BARGAINING

by

Roberto Serrano

Department of Economics, Brown University

May 2005 (to appear in *The New Palgrave Dictionary of Economics*, 2nd edition, McMillan, London)

Abstract: This article is a survey on bargaining theory. The focus is the game theoretic approach to bargaining, both on its axiomatic and strategic counterparts. The application of bargaining theory to large markets and its connections with competitive allocations are also discussed.

Journal of Economic Literature classification: C7.

Bargaining

In the simplest definition, we refer to bargaining as a socioeconomic problem involving two parties, who can cooperate towards the creation of a commonly desirable surplus, over whose distribution both parties are in conflict.

The nature of the cooperation in the agreement and the relative positions of the two parties in the status-quo before agreement takes place will influence the way in which the created surplus is divided. Many social, political and economic problems of relevance fit this definition: a buyer and a seller trying to transact a good for money, a firm and a union sitting at the negotiation table to sign a labor contract, a couple deciding how to split the intra-household chores, two unfriendly countries trying to reach a lasting peace agreement, or negotiations out of court between two litigating parties.

In all these cases three basic ingredients are present: (1) the status-quo situation, or the disagreement point, i.e., the arrangement that is expected to take place if an agreement is not reached; (2) the presence of mutual gains from cooperation; and (3) the multiplicity of possible cooperative arrangements, which will translate in different ways to split the resulting surplus.

If the situation involves more than two parties, matters are different, as advanced in von Neumann and Morgenstern (1944). Indeed, in addition to the possibilities already identified of either disagreement or agreement among all parties, it is now conceivable that an agreement be reached among some, but not all, of the parties. In multilateral settings, we are therefore led to distinguish pure bargaining problems, in which partial agreements of this kind are not possible because subcoalitions have no more power than individuals alone, from coalitional bargaining problems (or simply coalitional problems), in which partial agreements become a real issue in formulating threats and predicting outcomes. One example of the former could be a round of talks among countries in order to reach an international trade treaty in which each country has veto power, where a situation of voting in legislatures would be an example of the latter. In this article we shall concentrate on pure bargaining problems, leaving the description of coalitional problems to other entries in the dictionary (such as the ones corresponding to coalitional games, the core or the Shapley value). The current article will not be concerned with the vast informal literature on bargaining either, which performs case studies and tries to teach bargaining skills for the "real world" (for this purpose, the reader is referred to Raiffa (1982) for a classic reference.)

1. Approaches to bargaining before game theory. Before the adoption of game theoretic techniques, bargaining problems (also called bilateral monopolies at the time) were deemed indeterminate by economics. This was certainly the position stated by important economic theorists, including Edgeworth (1881) and Hicks (1932). More specifically, it was believed that the solution to a bargaining problem must satisfy both individual rationality and collective rationality properties: the former means that neither party should end up worse than at the status-quo and the latter refers to Pareto efficiency. Typically, the set of individually rational and Pareto efficient agreements is very large in a bargaining problem, and these theorists were inclined to think that theoretical arguments could not go further than this in getting a prediction. To be able to obtain such a prediction, one would have to rely on extra-economic variables, such as the bargaining power and abilities of either party, the psychological state of mind in negotiations, the religious beliefs of each party, the weather, and so on.

A precursor to the game theoretic study of bargaining, at least in the attempt to provide a more determinate prediction, is the analysis of Zeuthen (1930). This Danish economist formulated a principle by which the solution to a bargaining problem be dictated by the two parties' risk attitudes (given the probability of breakdown of negotiations following the adoption of a tough position at the bargaining table). The reader is referred to Harsanyi (1987) for a version of Zeuthen's principle and its connection with Nash's bargaining theory. The rest of the article turns to the game theoretic approaches to bargaining.

2. The axiomatic theory of bargaining. Nash (1950, 1953) are two seminal papers that constitute the birth of the formal theory of bargaining. Two assumptions are central in Nash's theory. First, bargainers are assumed to be fully rational individuals, and the theory is intended to yield predictions based exclusively on data relevant to them (in particular, the agents are equally skillful in negotiations, and the other extraneous factors mentioned above do not play a role).

Second, a bargaining problem is represented as a pair (S, d) in the utility space, where S is a compact and convex subset of $\mathbb{R}^2$ –the feasible set of utility pairs–, and $d \in \mathbb{R}^2$ is the disagreement utility point. Compactness follows from standard assumptions such as closed productions sets and bounded factor endowments, and convexity is obtained if one uses expected utility and lotteries over outcomes are allowed. Also, the set S must include points that dominate the disagreement point, i.e., there is a positive surplus to be enjoyed if agreement is reached and the question is how this surplus should be divided. As in most of game theory, by utility we mean von Neumann-Morgenstern expected utility; there may be underlying uncertainty, perhaps related to the probability of breakdown of negotiations. We shall normalize the disagreement utilities to 0 (this is without loss of generality if one uses expected utility because any positive affine transformation of utility functions represents the same preferences over lotteries). The resulting bargaining problem is called a normalized problem.

With this second assumption, Nash is implying that all relevant information to the solution of the problem must be subsumed in the pair (S, d). In other words, two bargaining situations that may include distinct details ought to be solved the same way if both reduce to the same pair (S, d) in utility terms. In spite of this, it is sometimes convenient to distinguish between feasible utility pairs (points in S) from feasible outcomes in physical terms (such as the splits of a pie, to be created after agreement).

Following each of the two papers by Nash (1950, 1953), bargaining theory is divided into two branches, the so-called axiomatic and strategic theories. The axiomatic theory, born with Nash (1950), which most authors identify with a normative approach to bargaining, proposes a number of properties that a solution to any bargaining problem should have, and proceeds to identify the solution that agrees with those principles. On the other hand, initiated in Nash (1953), the strategic theory is its positive counterpart: the usual approach here is the exact specification of the details of negotiation (timing of moves, information available, commitment devices, outside options and threats) and the identification of the behavior that would occur in those negotiation protocols. Thus, while the former stresses how bargaining *should* be resolved between rational parties according to some desirable principles, the latter describes how bargaining *could* evolve in a non-cooperative extensive form in the presence of common knowledge of rationality. Interestingly, the two theories connect and complement one another (for more on this connection, see the Nash program entry in this dictionary).

