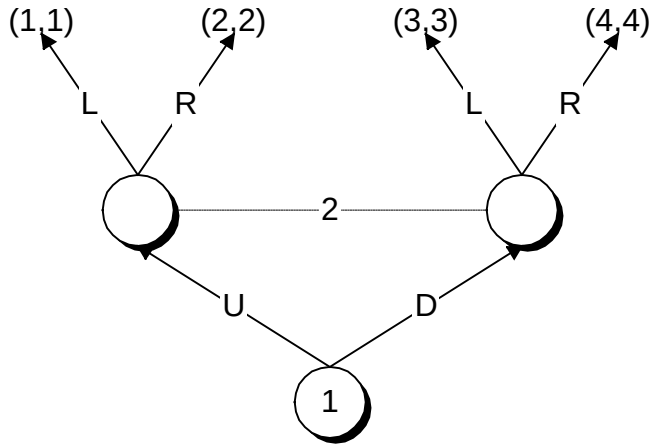


The Cold War

- player 1 is the Soviet Union
- *entry* corresponds to “invade Western Europe”
- *fight* means United States responds with strategic nuclear weapons – effectively destroying the entire world
- if the Soviet Union were to take over Western Europe it would hardly be rational for the United States to destroy the world
- Richard Nixon instructed Henry Kissinger to say to the Russians “I am sorry, Mr. Ambassador, but [the president] is out of control….you know Nixon is obsessed about Communism. We can't restrain him when he is angry – and he has his hand on the nuclear button.”

Information Sets and the Normal Form

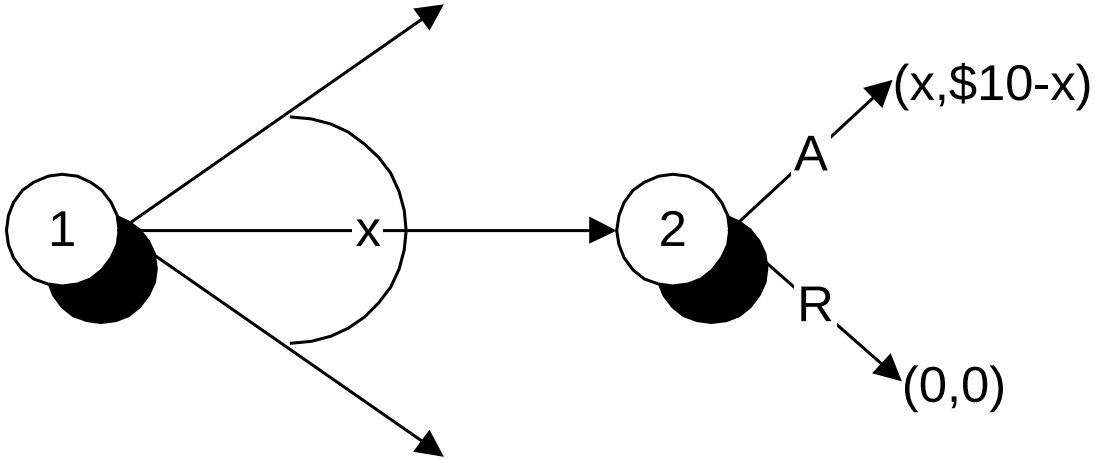
How can we represent a simultaneous move game as an extensive form?



- The dashed line represents an *information set*.
- A player knows what information set he is at, but not which node in the information set

Ultimatum Bargaining

extensive form



x is the demand by player 1 (in nickles)

subgame perfection player 2 accepts any demand less than $10

subgame perfection requires player 1 demand at least $9.95

Roth et al [1991]: ultimatum bargaining in four countries

pooled results of the final (of 10) periods of play in the 5 experiments with payoffs normalized to $10

Demand	Observations	Frequency of Observations	Accepted Demands	Probability of Acceptance
$5.00	37	28%	37	1.00
$6.00	67	52%	55	0.82
$7.00	26	20%	17	0.65

Does subgame perfection fail, or are the preferences wrong?

Best-Shot

- Sequential contributions of two players
- Only largest contribution counts

Contribution	***Public Benefit***
$0.00	$0.00
$1.64	$1.95
$3.28	$3.70
$4.10	$4.50
$6.50	$6.60

Best Responses

If your opponent doesn't contribute

Contribution	***Net Private Benefit***
$0.00	$0.00
$1.64	$0.31
$3.28*	$0.42*
$4.10	$0.40
$6.50	$0.10

if your opponent contributes something: optimal not to contribute at all

Analysis of Best Shot

- Player 1 contributes her opponent won't so he should put in $3.28 and get a net benefit of $0.42
- Player 1 doesn't contribute her opponent will put in $3.28 giving a benefit of $3.70
- So player 1 doesn't contribute, player 2 puts in $3.28
- Also a Nash equilibrium for Player 1 to put in $3.28 and Player 2 nothing
- Harrison and Hirshleifer found people played the subgame perfect equilibrium

