Economics 504: Final Exam, Spring 2010

Suggested Answers

1. (a) Efficient allocation: 1 gets A and 2 gets B. Without agent 1, both objects should be given to agent 3. So the pivotal transfer for agent 1 is $4 - 8 = -4$. Without agent 2, the best allocation gives total utility to be 8, so the pivotal transfer for agent 2 is $5 - 8 = -3$. Agent 3 pays 0 since she is not pivotal.

   (b) Efficient allocation is to give both object to agent 3. Agent 1 and 2 are not pivotal so they pay 0. Without agent 3 the best allocation generates total utility equal to 5, so the pivotal transfer to agent 3 is -5.

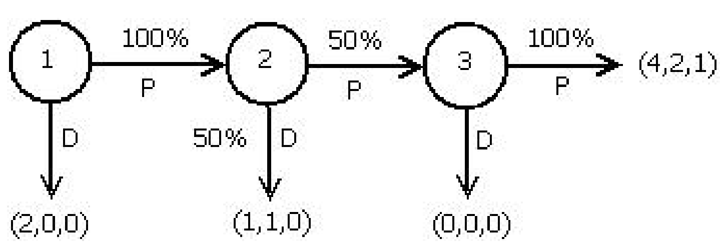
   (c) Efficient allocation: 1 gets A and 2 gets B. Again agent 3 pays 0 since her existence does not change the efficient allocation. Without agent the total utility of the efficient allocation is still 5, so agent 1 pays 0 as well. Same for agent 2 so she also pays 0.

2. Let $L = (p_1, L_1, p_2, L_2, \cdots, p_r, L_r)$ and $\hat{L} = (p_1, L_1, p_2, L_2, \cdots, p_i, \tilde{L}_i, \cdots, p_r, L_r)$, where $L_i \sim \tilde{L}_i$ for some i. To show the independence axiom we need $L \sim \hat{L}$. If the preference has a expected utility representation $U : \mathcal{L} \to \Re$, then

$$U(L) = \sum_{j=1}^{r} p_j U(L_j) = \sum_{j \neq i} p_j U(L_j) + p_i U(\tilde{L}_i) = U(\hat{L}),$$

where the second equality comes from $U(L_i) = U(\tilde{L}_i)$, since they are indifferent to the agent. But then $L \sim \hat{L}$.

3. Consider the following game.



The SPNE of this game is (P, P, P). The other Nash equilibrium outcome is D, which is supported by the stragety profiles $(D, Pr(P) = p, Pr(P) = q)$ in which

$(1-p)+4pq \leq 2$ so that player 1 does not deviate. However, the strategy profile indicated in the game tree is a (hetrogeneous) self-confirming equilibrium, since player 2 that plays D can hold a belief that player 3 will play D, which justifies her playing D. Notice that (P, D) is not a Nash equilibrium outcome.

4. (a) The Nash equilibria of the stage game are those in which the long-run player plays $Pr(nice) \leq 1/3$, and the short-run player plays *out*. In all Nash equilibria the long-run player gets 0.

   The pure Stackelberg strategy for the long-run player is to play *nice*, which gives her 4. The mixed Stackelberg strategy is to play $Pr(nice) = 1/3$, with payoff equal to 16/3. In both cases the short-run player plays *in*, which is her best response.

   The minmax payoff for the long-run player is 0 by the short-run player staying out.

   (b) Since the Nash equilibrium payoff is equal to the minmax, it is also the worst dynamic equilibrium payoff for all values of the discount factor. It is because playing static Nash strategy in every period is always an equilibrium, and players cannot get less than the minmax in equilibrium.

   (c) For large discount factor the best dynamic equilibrium payoff $\bar{v}$ is given by $\alpha = (Pr(nice) \geq 1/3, in)$. $\bar{v}$ is equal to 4, the worst in the support of the long-run player's actions. The threshold of discount factor is given by

$$4 \geq (1-\delta)6 + \delta W(mean) \geq (1-\delta)6,$$

   namely $\delta \geq 1/3$. For $\delta < 1/3$ the only dynamic payoff is the static Nash payoff 0.

   (d) If there is a probability $\mu > 0$ that the long-run player plays *nice* no matter what, then in any Nash equilibrium the long-run player's payoff is bounded below by $u_1^* = \delta^k \cdot 4$, where $k = \ln \mu / \ln(\frac{1}{3})$.[1]

   Consider any equilibrium strategy profile. The payoff of the long-run player

[1] Or $k = \max\{n \in N | n \leq \ln \mu / \ln(\frac{1}{3})\}$ if you like.

in equilibrium should be at least as large as that of playing *nice* in every period no matter what, which is a feasible but possibly suboptimal strategy. Denote h_t be the history that *nice* is been played in every period prior to time t. Let π_t be the total probability that the short-run player expects the long-run player to play *nice* at time t and history h_t. Note that if $\pi_t > 1/3$ than the short-run player strictly prefers to play *in*. Let $\pi(w^*|h_t)$ be the probability that the long-run player is the committed type that plays *nice* in history h_t. Notice also that we always have $\pi(w^*|h_t) \geq \pi_t$. Moreover, by Bayes rule,

$$\pi(w^*|h_t) = \frac{\pi(w^*|h_{t-1})}{\pi_t}.$$

Therefore if the short-run player plays *out* in h_t then $\pi_t \leq 1/3$ and $\pi(w^*|h_t) \geq 3\pi(w^*|h_{t-1})$. Hence in history h_t if the short-run player ever played *out* for n times then $\pi(w^*|h_t) \geq \mu/(\frac{1}{3})^n$. That implies $n \leq \ln \mu / \ln(\frac{1}{3})$.

Hence by always playing *nice* the long-run player ensures that the short-run player plays *out* for at most $k = \ln \mu / \ln(\frac{1}{3})$ times, which gives the long-run player at least $u_1^* = \delta^k \cdot 4$.

(e) The worst dynamic equilibrium payoff in the presence of moral hazard is still 0 for any discount factor and p, since always playing the static Nash equilibrium in every period is still an equilibrium.

(f) The best dynamic equilibrium payoff is solved by

$$\begin{aligned} \bar{v} \quad &= (1-\delta)4 + \delta[p\bar{v} + (1-p)w] \\ \bar{v} \quad &= (1-\delta)6 + \delta[(1-p)\bar{v} + pw] \\ s.t. \quad & 0 \leq w \leq \bar{v}, \end{aligned}$$

otherwise $\bar{v} = 0$.

Solving the above equations gives $\bar{v} = \frac{10p-6}{2p-1}$, and $w = \frac{10\delta p - 4\delta - 2}{\delta(2p-1)}$, where $\bar{v} \geq 0$ iff $p \geq \frac{3}{5}$, and $w \geq 0$ iff $\delta \geq \frac{1}{5p-2}$.

(g) Notice that $\frac{10p-6}{2p-1} < 4$ and $\frac{1}{5p-2} > \frac{1}{3}$ for all $\frac{1}{2} < p < 1$. So moral hazard is bad both in terms of a lower level of the best dynamic payoff and a higher threshold of δ.