2.1. The Nash bargaining solution. The first contribution to axiomatic bargaining theory was made by John Nash in his path-breaking paper published in 1950. Nash wrote it as a term paper in an international trade course that he was taking as an undergraduate at Carnegie, at the age of seventeen. At the request of his Carnegie economics professor, Nash mailed his term paper to John von Neumann, who had just published his monumental book with Oskar Morgenstern. John von Neumann may not have paid enough attention to a paper sent by an undergraduate at a different university, and nothing happened with the paper until Nash arrived in Princeton to begin his PhD in mathematics.

According to Nash (1950), a solution to bargaining problems is simply a function that assigns to each normalized utility possibility set S one of its feasible points (recall that the normalization of the disagreement utilities has already been performed). The interpretation is that the solution dictates a specific agreement to each possible bargaining situation. Examples of solutions are: (i) the disagreement solution, which assigns to each normalized bargaining problem the point (0,0), a rather pessimistic solution; and (ii) the dictatorial solution with bargainer 1 as the dictator, which assigns the point in the Pareto frontier of the utility possibility set in which agent 2 receives 0 utility. Surely, neither of these solutions looks very appealing: while the former is not Pareto efficient because it does not exploit the gains from cooperation associated with an agreement, the latter violates the most basic fairness principle by being so asymmetric.

Nash (1950) proceeds by proposing four desirable properties that a solution to bargaining problems should have.

1. Scale invariance or independence of equivalent utility representations. Since the bargaining problem is formulated in von Neumann-Morgenstern utilities, if utility functions are rescaled but they represent the same preferences, the solution should be rescaled in the same fashion. That is, no

fundamental change in the recommended agreement will happen following a renormalization of utility functions; the solution will simply rescale utilities accordingly.

2. Symmetry. If a bargaining problem is symmetric with respect to the 45 degree line, the solution must pick a point on it: in a bargaining situation in which each of the threats made by one bargainer can be countered by the other with exactly the same threat, both should be equally treated by the solution. This axiom is sometimes called "equal treatment of equals" and it ensures that the solution yields "fair" outcomes.

3. Pareto efficiency. The solution should pick a point of the Pareto frontier. As elsewhere in welfare economics, efficiency is the basic ingredient of a normative approach to bargaining; negotiations should yield an efficient outcome in which all gains from cooperation are exploited.

4. Independence of irrelevant alternatives (IIA). Suppose a solution picks a point from a given normalized bargaining problem. Consider now a new normalized problem, subset of the original, but containing the point selected earlier by the solution. Then, the solution must still assign the same point. That is, the solution should be independent of "irrelevant" alternatives: as in a constrained optimization program, the deleted alternatives are deemed irrelevant because they were not chosen when they were present, so their absence should not alter the recommended agreement.

With the aid of these four axioms, Nash (1950) proves the following result:

Theorem 1 There is a unique solution to bargaining problems that satisfies properties (1-4): it is the one that assigns to each normalized bargaining problem the point that maximizes the product of utilities of the two bargainers.

Today we refer to this solution as the Nash solution. Although some of the axioms have been the center of some controversy –especially, his fourth axiom of IIA–, the Nash solution has remained as the fundamental piece of this theory and its use in applications is pervasive.

Some features of the Nash solution ought to be emphasized:

- The theory can be extended to the multilateral case, in which there are $n \geq 3$ parties present in bargaining: in a multilateral problem, it continues to be true that the unique solution that satisfies (1-4) is the one prescribing that agreement in which the product of utilities is maximized. See Lensberg (1988) for an important alternative axiomatization.

- The theory is independent from the details of the negotiation specific protocols, since it is formulated directly in the space of utilities. In particular, it can be applied to problems where the utilities are derived from only one good or issue, as well as those where it comes from multiple goods or issues.

- Perhaps surprisingly because risk is not explicitly part of Nash's story, it is worth noting that the Nash solution punishes risk aversion. All other things equal, it will award a lower portion of the surplus to a risk averse agent. This captures an old intuition in previous literature that risk aversion is detrimental to a bargainer: afraid of the bargaining breakdown, the more risk averse a person is, the more he will concede in the final agreement. For example, suppose agents are bargaining over how to split a surplus of size 1. Let the utility functions be as follows: $u_1(x_1) = x_1^{\alpha}$ for $0 < \alpha \leq 1$, and $u_2(x_2) = x_2$, where x_1 and x_2 are the non-negative shares of the surplus, which add up to 1. The reader can calculate that the Pareto frontier of the utility possibility set corresponds to the agreements satisfying the equation $u_1^{1/\alpha} + u_2 = 1$. Therefore, the Nash solution awards the utility vector $(u_1^*, u_2^*) = ((\frac{\alpha}{\alpha+1})^{\alpha}, \frac{1}{\alpha+1})$, corresponding to shares of the surplus $(x_1, x_2) = (\frac{\alpha}{\alpha+1}, \frac{1}{\alpha+1})$. Note how the smaller α, the more risk averse bargainer 1 is.

- Zeuthen's principle turns out to be related to the Nash solution (see Harsanyi (1987)): in identifying the bargainer who must concede next, the Nash product of utilities of the two proposals plays a role. See Rubinstein, Safra and Thomson (1992) for a related novel interpretation of the Nash solution.

- The family of asymmetric Nash solutions has also been used in the literature, as a way to capture unequal bargaining powers. If the bargaining power of player i is $\beta_i \in [0,1]$, $\sum_i \beta_i = 1$, the asymmetric Nash solution with weights (β_1, β_2) is defined as the function that assigns to each normalized bargaining problem the point where $u_1^{\beta_1} u_2^{\beta_2}$ is maximized.

2.2. The Kalai-Smorodinsky bargaining solution. Several researchers criticized some of Nash's axioms, IIA especially. To see why, think of the following example, that begins with the consideration of a symmetric right angle triangle S with legs of length 1. Clearly, efficiency and symmetry alone determine that the solution must be the point $(1/2, 1/2)$. Next, chop off the top part of the triangle to get a problem $T \subset S$, in which all points where $u_2 > 1/2$ have been deleted. By IIA, the Nash solution applied to the problem T is still the point $(1/2, 1/2)$.

Kalai and Smorodinsky (1975) propose to retain the first three axioms of Nash's, but drop IIA. Instead, They propose an individual monotonicity axiom. To understand it, let $a_i(S)$ be the highest utility that agent i can achieve in the normalized problem S, and let us call it agent i's aspiration level. Let $a(S) = (a_1(S), a_2(S))$ be the utopia point, typically not feasible.

5. Individual monotonicity. If $T \subset S$ are two normalized problems, and $a_j(T) = a_j(S)$, the solution must award i a utility in S at least as high as in T.