Best Shot Information

- Harrison and Hirshleifer did not state what payoffs of other player was
- But players alternated between moving first and second, so presumably could figure this out

Prasnikar and Roth [1992]

- Never changed player role: always first or second
- Two versions: full information, only know own payoff

Experimental Results and Subgame Perfection

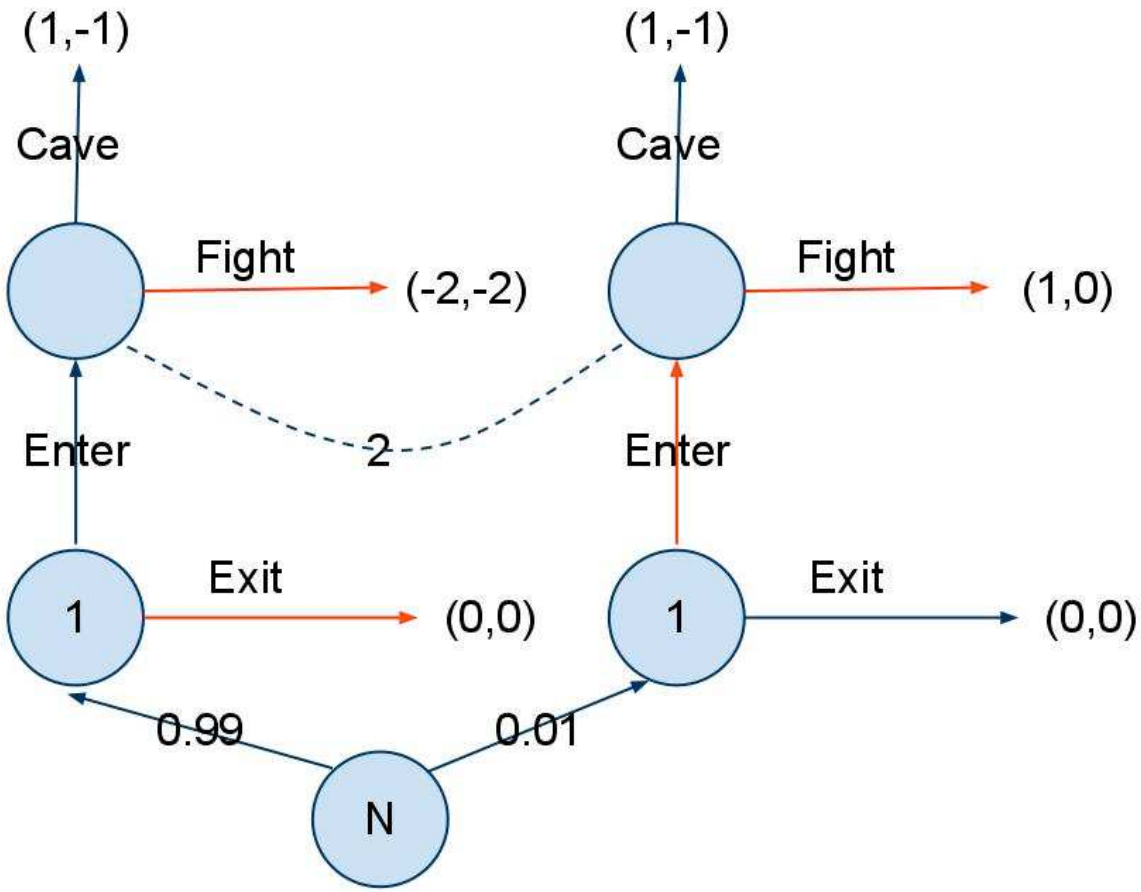
- Full information: like Harrison/Hirshleifer – subgame perfect, first doesn't donate; in final eight rounds first mover never made contribution
- Partial information: only get Nash - in bulk of matches one player contributing $3.28 and the other $0.00
- But: in over half of matches the contributing player was the first mover
- Not subgame perfect
- Note learning aspect: if I move first and kick in $3.28 my opponent will contribute nothing
- I never learn that had I not bothered to contribute my opponent would have put the $3.28 in for me
- If subgame perfection is theory of what happens when players are fully informed of the structure of the game: should not expect predictions to hold up when they are only half informed

Is Subgame Perfection Robust?

Do predictions of subgame perfection hold up when players are poorly informed about motives of opponents?

What if there is onlly a small departure from assumption of perfect information?