We can now state the Kalai-Smorodinsky theorem:

Theorem 2 There is a unique solution to bargaining problems that satisfies properties (1, 2, 3, 5): it is the one that assigns to each normalized bargaining problem the intersection point of the Pareto frontier and the straight line segment connecting 0 and the utopia point.

Note how the Kalai-Smorodinsky solution awards the point $(2/3, 1/3)$ to the problem T of the beginning of this subsection. In general, while the Nash solution pays attention to local arguments (it picks out the point of the smooth Pareto frontier where the utility elasticity $(du_2/u_2)/(du_1/u_1)$ is 1), the Kalai-Smorodinsky solution is mostly driven by "global" considerations, such as the highest utility each bargainer can obtain in the problem.

2.3. Other solutions. Although the two major axiomatic solutions are Nash's and Kalai-Smorodinsky's, authors have derived a plethora of other solutions also axiomatically (see, for example, Thomson (1994) for an excellent survey). Among them, one should perhaps mention the egalitarian solution, which picks out the point of the Pareto frontier where utilities are equal. This is based on very different principles, much more tied to ethics of a certain kind and less to the principles governing bargaining between two rational individuals. In particular, note how it is not invariant to equivalent utility representations, because of the strong interpersonal comparisons of utilities that it performs.

3. The strategic theory of bargaining. Now we are interested in specifying the details of negotiations. Thus, while we may lose the generality of the axiomatic approach, our goal is to study reasonable procedures and identify rational behavior in them. For this and the next section, some great references include Osborne and Rubinstein (1990) and Binmore, Osborne and Rubinstein (1992).

3.1. Nash's demand game. Nash (1953) introduces the first bargaining model expressed as a non-cooperative game. Nash's demand game, as it is often called, captures in crude form the force of commitment in bargaining. Both bargainers must demand simultaneously a utility level. If the pair of utilities is feasible, it is implemented; otherwise, there is disagreement and both receive 0. This game admits a continuum of Nash equilibrium outcomes, including every point of the Pareto frontier, as well as disagreement. The first message that emerges from Nash's demand game is the indeterminacy of equilibrium outcomes, commonplace in non-cooperative game theory. In the same paper, advancing ideas that would be developed a couple of decades later, Nash proposed a refinement of the Nash equilibrium concept based on the possibility of uncertainty around the true feasible set. The result was a selection of one Nash equilibrium outcome, which converges to the Nash solution agreement as uncertainty vanishes.

The model just described is referred to as Nash's demand game with fixed threats: following an incompatible pair of demands, the outcome is the fixed disagreement point. Nash (1953) also analyzed a variable threats model. In it, the stage of simultaneous demands is preceeded by another stage, in which bargainers choose threats. Given a pair of threats chosen in the first stage, the refinement argument is used to obtain the Nash solution of the induced problem in the ensuing subgame (where the threats determine an endogenous disagreement point). Solving the entire game is possible by backward induction, appealing to logic similar to that in von Neumann's minimax theorem; see Abreu and Pearce (2002) for a connection between the variable threats model and repeated games.

3.2. The alternating offers bargaining procedure. The following game describes elegantly a stylized protocol of negotiations over time. It was studied by Stahl (1972) under the assumption of an exogenous deadline (finite horizon game), and by Rubinstein (1982) in the absence of a deadline (infinite horizon game). Players 1 and 2 are bargaining over a surplus of size 1. The bargaining protocol is one of alternating offers. In period 0, player 1 begins by making a proposal, a division of the surplus, say $(x, 1-x)$, where $0 \leq x \leq 1$ represents the part of the surplus that she demands for herself. Player 2 can then either accept or reject this proposal. If he accepts, the proposal is implemented; if he rejects, a period must elapse for them to come back to the negotiation table, and at that time (period 1), the roles are reversed so that player 2 will make a new proposal $(y, 1-y)$, where $0 \leq y \leq 1$ is the fraction of surplus that he offers to player 1. Player 1 must then either accept the new proposal, in which case bargaining ends with $(y, 1-y)$ as the agreement; or reject it, in which case a period must elapse before player 1 makes a new proposal. In period 2, player 1 proposes $(z, 1-z)$, to which player 2 must respond, and so on. The T-period finite horizon game imposes the disagreement outcome, with zero payoffs, after T proposals have been rejected. On the other hand, in the infinite horizon version, there is always a new proposal in the next period after a proposal is rejected.

Both players discount the future at a constant rate. Let $\delta \in [0, 1)$ be the per period discount factor. To simplify, let us assume that utility is linear in shares of the surplus. Therefore, from a share x agreed in period t, a player derives a utility of $\delta^{t-1}x$. Note how utility is increasing in the share of the surplus (monotonicity), and decreasing in delay with which the agreement takes place (impatience).

A strategy for a player is a complete contingent plan of action to play the game. That is, a strategy specifies a feasible action every time a player is called upon to act in the game. In a dynamic game, Nash equilibrium does little to restrict the set of predictions: for example, it can be shown that in the alternating offers games, any agreement $(x, 1-x)$ in any period t, $0 \leq t \leq T < \infty$, can be supported by a Nash equilibrium; disagreement is also a Nash equilibrium outcome.

The prediction that game theory gives in a dynamic game of complete information is typically based on finding its subgame perfect equilibria. A subgame perfect equilibrium (SPE) in a two-player game is a pair of strategies, one for each player, such that the behavior specified by them is a best response to each other at every point in time (not only at the beginning of the game). By asking that players must choose a best response to each other at every instance that they are supposed to act, SPE rules out incredible threats: that is, at a SPE players have an incentive to carry out the threat implicit in their equilibrium strategy because it is one of the best responses to the behavior they expect the other player to follow at that point.

In the alternating offers games described above, there is a unique SPE, both in the finite and infinite horizon versions. The SPE in the finite horizon game is found by backward induction. For example, in the one-period game, the so-called "ultimatum game," the unique SPE outcome is the agreement on the split $(1, 0)$: since the outcome of a rejection is disagreement, the responder will surely accept any share of $\epsilon > 0$, which implies that in equilibrium the proposer ends up taking the entire surplus. Using this intuition, one can show that the outcome of the two-period game is the immediate agreement on the split $(1 - \delta, \delta)$: anticipating that if negotiations get to the final period, player 2 (the proposer in that final period) will take the entire surplus, player 1 convinces him not to get there simply by offering him the present discounted value of the entire surplus, i.e., δ, while she takes the rest. This logic continues and can be extended to any finite horizon. The sequence of SPE outcomes so obtained as the deadline $T \to \infty$ is shown to converge to the unique SPE of the infinite horizon game. This game, more challenging to solve since one cannot go to its last period to begin inducting backwards, was studied in Rubinstein (1982). We proceed to state its main theorem and discuss the properties of the equilibrium (see Shaked and Sutton (1984) for a simple proof).

Theorem 3 Consider the infinite horizon game of alternating offers, in which both players discount the future at a per period rate of $\delta \in [0, 1)$. There exists a unique SPE of this game: it prescribes immediate agreement on the division $(\frac{1}{1+\delta}, \frac{\delta}{1+\delta})$.

The first salient prediction of the equilibrium is that there will not be any delay in reaching an agreement. Complete information –each player knows the other player's preferences– and the simple structure of the game are key factors to explain this.

The equilibrium awards an advantage to the proposer, as expressed by the discount factor: note how the proposer's share exceeds the responder's by a factor of $1/\delta$. Given impatience, having to respond to a proposal puts an agent in a delicate position, since rejecting the offer entails time wasted until the next round of negotiations comes. This is the source of the proposer's advantage. Of course, this advantage is larger the larger the impatience of the responder: note how if $\delta = 0$ (extreme impatience), the equilibrium awards all the surplus to the proposer because her offer is virtually an ultimatum; on the other hand, as $\delta \to 1$, the first-mover advantage disappears and the equilibrium tends to equal split of the surplus.

To understand how the equilibrium works and in particular how the threats employed in it are credible, consider the SPE strategies. Both players use the same strategy, and it is the following: as a proposer, each player always asks for $1/(1+\delta)$ and offers $\delta/(1+\delta)$ to the other party; as a responder, a player accepts an offer as long as the share offered to the responder is at least $\delta/(1+\delta)$. Note how rejecting a share lower than $\delta/(1+\delta)$ is credible, in that its consequence, according to the equilibrium strategies, is to agree in the next period on a split that awards the rejecting player a share of $1/(1+\delta)$, whose present discounted value at the time the rejection occurs is exactly $\delta/(1+\delta)$.

To appreciate the difference with Nash equilibrium, let us argue for example that the split $(0, 1)$ cannot happen in a SPE. This agreement happens in a Nash equilibrium, supported by strategies that ask player 1 to offer the whole pie to player 2, and player 2 to reject any other offer. However, the threat embodied in player 2's strategy is not credible: when confronted with an offer $(\epsilon, 1-\epsilon)$ for $\delta < 1-\epsilon < 1$, player 2 will have to accept it, contradicting his strategy. Can the reader argue why the Nash equilibrium split $(1, 0)$ is not a SPE outcome either (because to do so one would need to employ incredible threats)? Rubinstein (1982) shows that the same non-credible threats are associated with any division of the pie other than the one identified in the theorem.

The Rubinstein-Stahl alternating offers game provides an elegant model of how negotiations may take place over time, and its applications are numerous, including bargaining problems pertaining to international trade, industrial organization, or political economy. However, unlike Nash's axiomatic theory, its predictions are sensitive to details. This is no doubt one of its strengths because one can calibrate how those details may influence the theory's prediction, but it is also its weakness in terms of lack of robustness in predictive power.

3.3. Incomplete information. In a static framework, Chatterjee and Samuelson (1983) study a double auction. A buyer and a seller are trying to transact a good. Each proposes a price, and trade takes place at the average of the two prices if and only if the buyer's price exceeds the seller's. Each trader knows his own valuation for the good. However, there is incomplete information on each side concerning the other side's valuation. It can be shown that in any equilibrium of this game there are inefficiencies: given certain ex-post valuations of buyer and seller, there should be trade, yet it is precluded because of the incompleteness of information, which leads traders to play "too tough."

Let us now turn to bargaining over time. As pointed out above, one prediction of the Rubinstein-Stahl model is immediate agreement. This may clash with casual observation, by simply noting the existence of strikes, lockouts and long periods of disagreement in many actual negotiations. As a consequence, researchers have suggested the construction of models in which inefficiencies, in the form of delay in agreement, occur in equilibrium. The main feature of bargaining models with this property is incomplete information. (For delay in agreement that does not rely on incomplete information, see Fernandez and Glazer (1991), Avery and Zemsky (1994), and Busch and Wen (1995).)

If parties do not know each other's preferences (impatience rate, per period fixed cost of hiring a lawyer, profitability of the agreement, and so on), the actions taken by the parties in the bargaining game may be intended to elicit part of the information that they do not have, or perhaps they may be meant to reveal or misrepresent some of the information privately held.

One technical remark is in order. The typical approach is to reduce the uncertainty to a game of imperfect information, through the specification of types in the sense of Harsanyi (1967, 1968). In such games, SPE no longer constitutes an appropriate refinement of Nash equilibrium. The relevant equilibrium notions are perfect Bayesian equilibrium and sequential equilibrium, and in them, the off-equilibrium path beliefs play an important role in sustaining outcomes. Moreover, these concepts are often incapable of yielding a determinate prediction in many games, and authors have in these cases resorted to further refinements. One problem of the refinements literature, though, is that it lacks strong foundations. Often the successful use of a given refinement in a game is accompanied by a bizarre prediction when the same concept is used in other games. Therefore, one should interpret these findings as showing the possibilities that equilibrium can offer in these contexts, but the theory here is far from giving a determinate answer.

Rubinstein (1985) studies an alternating offers procedure in which there is one-sided incomplete information (i.e., while player 1 has uncertainty regarding player 2's preferences, player 2 is fully informed). Suppose there are two types of player 2: one of them is "weaker" than player 1, while the other is "stronger" (in terms of impatience, or per period costs). This game admits many equilibria, and they differ as a function of parameter configurations. There are pooling equilibria, in which an offer from player 1 is accepted immediately by both types of player 2. More relevant to the current discussion, there are also separating equilibria, in which player 1's offer is accepted by the weak type of player 2, while the strong type signals his true preferences by rejecting the offer and imposing delay in equilibrium. These equilibria are also used to construct other equilibria with more periods of delay in agreement. Some authors argue (Gul and Sonnenschein (1988)) that long delays in equilibrium are the product of strong non-stationary behavior (that is, a player behaves very differently in and out of equilibrium, as a function of changes in his beliefs). They show that imposing stationary behavior limits the delay in agreement quite significantly. One advantage of stationary equilibria is their simplicity, but one problem with them is that stationarity is also imposed on beliefs (players hold beliefs that are independent of the history of play).