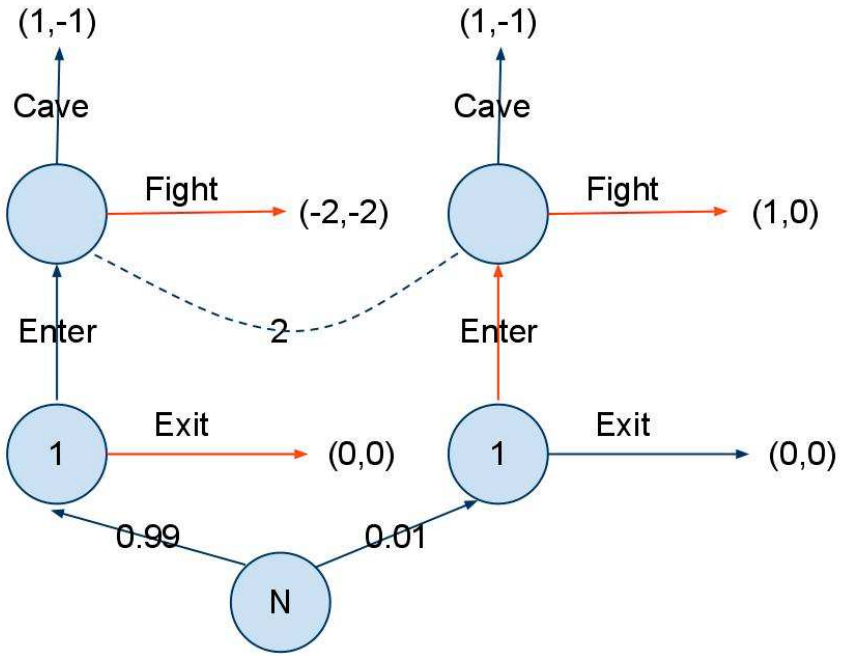
Elaborated Selten Game



Nature's Move and Information

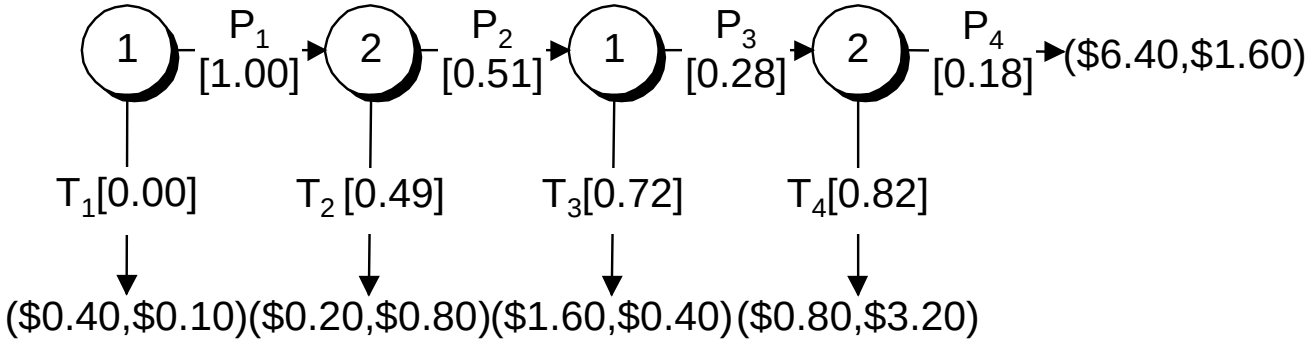
- Notice "Nature" as a player to represent random events
- Notice information set: player 1 knows which game Nature chose, player 2 does not
- Example of "Bayesian Game" player 1 learns his "type" which is private information
- moves of Nature labeled with probabilities
- with probability 0.99 Nature chooses the Selten game
- with probability 0.01 Nature chooses an alternative game
- study problem of players not knowing structure of the game by making it an explicit part of the game

Analysis of Game: Strict Nash Equilibrium



- No subgames
- Subgame perfection can only serve as a tie-breaker
- Players indifferent about “off the equilibrium path”

Grab a Dollar



- Grab a Dollar versus Centipede
- Subgame perfection: dramatic failure
- Nash Equilibrium

What failed? Preferences versus Subgame Perfection

- Final stage: give up $1.60 in order to increase the payment to player 1 by $5.60?
- 18% of player 2's altruistic enough to choose latter
- 18% of player 2's giving money changes strategic nature of play
- what should a selfish player 1 do on the third move?
- grabs get $1.60; pass have 18% chance of $6.40 and 82% chance of getting $0.80; expected earning of slightly over $1.80 by passing
- always best to stay in as long as possible and hope you get lucky
- What is true mistake?
- Could you know you are mistaken?

How to Model?

- High conditional probability of passing in final round of 18%
- Viewed from beginning of the game expected loss is mild
- Viewed from the perspective of the last round it is fairly high
- Quantal response has a hard time coping with this, because it treats all errors as equal
- Need an explicit theory of altruism