The analysis is simpler and multiplicity of equilibrium is less of a problem in games in which the uninformed party makes all the offers. Consider, for example, a version of the model in Sobel and Takahashi (1983). The two players are a firm and a union. The firm is fully informed, while the union does not know the true profitability of the firm. The union makes all offers in these wage negotiations, and there is discounting across periods. In equilibrium, different types of the firm accept offers at different points in time: firms whose profitability is not that high can afford to reject the first high wage offers made by the union to signal their private information, while very profitable firms cannot because delay in agreement hurts them too much.

Most papers have studied the case of private values asymmetric information (if a player knows her type, she knows her preferences), although the correlated values case has also been analyzed (there, knowing one's type is not sufficient to know one's utility function) – see Evans (1989) and Vincent (1989). The case of two-sided asymmetric information, in which neither party is fully informed, has been treated, for example, in Watson (1998). In all these results, one is able to find equilibria with significant delay in agreement, implying the consequent inefficiencies. Uncertainty may also be about the rationality of the opponent: for example, one may be bargaining with a "behavioral type" who has an unknown threshhold below which he will reject all proposals (see Abreu and Gul (2000)).

A more general approach is adopted by the mechanism design papers. The focus is not simply on explaining delay as an equilibrium phenomenon in a given extensive form. Rather, the question is whether inefficiencies are a consequence of equilibrium behavior in any bilateral bargaining game with incomplete information. The classic contribution to this problem is the paper by Myerson and Satterthwaite (1983). In a bilateral trading problem in which there is two-sided private values asymmetric information and the types for each trader are drawn independently from overlapping intervals, there does not exist any budget-balanced mechanism satisfying incentive compatibility, interim individual rationality and ex-post efficiency. All these are desirable properties for a trading mechanism. Budget balance implies that payoffs cannot be increased with outside funds. Incentive compatibility requires that each type has no incentive to misrepresent his information. Interim individual rationality means that no type be worse off trading than not trading. Finally, ex-post efficiency imposes that trade take place if and only if positive gains from trade exist. This impossibility result is a landmark of the limitations of bargaining under incomplete information, and has generated an important literature that explores ways to overcome it (see for example Gresik and Satterthwaite (1989) and Satterthwaite and Williams (1989)).

3.4. Indivisibilities in the units. One important way in which Rubinstein's result is not robust happens when there are only a finite set of possible offers to be made (see van Damme, Selten and Winter (1990) and Muthoo (1991)). Indivisibilities make it impossible for an exact adjustment of offers to leave the responder indifferent; as a result, multiple and inefficient equilibria appear. The issue concerns how fine the grid of possible instantaneous offers is with respect to the time grid at which bargaining takes place. If the former is finer than the latter, Rubinstein's uniqueness goes through, while it does not otherwise. There will be circumstances for which one or the other specification of negotiation rules will be more appropriate.

3.5. Multi-issue bargaining. The following preliminary observation is worth making: if offers are made in utility space or all issues must be bundled in every offer, Rubinstein's result obtains. Thus, the literature on multi-issue bargaining has looked at procedures that depart from these assumptions.

The first generation of papers with multiple issues assumed that the agenda, i.e., the order in which the different issues are brought to the table, was exogenously given. Since each issue is bargained over one at a time, Rubinstein's uniqueness and efficiency result obtains, simply proceeding by backward induction on the issues. Fershtman (1990, 2000) and Busch and Horstmann (1997) study such games, from which one learns the comparative statics of equilibrium when agendas are exogenously fixed. The next group of papers study more realistic games where the agenda is chosen endogenously by the players. The main lesson from this line of work is that restricting the issues that a proposer can bring to the table is a source of inefficiencies. Inderst (2000) and In and Serrano (2003) study a procedure where agenda is totally unrestricted, i.e., the proposer can make offers on any subset of remaining issues, and by exploiting tradeoffs in the marginal rates of substitution between issues, Rubinstein's efficiency result is also found. In contrast, Lang and Rosenthal (2001) and In and Serrano (2004) construct multiple and inefficient equilibria (including those with arbitrarily long delay in agreement) when agenda restrictions are imposed. Finally, Weinberger (2000) considers multi-issue bargaining when the responder can accept selectively subsets of proposals and also finds inefficiencies if issues are indivisible.

3.6. Multilateral bargaining. Even within the case of pure bargaining problems, one needs to make a distinction between different ways to model negotiations. The first extension of the Rubinstein game to this case is due to Shaked, as reported in Osborne and Rubinstein (1990); see also Herrero (1985). Today we refer to the Shaked/Herrero game as the unanimity game. In it, one of the players, say player 1, begins by making a public proposal to the others. A proposal is a division of the unit of surplus available when agreement is reached. Players $2, \dots, n$ then must accept or reject this proposal. If all agree, it is implemented immediately. If at least one of them rejects, time elapses and the next period another player, say player 2, will make a new proposal, and so on. Note how these rules reduce to Rubinstein's when there are only two players. However, the prediction emerging from this game is dramatically different. For values of the discount factor that are sufficiently high (if $\delta \geq 1/(n-1)$), every feasible agreement can be supported by a SPE, and in addition, equilibria with an arbitrary number of periods of delay in agreement show up. The intuition for this extreme result is that the unanimity required by the rules in order to implement an agreement facilitates a plethora of equilibrium behaviors. For example, let us see how in the case of $n = 3$ it is possible to sustain an agreement where all the surplus goes to player 3. If player 2 rejects it, the same split will be repeated in the continuation, so it is pointless to reject. If player

1 changes her proposal to try to obtain a gain, it will be rejected by that responder who in the proposal receives less than 1/2 (there must be at least one). This rejector can be bribed with receiving the entire surplus in the continuation, whose present discounted value is at least 1/2 (recall $\delta \geq 1/2$), thereby rendering his rejection credible. Of course, the choice of player 3 as the one receiving the entire surplus is entirely arbitrary, and therefore, one can see how extreme multiplicity of equilibrium is a phenomenon inherent to the unanimity game. This multiplicity relies on non-stationary strategies, as it can be shown that there is a unique stationary SPE.

An alternative extension of the Rubinstein rules to multilateral settings is given by exit games; see Jun (1987), Chae and Yang (1994), Krishna and Serrano (1996). As an illustration, let us describe the negotiation rules of the Krishna-Serrano game. Player 1 makes a public proposal, a division of the surplus, and the others must respond to it. Those who accept it leave the game with the shares awarded by the proposer, while the rejectors continue to bargain with the proposer over the part of the surplus that has not been committed to any player. A new proposal comes from one of the rejectors, and so on. These rules also reduce to Rubinstein's if $n = 2$, but now the possibility of exiting the game by accepting a proposal has important implications for the predictive power of the theory. Indeed, Rubinstein's uniqueness is restored and the equilibrium found inherits the properties of that of Rubinstein's, including its immediate agreement and the proposer's advantage (the equilibrium shares are $1/[1 + (n - 1)\delta]$ for the proposer and $\delta/[1 + (n - 1)\delta]$ for each responder). Note how, given that the others accept, each responder is *de facto* immersed in a two-player Rubinstein game, so in equilibrium he receives a share that makes him exactly indifferent between accepting and rejecting: this explains the ratio $1/\delta$ between the proposer's and each responder's equilibrium shares. The sensitivity of the result to the exact specification of details is emphasized in other papers. Vannetelbosch (1999) shows that uniqueness obtains in the exit game even with a notion of rationalizability, weaker than SPE; and Huang (2002) establishes that uniqueness is still the result in a model that combines unanimity and exit, since offers can be made both conditional and unconditional to each responder. Baliga and Serrano (1995, 2001) introduce imperfect information in the unanimity and exit games (offers are not public, but made in personalized envelopes), and multiplicity is found in both, based on multiple off-equilibrium path beliefs. Merlo and Wilson (1995) propose a stochastic specification and also find uniqueness of the equilibrium outcome. In a model often used in political applications, Baron and Ferejohn (1989) study a procedure with random proposer in which the proposals are adopted if approved by simple majority (between the unanimity and exit procedures described).

4. Bargaining and markets. Bargaining theory provides a natural approach to understand how prices may emerge in markets as a consequence of the direct interaction of agents. One can characterize the outcomes of models in which the interactions of small groups of agents are formulated as bargaining games, and compare them to market outcomes such as competitive equilibrium allocations. If a connection between the two is found, one is giving an answer to the long standing question of the origin of competitive equilibrium prices, without having to resort to the story of the Walrasian auctioneer. If not, one can learn the importance of the frictions in the model that may be preventing such a connection. Both kinds of results are valuable for economic theory.

4.1. Small markets. Models have been explored in which two agents are bargaining, but at least one of them may have an outside option (see Binmore, Shaked and Sutton (1988)). Thus, the bargaining pair is part of a larger economic context, which is not explicitly modeled. In the simplest specification, uniqueness and efficiency of the equilibrium is found. In the equilibrium, the outside option is used if it pays better than the Rubinstein equilibrium, while it is ignored otherwise. Jehiel and Moldovanu (1995) show that delays may be part of the equilibrium when the agreement between a seller and several buyers is subject to externalities among the buyers: a buyer may have an incentive to reject an offer in the hope of making a different buyer accept the next offer and free-ride from that agreement. In general, these markets involving a small number of agents do not yield competitive allocations because market power is retained by some traders (see Rubinstein and Wolinsky (1990)).

4.2. Large markets under complete information. The standard model assumes a continuum of agents who are matched at random, typically in pairs, to perform trade of commodities. If a pair of agents agrees on a trade, they break the match. In simpler models, all traders leave the market after they trade once. In the more general models agents may choose either to leave and consume, or stay in the market to be matched anew. Some authors have studied steady-state versions, in which the measure of traders leaving the market every period is offset exactly by the same measure of agents entering the market. In contrast, non-steady state models do not keep the measure of active traders constant (one prominent class of non-steady state models is that of one-time entry, in which after the initial period, there is no new entry; certain transacting agents exit every period, so the market size dwindles over time). The analysis has been performed with discounting (where δ is the common discount factor that is thought of as being near 1) or without it: in both cases the idea is to describe frictionless or almost frictionless conditions (see, for example, Muthoo (1993) for a paper that considers several frictions and the outcomes that result when some, but not all, of them are removed.)

The first models were introduced by Diamond and Maskin (1979), Diamond (1981), and Mortensen (1982), and they used the Nash solution to solve each bilateral bargaining encounter. Later each pairwise meeting has been modeled adopting a procedure from the strategic theory.

The most general results in this area are provided by Gale (1986a, b, c, 1987). First, in a partial equilibrium setup, a market for an indivisible good is analyzed in Gale (1987), both under steady state and non-steady state assumptions. The result is that all equilibrium outcomes yield trade at the competitive price when discounting is small: in all equilibria trade tends to take place at only one price, and that price must be the competitive price because it is the one that maximizes each trader's expected surplus. This generalizes a result of Binmore and Herrero (1988) and clarifies an earlier claim made by Rubinstein and Wolinsky (1985). Rubinstein and Wolinsky analyzed the market in steady state and claimed that the market outcome was different from the competitive one. Their claim is justified if one measures the sets of traders in terms of the stocks present in the market, but Gale (1987) argues convincingly that, given the steady state imposed on the solution concept, it is the flow of agents into the market every period, not the total stock, that should comprise the relevant demand and supply curves. When this is taken into account, all prices are competitive because the measure of transacting sellers is the same as that of the transacting buyers.

In a more general model, Gale (1986a, b, c) studies an exchange economy with an arbitrary number of divisible goods. Now there is no discounting and agents can trade as many periods as they wish before they leave the market place. Only after an agent rejects a proposal can he leave the market. Under a number of technical assumptions, Gale shows once again that all the equilibrium outcomes of his game are Walrasian:

Theorem 4 At every market equilibrium, each agent leaves the market with the bundle x_k with probability 1, where the list of such bundles is a Walrasian allocation of the economy.

Different versions of this result are proved in Gale (1986a, c) and in Osborne and Rubinstein (1990). Also, Kunimoto and Serrano (2004) obtain the same result under substantially weaker assumptions on the economy, thereby emphasizing the robustness of the connection between the market equilibria of this decentralized exchange game and the Walrasian allocations of the economy. There are two key steps in this argument: first, one establishes that, since pairs are trading, pairwise efficiency obtains, which under some conditions leads to Pareto efficiency; and second, the equilibrium strategies imply budget balance so that each agent cannot end up with a bundle that is worth more than his initial endowment (given prices supporting the equilibrium allocation, already known to be efficient).

Dagan, Serrano and Volij (2000) also show a Walrasian result, but in their game the trading groups are coalitions of any finite size: in their proof, the force of the core equivalence theorem is exploited. One final comment is pertinent at this point. Some authors (Gale (2000)) question the use of coalitions of any finite size in the trading procedure because the "large" size of some of those groups seems to clash with the "decentralized" spirit of these mechanisms. On the other hand, one can also argue that for the procedure to allow trade only in pairs, some market authority must be keeping track of this, making sure that coalitions of at least three agents are "illegal." Both trading technologies capture appealing aspects of decentralization, depending on the circumstances, and the finding is that either one yields a robust connection with the teachings of general equilibrium theory in frictionless environments. This is one more instance of the celebrated equivalence principle: in models involving a large number of agents, game theoretic predictions tend to converge, under some conditions, to the set of competitive allocations.

4.3. Large markets under incomplete information. If the asymmetric information is of the private values type, the same equivalence result is obtained between equilibria of matching and bargaining models and Walrasian allocations. This message is found, for example, in Rustichini, Satterthwaite and Williams (1994), Gale (1987) and Serrano (2002). In the latter model, for instance, some non-Walrasian outcomes are still found in equilibrium, but they can be explained by features of the trading procedure that one could consider as frictions, such as a finite set of prices and finite sets of traders' types.

The result is quite different when asymmetric information goes beyond private values. For example, Wolinsky (1990) studies a market with pairwise meetings in which there is uncertainty regarding the true state of the world (which determines the true quality of the good being traded). Some traders know the state, while others do not, and there are uninformed traders among buyers and sellers (two-sided asymmetric information). The analysis is performed in steady-state. To learn the true state, uninformed traders sample agents of the opposite side of the market. However, each additional meeting is costly due to discounting. The relevant question is whether information will get revealed from the informed to the uninformed when discounting is removed. Wolinsky's answer is in the negative: as the discount factor $\delta \to 1$, a non-negligible fraction of uninformed traders transacts at a price that is not ex-post individually rational. It follows that the equilibrium outcomes do not approximate those given by a fully revealing rational expectations equilibrium (REE). The reason for this result is that, while as $\delta \to 1$ sampling becomes cheaper and therefore each uninformed trader samples more agents, this is true on both sides so that uninformed traders end up trying to learn from agents that are just as uninformed as they are. Serrano and Yosha (1993) overturn this result when asymmetric information is one-sided: in this case, although the noise force behind Wolinsky's result is not operative because of the absence of uninformed traders on one side, there is a negative force that works against learning, and that is the fact that misrepresenting information becomes cheaper for informed traders as $\delta \to 1$. The analysis in Serrano and Yosha's paper shows that, under steady-state restrictions, the learning force is more powerful than the misrepresentation one, and convergence to REE is attained. Finally, Blouin and Serrano (2001) perform the analysis without the strong steady-state assumption, and show that with both information structures (one-sided and two-sided asymmetries) the result is negative: Wolinsky's noise force in the two-sided case continues to be crucial, while misrepresentation becomes very powerful in the one-sided model because of the lack of fresh uninformed traders. In these models, agents have no access to aggregate market signals; information is heavily restricted because agents observe only their own private history. It would be interesting to analyze other procedures where information may flow more easily.

Roberto Serrano

References

Abreu, D. and F. Gul (2000). Bargaining and reputation. Econometrica 68, 85-118.

Abreu, D. and D. Pearce (2002). Bargaining, reputation and equilibrium selection in repeated games. Mimeo, Princeton University.

Avery, C. and P. B. Zemsky (1994). Money burning and multiple equilibria in bargaining. Games and Economic Behavior 7, 154-168.

Baliga, S. and R. Serrano (1995). Multilateral bargaining with imperfect information. Journal of Economic Theory 67, 578-589.

Baliga, S. and R. Serrano (2001). Multilateral negotiations with private side-deals: a multiplicity example. Economics Bulletin 3, No. 1, 1-7.

Baron, D. and J. Ferejohn (1989). Bargaining in legislatures. American Political Science Review 83, 1181-1206.

Binmore, K. G. and M. Herrero (1988). Matching and bargaining in dynamic models. Review of Economic Studies 55, 17-31.

Binmore, K. G., M. J. Osborne and A. Rubinstein (1992). Non-cooperative models of bargaining. In R. J. Aumann and S. Hart (eds.) Handbook of Game Theory with Economic Applications, Vol. 1. Elsevier, New York.

Binmore, K. G., A. Shaked and J. Sutton (1988). An outside option experiment. Quarterly Journal of Economics 104, 753-770.

Blouin, M. R. and R. Serrano (2001). A decentralized market with common values uncertainty: non-steady states. Review of Economic Studies 68, 323-346.

Busch, L.-A. and I. J. Horstmann (1997). Bargaining frictions, bargaining procedures and implied costs in multiple-issue bargaining. Economica 64, 669-680.

Busch, L.-A. and Q. Wen (1995). Perfect equilibria in a negotiation model. Econometrica 63, 545-565.

Chae, S. and J.-A. Yang (1994). An n-person pure bargaining game. Journal of Economic Theory 62, 86-102.

Chatterjee, K. and W. Samuelson (1983). Bargaining under incomplete information. Operations Research 31, 835-851.

Dagan, N., R. Serrano and O. Volij (2000). Bargaining, coalitions and competition. Economic Theory 15, 279-296.

Diamond, P. (1981). Mobility costs, frictional unemployment, and efficiency. Journal of Political Economy 89, 798-812.

Diamond, P. and E. Maskin (1979). An equilibrium analysis of search and breach of contract I: steady states. Bell Journal of Economics 10, 282-316.

Edgeworth, F. Y. (1881). Mathematical Psychics. Kegan Paul Publishers, London, reprinted in 2003, P. Newman (ed.) F. Y. Edgeworth's Mathematical Psychics and Further Papers on Political Economy. Oxford University Press.

Evans, R. (1989). Sequential bargaining with correlated values. Review of Economic Studies 56, 499-510.

Fernandez, R. and J. Glazer (1991). Striking for a bargain between two completely informed agents. American Economic Review 81, 240-252.

Fershtman, C. (1990). The importance of the agenda in bargaining. Games and Economic Behavior 2, 224-238.

Fershtman, C. (2000). A note on multi-issue two-sided bargaining: bilateral procedures. Games and Economic Behavior 30, 216-227.

Gale, D. (1986a). Bargaining and competition. Part I: characterization. Econometrica 54, 785-806.

Gale, D. (1986b). Bargaining and competition. Part II: existence. Econometrica 54, 807-818.

Gale, D. (1986c). A simple characterization of bargaining equilibrium in a large market without the assumption of dispersed characteristics. Working Paper 86-05, CARESS, University of Pennsylvania.

Gale, D. (1987). Limit theorems for markets with sequential bargaining. Journal of Economic Theory 43, 20-54.

Gale, D. (2000). Strategic Foundations of General Equilibrium - Dynamic Matching and Bargaining Games -. Cambridge University Press.

Gresik, T. and M. A. Satterthwaite (1989). The rate at which a simple market converges to efficiency as the number of traders increases: an asymptotic result for optimal trading mechanisms. Journal of Economic Theory 48, 304-332.

Gul, F. and H. Sonnenschein (1988). On delay in bargaining with one-sided uncertainty. Econometrica

56, 601-611.

Harsanyi, J. C. (1967, 1968). Games with incomplete information played by Bayesian players. Parts I, II and III. Management Science 14, 159-182, 320-334 and 486-502.

Harsanyi, J. C. (1987). Bargaining. In J. Eatwell, M. Milgate and P. Newman (eds.) The New Palgrave Dictionary of Economics, McMillan, London.

Herrero, M. (1985). N-player bargaining and involuntary underemployment. Chapter IV in A Strategic Bargaining Approach to Market Institutions. Ph.D. thesis, London School of Economics.

Hicks, J. R. (1932). The Theory of Wages. London, McMillan; New York, St. Martin Press, 1963.

Huang, C.-Y. (2002). Multilateral bargaining: conditional and unconditional offers. Economic Theory 20, 401-412.

In, Y. and R. Serrano (2003). Agenda restrictions in multi-issue bargaining (II): unrestricted agendas. Economics Letters 79, 325-331.

In, Y. and R. Serrano (2004). Agenda restrictions in multi-issue bargaining. Journal of Economic Behavior and Organization 53, 385-399.

Inderst, R. (2000). Multi-issue bargaining with endogenous agenda. Games and Economic Behavior 30, 64-82.

Jehiel, P. and B. Moldovanu (1995). Cyclical delay in bargaining with externalities. Review of Economic Studies 62, 619-637.

Jun, B. H. (1987). A structural consideration on 3-person bargaining. Chapter III in Essays on Topics in Economic Theory. Ph. D. Thesis, Department of Economics, University of Pennsylvania.

Kalai, E. and M. Smorodinsky (1975). Other solutions to Nash's bargaining problem. Econometrica 43, 513-518.

Krishna, V. and R. Serrano (1996). Multilateral bargaining. Review of Economic Studies 63, 61-80.

Kunimoto, T. and R. Serrano (2004). Bargaining and competition revisited. Journal of Economic Theory 115, 78-88.

Lang, K. and R. W. Rosenthal (2001). Bargaining piecemeal or all at once? Economic Journal 111, 526-540.

Lensberg, T. (1988). Stability and the Nash solution. Journal of Economic Theory 45, 330-341.

Merlo, A. and C. Wilson (1995). A stochastic model of sequential bargaining with complete information. Econometrica 63, 371-399.

Mortensen, D. (1982). Property rights and efficiency in mating, racing, and related games. American Economic Review 72, 968-979.

Muthoo, A. (1991). A note on bargaining over a finite set of feasible agreements. Economic Theory 1, 290-292.

Muthoo, A. (1993). Sequential bargaining and competition. Economic Theory 3, 353-363.

Myerson, R. B. and M. A. Satterthwaite (1983). Efficient mechanisms for bilateral trading. Journal of Economic Theory 29, 265-281.

Nash, J. F. (1950). The bargaining problem. Econometrica 18, 155-162.

Nash, J. F. (1953). Two person cooperative games. Econometrica 21, 128-140.

Osborne, M. J. and A. Rubinstein (1990). Bargaining and Markets. Academic Press, San Diego, CA, available at http://www.economics.utoronto.ca/osborne/bm/

Raiffa, H. (1982). The Art and Science of Negotiation. Harvard University Press, Cambridge.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. Econometrica 50, 97-109.

Rubinstein, A. (1985). A bargaining model with incomplete information on time preferences. Econometrica 53, 1151-1172.

Rubinstein, A. and A. Wolinsky (1985). Equilibrium in a market with sequential bargaining. Econometrica 53, 1133-1150.

Rubinstein, A. and A. Wolinsky (1990). Decentralized trading, strategic behavior and the Walrasian outcome. Review of Economic Studies 57, 63-78.

Rubinstein, A., Z. Safra and W. Thomson (1992). On the interpretation of the Nash bargaining solution and its extension to non-expected utility preferences. Econometrica 60, 1171-1186.

Rustichini, A., M. A. Satterthwaite and S. R. Williams (1994). Convergence to efficiency in a simple market with incomplete information. Econometrica 62, 1041-1063.

Satterthwaite, M. A. and S. R. Williams (1989). The rate of convergence to efficiency in the buyer's bid double auction as the market becomes large. Review of Economic Studies 56, 477-498.

Serrano, R. (2002). Decentralized information and the Walrasian outcome: a pairwise meetings market with private values. Journal of Mathematical Economics 38, 65-89.

Serrano, R. and O. Yosha (1993). Information revelation in a market with pairwise meetings: the one-sided information case. Economic Theory 3, 481-499.

Shaked, A. and J. Sutton (1984). Involuntary unemployment as a perfect equilibrium in a bargaining model. Econometrica 52, 1351-1364.

Sobel, J. and I. Takahashi (1983). A multi-stage model of bargaining. Review of Economic Studies 50, 411-426.

Stahl, I. (1972). Bargaining Theory. Stockholm School of Economics.

Thomson, W. (1994). Cooperative models of bargaining. In R. J. Aumann and S. Hart (eds.) Handbook of Game Theory with Economic Applications, Vol. 2. Elsevier, New York.

van Damme, E., R. Selten and E. Winter (1990). Alternating bid bargaining with a smallest money unit. Games and Economic Behavior 2, 188-201.

Vannetelbosch, V. (1999). Rationalizability and equilibrium in n-person sequential bargaining. Economic Theory 14, 353-371.

Vincent, D. (1989). Bargaining with common values. Journal of Economic Theory 48, 47-62.

von Neumann, J. and O. Morgenstern (1944). Theory of Games and Economic Behavior. Princeton University Press.

Watson, J. (1998). Alternating-offer bargaining with two-sided incomplete information. Review of Economic Studies 65, 573-594.

Weinberger, C. J. (2000). Selective acceptance and inefficiency in a two-issue complete information bargaining game. Games and Economic Behavior 31, 262-293.

Wolinsky, A. (1990). Information revelation in a market with pairwise meetings. Econometrica 58, 1-23.

Zeuthen, F. (1930). Problems of Monopoly and Economic Warfare. London, Kegan Paul.